



Fiber direction and stacking sequence design for bicycle frame made of carbon/epoxy composite laminate

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ARTICLE INFO

Article history:

Received 4 August 2009

Accepted 20 October 2009

Available online 23 October 2009

Keywords:

Bicycle frame

Composite laminates

Stacking sequences

Finite element

ABSTRACT

According to the maximum stress theory and the results of strength-to-stress ratios, the fiber direction and stacking sequence design for the bicycle frame made of the carbon/epoxy composite laminates have been discussed in this paper. Three testing methods for the bicycle frame, i.e. torsional, frontal, and vertical loadings, are adopted in the analysis. From the finite element results, the stacking sequences $[0/90/90/0]_s$ and $[0/90/45/-45]_s$ are the good designs for the composite bicycle frames. On the contrary, the uni-directional laminates, i.e. $[0/0/0/0]_s$, $[90/90/90/90]_s$, $[45/45/45/45]_s$, and $[-45/-45/-45/-45]_s$, are the bad designs. In addition, weak regions of failure occur at the fillets and connections of the frame, i.e. the stress concentration regions. All weak points occur at the inner or outer layer of the laminated composite tube. The 0° -ply and 90° -ply located on the inner and outer layer of the tube can effectively resist the higher stress at its location.

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1. Introduction

Composite materials which are composed of reinforced fibers and plastics matrix have high strength-to-weight and stiffness-to-weight ratios. They have unique advantages over monolithic materials, such as high strength, high stiffness, long fatigue life, low density, corrosion resistance, wear resistance, and environmental stability [1]. Due to above characteristics, the laminated fiber-reinforced composite materials such as carbon/epoxy or glass/polyester composites are widely applied in aircraft, aerospace, military, automotive, marine, and sports structures [1,2].

The bicycles are popular sports equipments or traffic tools. The frame of the bicycle is the main structure to support the external loads. Traditional materials of the bicycle frame are the steel or aluminum alloy. For the purpose of reducing weight, the carbon/epoxy composite materials are now widely used to make the bicycle frames. An example of the carbon/epoxy bicycle frame [1] only weights 1.36 kg, which is much less than the 5 kg weight of the corresponding steel frame.

In the design process of the bicycle, the structural analysis of the frame or other parts is a very important stage. With the aid of theoretical or numerical calculations, the strength and stiffness of the bicycle structures can be predicted and modified to the

optimal design before the manufacture of the prototype and commercial products. The finite element method is one of the numerical calculations applied in various physical problems. It usually plays a major role to calculate the stress and deformation of the structures.

In 1986, the finite element method was applied in the design of the steel and aluminum bicycle frames [3]. The Euler beam elements (or frame elements) were adopted in the simplified model of the whole bicycle frame. The deflection, von Mises stress and strain energy of the frame under various loading conditions were obtained. The design strength, riding performance and weight reduction of the bicycle have been considered and discussed [3].

The finite element method was also adopted to analyze the structural behaviors of the composite bicycle frames [4,5]. The shell elements were used to model the composite bicycle frame [4]. In that study, two types of shapes of the graphite/epoxy composite frame were analyzed under three loading conditions. The 0° fiber direction corresponds roughly to a line which follows the shape of the bicycle from the front tube to the rear dropouts. The stacking sequences $[0_2/90]_s$ and $[0_2/90_2/0]_s$ were used, respectively, in the low and high loaded regions of the frame [4].

The single-layer equivalent model was adopted to simulate the multi-ply composite laminate of the bicycle frame [5]. The effective material constants of the 8-ply carbon/epoxy laminate were obtained by the mathematical transformation. Under the torsional loading, the results showed that the stacking $[0/+45/-45/0]_s$ can cause the highest stiffness [5]. In addition, higher stresses happened on the connected regions and fillets of the frame tubes.

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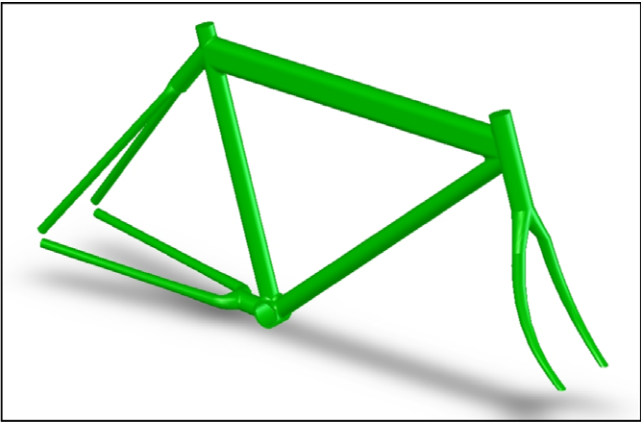


Fig. 1. CAD model of bicycle frame.

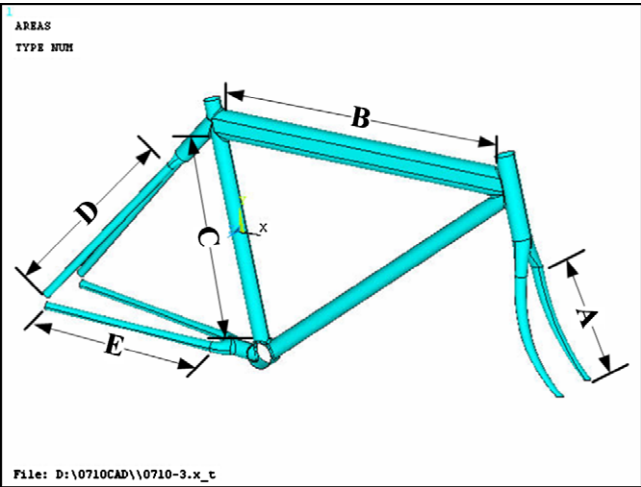


Fig. 2. Main dimensions of bicycle frame.

It is important to find the better fiber direction and stacking sequence of the composite laminate of the bicycle frame. A US Patent [6] suggested an example of $[0/\pm 45/90]_2$ for the stacking sequence.

Table 1
Main dimensions of bicycle frame.

Part	Length (mm)	Diameter (mm)
A	300	12–18
B	483	45
C	420	25
D	300	13
E	320	13–14

On the other hand, for the bonded repair, the ply orientations 0° , $+45^\circ$, -45° , and 90° of the laminated composite patch were considered to repair the crack in the aircraft component [7]. When the cracked component is subjected to variable flight loadings, the patch's fiber directions 90° and $\pm 45^\circ$ related to the crack direction are the optimal design for the bonded repair [7,8]. Similar to the crack repair, the optimal ply design of the composite bicycle frame needs to be obtained to support various loading conditions.

In this paper, the fiber direction and stacking sequence design for the bicycle frame made of the carbon/epoxy composite laminates will be discussed. Under torsional, frontal, and vertical loadings, the normal and shear stresses with respect to the principal material coordinate system of each ply will be obtained from the finite element analyses. The maximum stress theory [1,9] is used to be the failure criterion. The strength-to-stress ratio R is defined as the design parameter for the optimal selection from 33 stacking sequences of laminates. The larger value of R implies the higher safety factor of the frame structure. Fig. 1 shows the SolidWorks [10] CAD model of the bicycle frame in this study. The finite element software ANSYS [11] will be used to analyze the stress field and structural behaviors.

2. Problem definitions

2.1. Bicycle frame and composite laminates

According to the CNS standard [12], main dimensions of the bicycle frame are shown in Fig. 2 and Table 1. The bicycle frame consists of many tubes made of carbon-fiber reinforced (carbon/epoxy) composite laminates. The composite structures of the tubes, fillets, and connection regions are all the 8-ply laminate. The thickness of each ply is 0.3 mm.

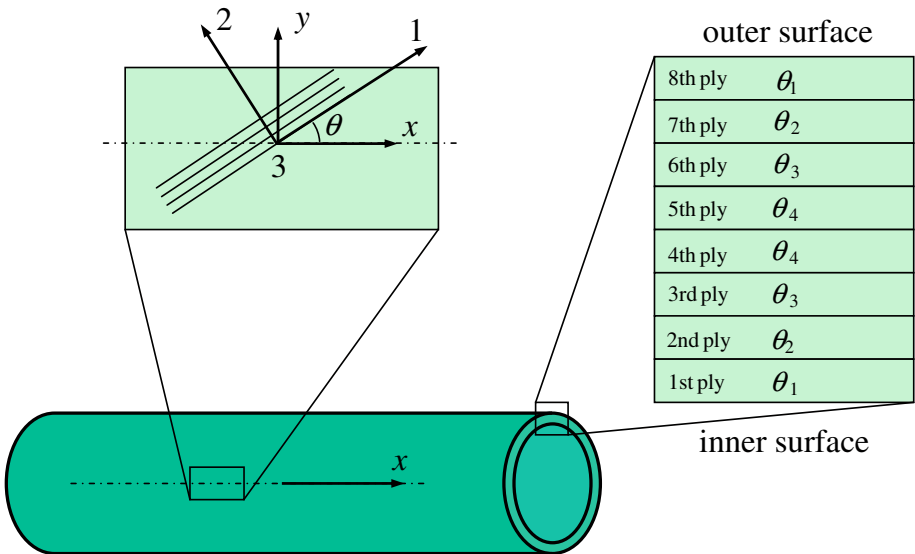


Fig. 3. 8-ply laminated tube.

Table 2

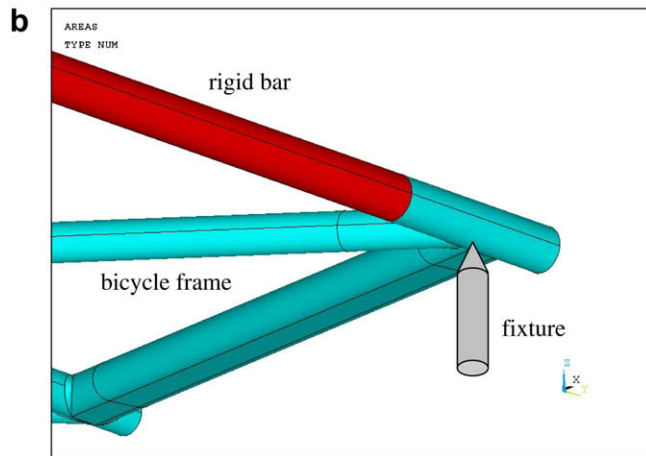
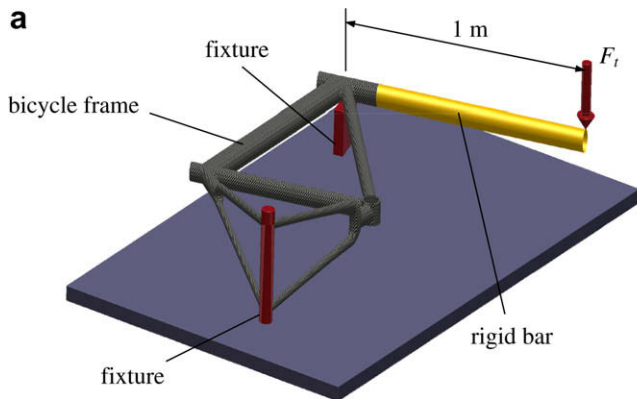
Stacking sequences of laminates in this study.

Case	Stacking	Case	Stacking
1	$[0/90/45/-45]_s$	13	$[-45/0/90/45]_s$
2	$[0/45/90/-45]_s$	14	$[-45/90/0/45]_s$
3	$[0/-45/90/45]_s$	15	$[-45/45/0/90]_s$
4	$[0/90/-45/45]_s$	16	$[-45/0/45/90]_s$
5	$[0/45/-45/90]_s$	17	$[-45/90/45/0]_s$
6	$[0/-45/45/90]_s$	18	$[-45/45/90/0]_s$
7	$[90/45/-45/0]_s$	19	$[45/0/90/-45]_s$
8	$[90/-45/45/0]_s$	20	$[45/90/0/-45]_s$
9	$[90/0/45/-45]_s$	21	$[45/-45/0/90]_s$
10	$[90/45/0/-45]_s$	22	$[45/0/-45/90]_s$
11	$[90/-45/0/45]_s$	23	$[45/90/-45/0]_s$
12	$[90/0/-45/45]_s$	24	$[45/-45/90/0]_s$

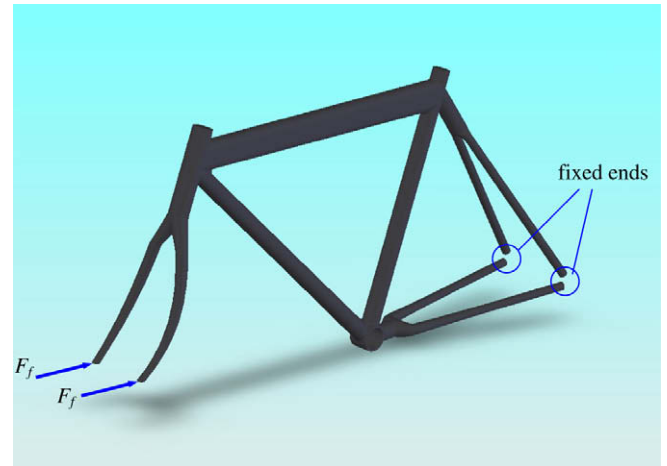
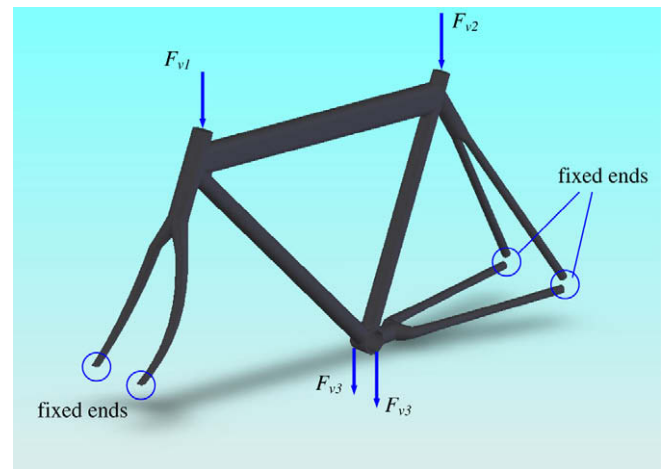
Table 3

Additional stacking sequences of laminates in this study.

Case	Stacking	Case	Stacking
A	$[0/90/0/0]_s$	F	$[0/0/0/0]_s$
B	$[0/0/90/0]_s$	G	$[90/90/90/90]_s$
C	$[0/90/90/0]_s$	H	$[45/45/45/45]_s$
D	$[90/0/0/90]_s$	I	$[-45/-45/-45/-45]_s$
E	$[0/45/-45/0]_s$		

**Fig. 4.** Torsional loading test [5].

In Fig. 3, the coordinate 1–2–3 is the principal material coordinate system (PMCS) of each ply. The 1-axis is along the fiber direction. The other coordinate x – y – z is the reference coordinate system (RCS) used to define the fiber angle. The fiber angle θ is defined as

**Fig. 5.** Frontal loading test.**Fig. 6.** Vertical loading test.**Table 4**

Material constants of carbon/epoxy composite [5] (the subscript 1 is the fiber axis).

E_1	E_2	E_3
Young's modulus (GPa)		
162	14.9	14.9
ν_{12}	ν_{13}	ν_{23}
Poisson's ratio		
0.283	0.283	0.386
G_{12}	G_{13}	G_{23}
5.7	5.7	5.4
Shear modulus (GPa)		

the angle between the 1-axis and x -axis. In this study, all stacking sequences denoted as $[\theta_1/\theta_2/\theta_3/\theta_4]_s$ are symmetrical as shown in Fig. 3.

In Tables 2 and 3, there are totally 33 stacking sequences of laminates considered in this study. The stacking sequences in Table 2 are selected from four main fiber directions, i.e. $\theta = 0^\circ$, $+45^\circ$, -45° , and 90° which have been adopted in the past references [6,7]. In Table 3, additional stacking sequences are also considered. The stacks of Cases A, B, C, and D are similar to those used by Lessard et al. [4]. Case E, i.e. $[0/45/-45/0]_s$ is the optimal design suggested

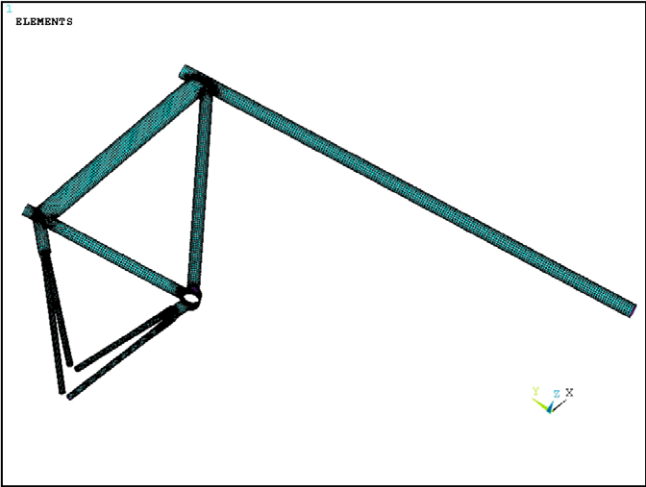


Fig. 7. Finite element model for torsional loading test.

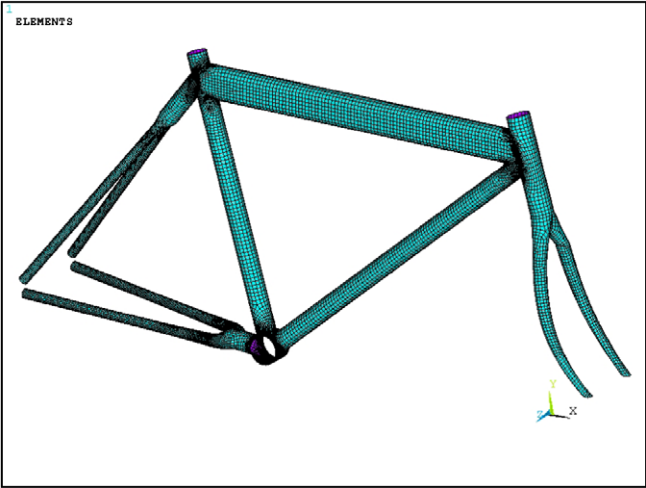


Fig. 8. Finite element model for frontal or vertical loading test.

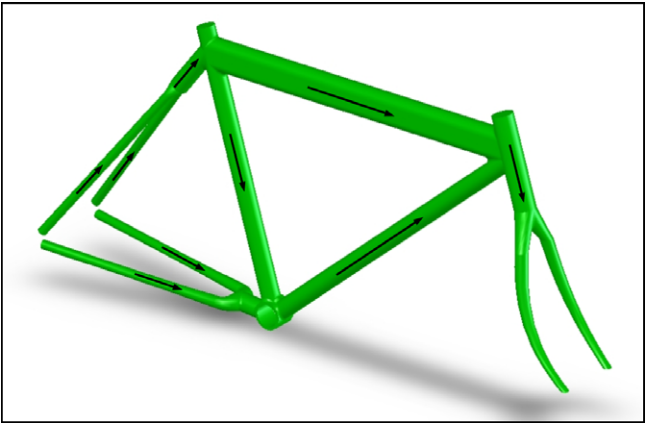


Fig. 9. Directions of x-axes of RCS for each tube.

by Liao [5] for the higher stiffness under the torsional loading. The uni-directional laminates, i.e. Case F, G, H, and I, are also considered in the analysis.

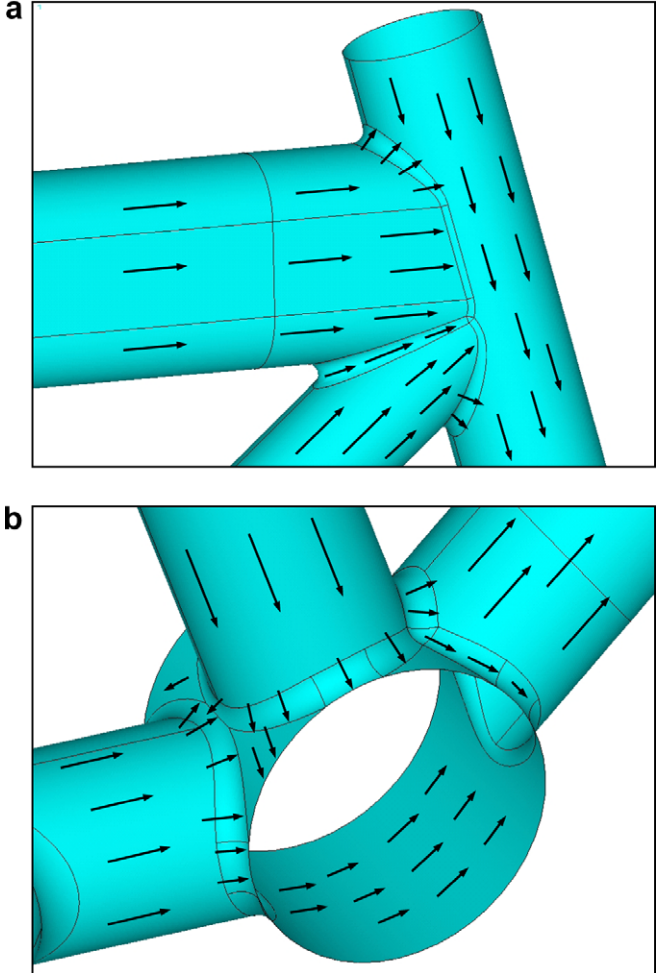


Fig. 10. Directions of x-axes of RCS on local fillets.

Table 5
Strength values of carbon/epoxy composite [14].

Strength	Value (MPa)
S_{1t}	1760
S_{1c}	1570
S_{2t}	80
S_{2c}	80
S_{3t}	80
S_{3c}	80
S_{23}	98
S_{31}	98
S_{12}	98

2.2. Three testing methods

The boundary and loading conditions for the finite element analyses are based on the testing methods of the bicycle frame. According to past Refs. [5,13], three testing methods, i.e. torsional, frontal and vertical loadings, are considered in this paper. To find the better stacking design of each test, these three tests are analyzed and discussed separately.

The torsional loading test was proposed by Liao [5]. As shown in Fig. 4, the front fork of the bicycle is replaced by a rigid bar for the test. The bicycle frame is supported by two fixtures. The fixture near the rigid bar is a single-point support. The static side force $F_t = 10 \text{ kgf}$ (98 N) is applied on the end of the rigid bar.

Table 6

Stresses of Case 1 with [0/90/45/−45]_s under torsional loading.

		σ_1 (Pa)		σ_2 (Pa)		σ_3 (Pa)		τ_{12} (Pa)		τ_{23} (Pa)		τ_{31} (Pa)	
		Min	Max	Min	Max	Min	Max	Min	Max	Min	Max	Min	Max
8th-ply	TOP	−0.250E9	0.197E9	−0.222E8	0.239E8	0	0	−0.920E7	0.121E8	−0.238E−7	0.133E−7	−0.219E−7	0.212E−7
	MID	−0.243E9	0.193E9	−0.196E8	0.209E8	0	0	−0.862E7	0.107E8	−0.236E7	0.239E7	−0.288E7	0.305E7
	BOT	−0.237E9	0.189E9	−0.177E8	0.180E8	0	0	−0.857E7	0.968E7	−0.472E7	0.478E7	−0.577E7	0.610E7
7th-ply	TOP	−0.205E9	0.205E9	−0.201E8	0.161E8	0	0	−0.968E7	0.857E7	−0.578E7	0.547E7	−0.499E7	0.504E7
	MID	−0.201E9	0.199E9	−0.196E8	0.157E8	0	0	−0.945E7	0.853E7	−0.727E7	0.737E7	−0.680E7	0.686E7
	BOT	−0.197E9	0.192E9	−0.194E8	0.153E8	0	0	−0.921E7	0.849E7	−0.876E7	0.932E7	−0.862E7	0.867E7
6th-ply	TOP	−0.133E9	0.182E9	−0.178E8	0.148E8	0	0	−0.872E7	0.106E8	−0.864E7	0.916E7	−0.979E7	0.869E7
	MID	−0.127E9	0.163E9	−0.172E8	0.128E8	0	0	−0.856E7	0.105E8	−0.974E7	0.103E8	−0.109E8	0.976E7
	BOT	−0.126E9	0.161E9	−0.170E8	0.109E8	0	0	−0.840E7	0.104E8	−0.108E8	0.114E8	−0.121E8	0.108E8
5th-ply	TOP	−0.198E9	0.125E9	−0.911E7	0.141E8	0	0	−0.104E8	0.840E7	−0.115E8	0.103E8	−0.120E8	0.114E8
	MID	−0.194E9	0.123E9	−0.895E7	0.140E8	0	0	−0.103E8	0.824E7	−0.118E8	0.106E8	−0.124E8	0.118E8
	BOT	−0.190E9	0.120E9	−0.908E7	0.139E8	0	0	−0.103E8	0.818E7	−0.122E8	0.109E8	−0.127E8	0.122E8
4th-ply	TOP	−0.190E9	0.120E9	−0.908E7	0.139E8	0	0	−0.103E8	0.818E7	−0.122E8	0.109E8	−0.127E8	0.122E8
	MID	−0.187E9	0.118E9	−0.921E7	0.139E8	0	0	−0.102E8	0.821E7	−0.118E8	0.106E8	−0.123E8	0.119E8
	BOT	−0.192E9	0.118E9	−0.934E7	0.147E8	0	0	−0.101E8	0.825E7	−0.114E8	0.102E8	−0.119E8	0.115E8
3rd-ply	TOP	−0.120E9	0.167E9	−0.167E8	0.943E7	0	0	−0.825E7	0.101E8	−0.109E8	0.113E8	−0.121E8	0.108E8
	MID	−0.118E9	0.177E9	−0.179E8	0.112E8	0	0	−0.838E7	0.999E7	−0.978E7	0.101E8	−0.109E8	0.970E7
	BOT	−0.117E9	0.188E9	−0.191E8	0.137E8	0	0	−0.858E7	0.990E7	−0.881E7	0.897E7	−0.970E7	0.861E7
2nd-ply	TOP	−0.190E9	0.211E9	−0.179E8	0.121E8	0	0	−0.964E7	0.876E7	−0.803E7	0.947E7	−0.892E7	0.875E7
	MID	−0.194E9	0.217E9	−0.177E8	0.125E8	0	0	−0.104E8	0.891E7	−0.636E7	0.751E7	−0.709E7	0.693E7
	BOT	−0.201E9	0.223E9	−0.175E8	0.144E8	0	0	−0.111E8	0.905E7	−0.469E7	0.556E7	−0.525E7	0.512E7
1st-ply	TOP	−0.208E9	0.136E9	−0.181E8	0.191E8	0	0	−0.905E7	0.111E8	−0.498E7	0.485E7	−0.586E7	0.495E7
	MID	−0.206E9	0.146E9	−0.252E8	0.321E8	0	0	−0.110E8	0.141E8	−0.249E7	0.242E7	−0.293E7	0.248E7
	BOT	−0.204E9	0.167E9	−0.328E8	0.499E8	0	0	−0.255E8	0.294E8	0	0	0	0
Min. value of R		R_{1cm}	R_{1tm}	R_{2cm}	R_{2tm}	–	–	R_{12m}		R_{23m}		R_{31m}	
		6.28	7.89	2.44	1.60	–	–	3.33		8.03		7.72	

The frontal loading test as shown in Fig. 5 is similar to the impact test method in [13]. In this study, the static frontal load $F_f = 490$ N is applied on each side of the front tube. The frame is fixed at both rear dropouts.

The vertical loading test as shown in Fig. 6 is similar to the loads of the vibration proof in the JIS [13]. As shown in Fig. 6, the loads are applied on the head part, seat part, and bottom bracket part. In this study, the static vertical loads are $F_{v1} = 6$ kgf (58.8 N), $F_{v2} = 67$ kgf (656.6 N) and $F_{v3} = 13.5$ kgf (132.3 N). The total vertical load is 100 kgf. The frame is fixed at rear dropouts and front tube ends.

Under torsional, frontal, and vertical loadings, the normal and shear stresses with respect to the PMCS of each ply will be obtained from the finite element analyses.

3. Methods of analyses

3.1. Orthotropic material property

Under the Cartesian coordinate 1–2–3, the constitutive equation of the orthotropic material such as the carbon/epoxy composite is [1]:

$$\begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \tau_{23} \\ \tau_{31} \\ \tau_{12} \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ C_{21} & C_{22} & C_{23} & 0 & 0 & 0 \\ C_{31} & C_{32} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ \gamma_{23} \\ \gamma_{31} \\ \gamma_{12} \end{bmatrix} \quad (1)$$

Above equation can be written as a simple form:

$$\{\sigma\} = [C]\{\varepsilon\} \quad (2)$$

where $\{\sigma\}$, $\{\varepsilon\}$, and $[C]$ are the stress, strain, and stiffness matrix, respectively. The compliance matrix $[S]$ is the inverse of $[C]$ as follows:

$$[S] = [C]^{-1} = \begin{bmatrix} \frac{1}{E_1} & -\frac{\nu_{21}}{E_2} & -\frac{\nu_{31}}{E_3} & 0 & 0 & 0 \\ -\frac{\nu_{12}}{E_1} & \frac{1}{E_2} & -\frac{\nu_{32}}{E_3} & 0 & 0 & 0 \\ -\frac{\nu_{13}}{E_1} & -\frac{\nu_{23}}{E_2} & \frac{1}{E_3} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{G_{23}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{G_{31}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{G_{12}} \end{bmatrix} \quad (3)$$

where E_i , G_{ij} and ν_{ij} are the Young's modulus, shear modulus and Poisson's ratio, respectively. The symmetrical matrix $[S]$ in Eq. (3) has nine independent material constants for the orthotropic material.

In the carbon/epoxy composite laminate, each ply has the orthotropic material property. The fiber directions of each ply can be different for practical applications. Table 4 lists the material constants of the carbon/epoxy composite [5]. In this table, the subscript 1 denotes the fiber axis. In Fig. 3, the Cartesian coordinate 1–2–3 is considered as the PMCS of each ply. Nine material constants in Table 4 will be used in the finite element analyses.

3.2. Finite element models

The finite element software ANSYS [11] is adopted in this study. Static structural analysis and linear elastic property are considered in this study. The SHELL91 elements are used to simulate the carbon/epoxy composite laminate of the bicycle frame. SHELL91 is the 8-noded high-order shell element based on the thick shell theory. In addition, the perfect bonding between plies is assumed in this analysis.

In Fig. 7, it shows the finite element model for the torsional loading test. It contains 31377 elements and 94548 nodes. Except for the rigid bar, the composite bicycle frame is modelled by the SHELL91 elements with 8-ply settings. The fiber directions of all plies can be changed to discuss its effects on the stress magnitude.

Table 7
Values of R_{itm} , R_{icm} , R_{ijm} , and R_{min} for different cases under torsional loading.

	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6	Case 7	Case 8	Case 9	Case 10	Case 11	Case 12	Case 13	Case 14	Case 15	Case 16	Case 17	Case 18	Case 19	Case 20	Case 21	Case 22	Case 23	Case 24
R_{icm}	6.28	6.33	5.92	6.01	6.21	6.08	6.62	6.85	6.59	6.68	6.79	6.31	5.75	5.06	6.38	6.01	5.11	6.13	6.56	7.13	6.68	6.38	6.94	6.79
R_{itm}	7.89	7.75	6.17	7.45	7.27	6.61	6.26	7.75	7.85	5.92	8.30	8.22	7.39	7.27	7.48	8.11	6.42	7.68	7.09	7.48	6.26	6.56	7.42	7.27
R_{2cm}	2.44	0.93	1.95	2.90	0.91	1.99	0.48	0.62	0.72	0.50	0.78	0.88	0.74	0.72	1.18	0.80	0.88	1.32	0.50	0.49	0.55	0.49	0.49	0.64
R_{2im}	1.60	1.18	1.03	1.60	1.06	0.92	0.77	0.68	1.03	0.89	0.69	0.91	0.68	0.89	1.03	0.68	1.08	1.22	0.79	0.71	0.85	0.81	0.65	0.75
R_{12m}	3.33	1.60	3.01	4.87	1.55	2.55	2.07	2.56	2.98	2.19	2.74	3.48	1.78	1.15	2.91	3.52	1.47	2.16	5.79	2.67	2.68	4.97	2.37	2.68
R_{23m}	8.03	7.42	7.42	7.96	7.84	7.77	7.96	7.77	7.77	7.71	7.25	7.77	7.10	7.05	7.59	7.59	7.71	7.84	7.48	7.59	7.77	7.90	8.09	7.96
R_{31m}	7.72	7.48	7.77	7.77	7.77	7.84	6.75	6.66	7.71	7.36	7.25	7.42	7.71	7.25	7.59	7.42	6.75	7.00	7.42	7.42	7.84	7.84	6.95	7.15
R_{min}	1.60	0.93	1.03	1.60	0.91	0.92	0.48	0.62	0.72	0.50	0.69	0.88	0.68	0.72	1.03	0.68	0.88	1.22	0.50	0.49	0.69	0.55	0.49	0.64

The rigid bar used for the side load is modelled by SHELL93 elements with isotropic material property. The Young's modulus of the rigid bar is 100 times as large as steel so as to simulate the rigid property.

Fig. 8 shows the finite element model for the frontal or vertical loading test. It contains 29177 elements and 87943 nodes. The rigid bar is replaced by the rigid frontal fork. The element type and Young's modulus of the rigid fork are the same as the rigid bar.

In Fig. 3, the PMCS, RCS, and fiber direction are defined. Fig. 9 shows the directions of x -axes of RCS for each tube. The x -axis is along the axial direction of the tube. Then the fiber angle of each ply can be defined with respect to the x -axis. In local fillets or tube connections, the x -axes are assigned such like streamlines as shown in Fig. 10. In ANSYS, the direction of the material property is assigned by the element coordinate system (ECS) of each element. The ECS and RCS must be the same in this study so that correct fiber directions can be defined by PMCS and RCS.

The boundary conditions are prescribed on the specified nodes of the finite element model. For the shell elements, the fixed constraints on the node are zero displacements and zero rotations. The external load on the tube end are prescribed distributively on the nodes.

3.3. Failure criterion

The maximum stress theory [1,9] is used to be the failure criterion in this study. The failure occurs when at least one stress component along one axis of the PMCS exceeds the corresponding strength in that direction [1]. Considering the stresses $\sigma_1, \sigma_2, \sigma_3, \tau_{23}, \tau_{31}$ and τ_{12} based on the PMCS in Fig. 3, the maximum stress theory is expressed as follows [1]:

$$\sigma_1 = \begin{cases} S_{1t} & \text{when } \sigma_1 > 0 \\ -S_{1c} & \text{when } \sigma_1 < 0 \end{cases} \quad (4)$$

$$\sigma_2 = \begin{cases} S_{2t} & \text{when } \sigma_2 > 0 \\ -S_{2c} & \text{when } \sigma_2 < 0 \end{cases} \quad (5)$$

$$\sigma_3 = \begin{cases} S_{3t} & \text{when } \sigma_3 > 0 \\ -S_{3c} & \text{when } \sigma_3 < 0 \end{cases} \quad (6)$$

$$|\tau_{23}| = S_{23} \quad (7)$$

$$|\tau_{31}| = S_{31} \quad (8)$$

$$|\tau_{12}| = S_{12} \quad (9)$$

where S_{it} , S_{ic} , and S_{ij} ($i, j = 1, 2, 3$) are the tensile, compressive, and shear strength values, respectively. The strength values of T300 carbon/epoxy composites in Table 5 [14] are used in this study.

3.4. Design parameter

In this paper, the strength-to-stress ratio R is defined as follows

$$R_{it} = |S_{it}|/|\sigma_i| \quad \text{when } \sigma_i > 0, \quad i = 1, 2, 3 \quad (10)$$

$$R_{ic} = |S_{ic}|/|\sigma_i| \quad \text{when } \sigma_i < 0, \quad i = 1, 2, 3 \quad (11)$$

$$R_{ij} = |S_{ij}|/|\tau_{ij}|, \quad i, j = 1, 2, 3 (i \neq j) \quad (12)$$

From the finite element results of each case, all stress components $\sigma_1, \sigma_2, \sigma_3, \tau_{23}, \tau_{31}$ and τ_{12} in each ply of the carbon/epoxy laminate can be obtained. Then the values of R for all stresses in the bicycle frame can be calculated. Also, the minimum values of R_{it} , R_{ic} , and R_{ij} of each stress component, i.e. R_{itm} , R_{icm} and R_{ijm} , can be obtained, respectively.

For a bicycle frame using specified carbon/epoxy laminate $[\theta_1/\theta_2/\theta_3/\theta_4]_s$, the global minimum value of R can be found as

$$R_{min} = \min[R_{itm}, R_{icm}, R_{ijm}] \quad (13)$$

Table 8Values of R_{ltm} , R_{lcm} , R_{ljm} , and R_{min} for different cases under torsional loading.

	Case 1	Case A	Case B	Case C	Case D	Case E	Case F	Case G	Case H	Case I
R_{lcm}	6.28	4.28	3.96	5.99	6.23	5.71	5.26	6.31	6.54	5.64
R_{ltm}	7.89	4.75	4.29	7.12	7.21	7.33	6.76	5.11	6.01	7.09
R_{2cm}	2.44	1.95	1.36	2.98	1.06	0.93	0.66	0.97	0.42	0.63
R_{2tm}	1.60	1.92	1.28	2.96	1.33	1.18	0.58	0.95	0.72	0.48
R_{12m}	3.33	1.38	1.27	1.78	1.81	1.52	0.91	1.55	1.83	1.21
R_{23m}	8.03	2.31	2.31	3.03	3.20	7.10	1.89	8.44	6.28	6.32
R_{31m}	7.72	2.55	2.33	3.06	2.85	4.94	2.03	4.08	4.18	4.43
R_{min}	1.60	1.38	1.27	1.78	1.06	0.93	0.58	0.95	0.42	0.48

Table 9Values of R_{ltm} , R_{lcm} , R_{ljm} , and R_{min} for different cases under frontal loading.

	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6	Case 7	Case 8	Case 9	Case 10	Case 11	Case 12	Case 13	Case 14	Case 15	Case 16	Case 17	Case 18	Case 19	Case 20	Case 21	Case 22	Case 23	Case 24
R_{lcm}	1.91	1.64	1.61	1.80	1.57	1.55	1.73	1.77	1.76	1.73	1.81	1.91	1.64	1.74	1.64	1.61	1.76	1.67	1.59	1.65	1.60	1.55	1.64	1.63
R_{ltm}	2.79	2.46	3.08	2.83	2.34	2.69	3.51	3.62	2.97	3.45	3.13	3.09	2.82	2.96	2.82	2.73	3.32	3.16	2.20	3.23	2.69	2.03	3.14	3.13
R_{2cm}	0.76	0.55	0.45	0.68	0.51	0.44	0.54	0.61	0.77	0.58	0.68	0.86	0.46	0.52	0.75	0.51	0.64	0.83	0.52	0.54	0.84	0.63	0.60	0.85
R_{2tm}	0.88	0.64	0.85	1.01	0.58	0.76	0.84	0.85	1.04	0.90	0.82	1.09	0.65	0.72	1.14	0.77	0.88	1.22	0.60	0.67	1.12	0.71	0.78	1.13
R_{12m}	1.94	1.60	1.70	1.95	1.81	1.78	2.14	2.11	2.08	1.92	1.87	2.05	2.04	2.07	1.72	2.14	2.00	2.33	2.02	2.12	2.02	2.00	2.02	2.64
R_{23m}	1.76	1.69	1.74	1.87	1.77	1.85	1.57	1.51	1.74	1.63	1.63	1.88	1.68	1.61	1.71	1.81	1.52	1.57	1.63	1.59	1.81	1.76	1.62	1.64
R_{31m}	1.78	1.79	1.66	1.75	1.72	1.63	1.94	1.72	1.75	1.89	1.71	1.71	1.57	1.79	1.53	1.53	1.79	1.57	1.77	1.94	1.61	1.72	1.96	1.64
R_{min}	0.76	0.55	0.45	0.68	0.51	0.44	0.54	0.61	0.77	0.58	0.68	0.86	0.46	0.52	0.75	0.51	0.64	0.83	0.52	0.54	0.84	0.63	0.60	0.85

Table 10Values of R_{ltm} , R_{lcm} , R_{ljm} , and R_{min} for different cases under frontal loading.

	Case 12	Case A	Case B	Case C	Case D	Case E	Case F	Case G	Case H	Case I
R_{lcm}	1.91	1.71	1.62	2.13	2.13	1.46	2.69	2.45	0.57	1.93
R_{ltm}	3.09	2.75	2.84	2.64	3.07	2.31	2.45	4.07	1.67	3.37
R_{2cm}	0.86	0.66	0.42	0.81	0.81	0.42	0.25	0.18	0.17	0.16
R_{2tm}	1.09	0.74	0.47	0.81	0.99	0.61	0.36	0.41	0.27	0.32
R_{12m}	2.05	1.39	1.12	1.28	1.28	1.84	0.93	0.74	0.83	0.53
R_{23m}	1.88	1.81	1.85	1.68	1.71	1.99	2.15	2.82	1.70	1.78
R_{31m}	1.71	2.24	1.85	1.68	1.54	2.22	2.44	1.08	1.52	1.57
R_{min}	0.86	0.66	0.42	0.81	0.81	0.42	0.25	0.18	0.17	0.16

Table 11Values of R_{ltm} , R_{lcm} , R_{ljm} , and R_{min} for different cases under vertical loading.

	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6	Case 7	Case 8	Case 9	Case 10	Case 11	Case 12	Case 13	Case 14	Case 15	Case 16	Case 17	Case 18	Case 19	Case 20	Case 21	Case 22	Case 23	Case 24
R_{lcm}	39.8	37.7	34.6	38.4	37.7	36.0	48.6	49.2	41.0	41.9	44.6	41.3	37.6	44.4	42.8	40.2	44.2	42.5	39.1	41.9	44.3	41.5	46.7	44.1
R_{ltm}	60.9	64.7	64.7	60.6	70.4	68.4	56.7	56.5	58.8	56.9	56.7	58.6	62.4	56.7	61.9	65.1	57.5	60.4	62.4	57.1	64.4	67.9	57.8	64.4
R_{2cm}	20.1	11.9	12.1	19.1	10.5	10.0	12.3	11.9	16.5	13.1	12.9	16.4	10.0	10.9	16.8	11.4	12.3	16.2	11.6	11.7	16.8	12.6	12.8	16.4
R_{2tm}	21.8	15.2	16.6	21.5	15.2	14.1	13.3	12.2	24.5	18.6	13.4	18.0	12.6	13.0	20.0	14.7	14.4	19.5	12.9	13.9	20.2	14.8	15.0	18.8
R_{12m}	32.6	40.0	32.0	33.9	44.7	36.9	48.0	52.6	36.0	42.4	46.0	36.8	26.5	16.1	41.1	43.1	17.5	38.5	45.5	46.4	41.3	43.1	42.7	40.6
R_{23m}	33.1	33.9	32.5	31.9	36.1	33.5	24.8	23.3	33.2	27.3	24.7	32.1	32.5	24.5	25.9	33.5	23.0	23.6	34.1	28.1	26.7	35.5	25.3	24.5
R_{31m}	29.6	27.3	25.9	31.5	25.9	25.3	34.2	33.6	30.2	30.8	31.7	32.1	24.8	28.9	23.1	24.0	28.8	23.7	26.6	30.0	23.8	25.5	31.7	24.6
R_{min}	20.1	11.9	12.1	19.1	10.5	10.0	12.3	11.9	16.5	13.1	12.9	16.4	10.0	10.9	16.8	11.4	12.3	16.2	11.6	11.7	16.8	12.6	12.8	16.4

Table 12Values of R_{ltm} , R_{lcm} , R_{ljm} , and R_{min} for different cases under vertical loading.

	Case 1	Case A	Case B	Case C	Case D	Case E	Case F	Case G	Case H	Case I
R_{lcm}	39.8	37.8	30.4	37.1	41.6	32.6	33.1	33.7	47.1	32.2
R_{ltm}	60.9	57.7	53.5	64.5	66.9	67.4	54.3	55.2	67.7	64.9
R_{2cm}	20.1	16.6	10.1	20.1	16.1	8.74	3.86	3.84	4.44	3.33
R_{2tm}	21.8	21.3	13.9	23.8	19.9	15.7	7.01	4.14	5.71	8.03
R_{12m}	32.6	27.1	17.4	28.9	30.3	41.7	15.3	13.8	13.6	15.7
R_{23m}	33.1	20.7	21.8	20.8	22.1	28.1	20.3	41.0	24.1	21.6
R_{31m}	29.6	26.0	22.0	20.9	19.5	31.1	31.5	17.0	28.5	22.2
R_{min}	20.1	16.6	10.1	20.1	16.1	8.74	3.86	3.84	4.44	3.33

For the design of the composite bicycle frame, R_{min} is the design parameter for the optimal selection from 33 stacking sequences

(in Tables 2 and 3) of laminates. The larger value of R_{min} implies the higher safety factor of the frame structure.

Table 13
Better and bad designs from 33 cases.

Test method	Better designs			Bad designs		
	Case	$R_{\min}$	Stacking	Case	$R_{\min}$	Stacking
Torsional loading test	C	1.78	[0/90/90/0] _s	H	0.42	[45/45/45/45] _s
	1	1.60	[0/90/45/−45] _s	I	0.48	[−45/−45/−45/−45] _s
	4	1.60	[0/90/−45/45] _s	7	0.48	[90/45/−45/0] _s
				20	0.49	[45/90/0/−45] _s
				23	0.49	[45/90/−45/0] _s
				10	0.50	[90/45/0/−45] _s
Frontal loading test				19	0.50	[45/0/90/−45] _s
	12	0.86	[90/0/−45/45] _s	I	0.16	[−45/−45/−45/−45] _s
	24	0.85	[45/−45/90/0] _s	H	0.17	[45/45/45/45] _s
	21	0.84	[45/−45/0/90] _s	G	0.18	[90/90/90/90] _s
	18	0.83	[−45/45/90/0] _s	F	0.25	[0/0/0/0] _s
	C	0.81	[0/90/90/0] _s			
	D	0.81	[90/0/0/90] _s			
	9	0.77	[90/0/45/−45] _s			
	1	0.76	[0/90/45/−45] _s			
	15	0.75	[−45/45/0/90] _s			
Vertical loading test	C	20.1	[0/90/90/0] _s	I	3.33	[−45/−45/−45/−45] _s
	1	20.1	[0/90/45/−45] _s	G	3.84	[90/90/90/90] _s
	4	19.1	[0/90/−45/45] _s	F	3.86	[0/0/0/0] _s
				H	4.44	[45/45/45/45] _s

Table 14
Locations of weak regions and layers.

Test method	Better designs					Bad designs				
	Case	$R_{\min}$	Weak region	Weak layer	Failure stress	Case	$R_{\min}$	Weak region	Weak layer	Failure stress
Torsional loading test	C	1.78	a	1st	τ_{12}	H	0.42	b	1st	σ_2
	1	1.60	b	1st	σ_2	I	0.48	b	1st	σ_2
	4	1.60	b	1st	σ_2	7	0.48	b	1st	σ_2
						20	0.49	b	1st	σ_2
						23	0.49	b	1st	σ_2
						10	0.50	b	1st	σ_2
Frontal loading test						19	0.50	b	1st	σ_2
	12	0.86	a	8th	σ_2	I	0.16	a	8th	σ_2
	24	0.85	a	8th	σ_2	H	0.17	a	8th	σ_2
	21	0.84	a	1st	σ_2	G	0.18	a	8th	σ_2
	18	0.83	a	1st	σ_2	F	0.25	a	8th	σ_2
	C	0.81	a	8th	σ_2					
	D	0.81	a	8th	σ_2					
	9	0.77	a	8th	σ_2					
	1	0.76	a	8th	σ_2					
	15	0.75	a	8th	σ_2					
Vertical loading test	C	20.1	b	8th	σ_2	I	3.33	b	8th	σ_2
	1	20.1	b	1st	σ_2	G	3.84	b	8th	σ_2
	4	19.1	b	8th	σ_2	F	3.86	b	1st	σ_2
						H	4.44	b	8th	σ_2

4. Results and discussion

The outputs of stress components from the finite element results are based on the PMCS (coordinate 1–2–3) of each ply. σ_1 is the normal stress along the fiber direction. σ_2 and σ_3 are the normal stresses along the directions perpendicular to the fiber. τ_{23} , τ_{31} , and τ_{12} are the shear stresses on three different planes.

4.1. Results of torsional loading test

For Case 1 stacking under the torsional loading, six stress components with respect to PMCS of each ply are listed in Table 6. The notations “min” and “max” express, respectively, the minimum and maximum values of each stress component. In addition, the notations “TOP”, “MID”, and “BOT” express respectively the top, middle, and bottom surfaces of each ply for the stress output location. In the last row of Table 6, the values of R_{itm} , R_{icm} , and R_{ijm} for

each stress component are obtained. According to Eq. (13), $R_{\min}$ of Case 1 is 1.60.

According to the stresses of all cases, Tables 7 and 8 list all values of R_{itm} , R_{icm} , R_{ijm} , and $R_{\min}$ for 33 stacking cases under the torsional loading. It is noted that $R_{\min}$ is the design parameter of each case. The larger $R_{\min}$ implies the higher safety factor or better design of the frame structure. From Tables 7 and 8, for the torsional loading test, the optimal design is Case C, which has the largest value 1.78 of $R_{\min}$. The stacking sequence of Case C is [0/90/90/0]_s.

Moreover, all values of σ_3 in Table 6 are equal to zero due to the assumption of the shell theory. According to Fig. 3, the top surface of the 8th-ply and the bottom surface of the 1st-ply are traction-free surfaces. The shear stresses τ_{23} and τ_{31} on these two surfaces must be zero. In Table 6, some numerical values of τ_{23} and τ_{31} approach zero or are equal to zero.

The stacking [0/45/−45/0]_s is the optimal design suggested by Liao [5] for the higher frame stiffness under the torional loading

test. However, in this paper, this stacking is not the optimal design concerning the better structural strength.

4.2. Results of frontal loading test

Tables 9 and 10 list all values of R_{itm} , R_{icm} , R_{ijm} , and R_{min} for 33 stacking cases under the frontal loading. The optimal design is Case 12, which has the largest value 0.86 of R_{min} . The stacking sequence of Case 12 is $[90/0/-45/45]_s$.

4.3. Results of vertical loading test

Tables 11 and 12 list all values of R_{itm} , R_{icm} , R_{ijm} , and R_{min} for 33 stacking cases under the vertical loading. The optimal design cases are Case 1 and Case C, which have the largest value 20.1 of R_{min} . The stacking sequences of Case 1 and Case C are $[0/90/45/-45]_s$ and $[0/90/90/0]_s$, respectively.

4.4. Discussions of optimal and bad designs

From above results, the optimal stacking sequence of each loading test has been obtained. Because the bicycle frame may be subjected to various loads, the common optimal stacking sequence for all loading tests must be determined.

The better and bad designs from 33 cases are shown in Table 13. According to the results of the torsional and vertical loadings, Case C is the common optimal design. For the frontal loading, R_{min} of Case C is 0.81. It is close to 0.86 of Case 12 with 5.81% difference. Obviously, Case C with $[0/90/90/0]_s$ is the final selection for the common optimal stacking sequence under three loading tests.

From Table 13, Case 1 is the common optimal design for Case 1–24 under the torsional and vertical loadings. Under the frontal loading, R_{min} of Case 1 is 0.76. The difference of R_{min} between Case 1 and 12 is 11.6%. It can be considered that, for Case 1–24, Case 1 with $[0/90/45/-45]_s$ is the common optimal stacking sequence under three loading tests.

In Case 1 and C, 0° -ply and 90° -ply locate on the outside of the frame tube. These two plies can effectively resist the higher stress at its location.

The bad designs of stacking sequences are also shown in Table 13. Case F, G, H, and I are common bad designs under the frontal and vertical loadings. However, Case H or I are two worse cases under the torsional and frontal loadings. It can be seen that the uni-directional laminates, i.e. $[0/0/0/0]_s$, $[90/90/90/90]_s$, $[45/45/45/45]_s$ and $[-45/-45/-45/-45]_s$, are the bad designs for the composite bicycle frame.

The laminate consisting of 0° , 90° , 45° , and -45° plies is used for the bicycle frame design according to the suggestions by Chue and Liu [7] and Nelson et al. [6]. Chue and Liu [7] have proved the better performance of these four fiber angles for the bonded repair on a crack. Nelson et al. [6] only proposed that these four fiber angles can be used for the bicycle frame, but there is no structural analysis in their patent document. In this paper, the stacking $[0/90/45/-45]_s$ performs good structural strength against the external loads. It matches the suggestions by Chue and Liu [7] and Nelson et al. [6]. In addition, the stacking $[0/90/90/0]_s$ also performs good structural strength and matches the concept by Lessard et al. [4].

4.5. Locations of weak regions

Table 14 lists the locations of weak regions and layers of the composite bicycle frame under three tests. The weak regions associated with fillets and connections are marked on the frame in Fig. 11. The failure will occur first at the weak region.

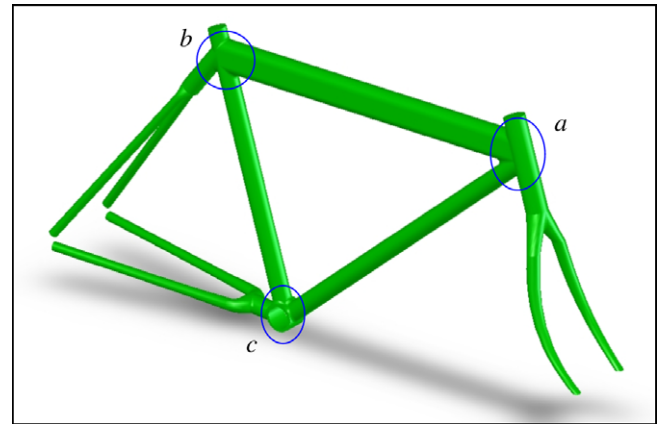


Fig. 11. Locations of weak regions.

Under the torsional loading, except for Case C, the first layer in the region *b* is the weak region and σ_2 is the failure stress. In Case C, the region *a* is the weak region and τ_{12} is the failure stress.

Under the frontal loading, the first or eighth layer in the region *a* is the weak region and σ_2 is the failure stress. However, under the vertical loading, the weak region changes its location to the region *b*.

From Table 5, the strength values of S_{2t} , S_{2c} , S_{3t} , S_{3c} and S_{ij} are much lower than S_{1t} and S_{1c} . So the stresses σ_2 and τ_{12} are the failure stresses.

All weak points occur at the first or eighth layer, i.e. the inner or outer layer of the laminated composite tube in Fig. 3. Both inner and outer layers endure higher bending stress and torsional shear stress.

5. Conclusions

In this paper, the fiber direction and stacking sequence design for the bicycle frame made of the carbon/epoxy composite laminates have been discussed. Under torsional, frontal, and vertical loadings, the normal and shear stresses with respect to the principal material coordinate system of each ply have been obtained from the finite element analyses.

According to the maximum stress theory and the results of strength-to-stress ratios, Case C with $[0/90/90/0]_s$ is the final selection for the common optimal stacking sequence under three loading tests. If the design selection is limited to only four plies 0° , 90° , 45° and -45° , Case 1 with $[0/90/45/-45]_s$ is the common optimal stacking sequence under three loading tests. To sum up, the stacking sequences $[0/90/90/0]_s$ and $[0/90/45/-45]_s$ are the good designs for the composite bicycle frames.

On the contrary, the uni-directional laminates, i.e. $[0/0/0/0]_s$, $[90/90/90/90]_s$, $[45/45/45/45]_s$, and $[-45/-45/-45/-45]_s$, are the bad designs. It can not be used in the composite bicycle frames to resist the external loads from different directions.

In addition, weak regions of failure occur at the fillets and connections of the frame, i.e. the stress concentration regions. All weak points occur at the inner or outer layer of the laminated composite tube. From Table 13, the 0° -ply and 90° -ply located on the inner and outer layer of the tube can effectively resist the higher stress at its location.

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