

Exam 1 (2019-20) Solutions/Mark Scheme

				Qu total
1a(i)	$V = 6t^{1/3} + 2t^{-3} - 4t^2$ $\frac{dv}{dt} = 2t^{-2/3} - 6t^{-4} - 8t$	3		
(ii)	$V = \frac{4t+3}{2t-2}$ Let $u = 4t+3$ $v = 2t-2$ $\frac{du}{dt} = 4$ $\frac{dv}{dt} = 2$ $\frac{dv}{dt} = \frac{(2t-2)(4) - (4t+3)(2)}{(2t-2)^2}$ $= \frac{8t-8-8t-6}{(2t-2)^2}$ $= \frac{-14}{(2t-2)^2}$	4		
(iii)	$V = (3+4t)^7$ Let $u = 3+4t$ so $v = u^7$ $\frac{du}{dt} = 4$ $\frac{dv}{du} = 7u^6$ $\frac{dv}{dt} = 4 \times 7u^6$ $= 28(3+4t)^6$	3		
(iv)	$V = t^2 \sin t$ Let $u = t^2$ $v = \sin t$ $\frac{du}{dt} = 2t$ $\frac{dv}{dt} = \cos t$ $\frac{dv}{dt} = (t^2)(\cos t) + (\sin t)(2t)$ $= t^2 \cos t + 2t \sin t$	4	14	

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1b	$s = 2t^3 - 13t^2 + 24t + 16$ $V = \frac{ds}{dt} = 6t^2 - 26t + 24$ (i) when $t = 3.5$ $V = 6(3.5)^2 - 26(3.5) + 24$ $V = 6.5 \text{ m/s.}$ (ii) $0 = 6t^2 - 26t + 24$ $0 = 3t^2 - 13t + 12$ $t = \frac{13 \pm \sqrt{(-13)^2 - 4(3)(12)}}{2(3)}$ $= \frac{13 \pm \sqrt{169 - 144}}{6}$ $= \frac{13 \pm \sqrt{25}}{6}$ $= 3 \text{ or } 1\frac{1}{3}$ (iii) $a = \frac{d^2s}{dt^2} = 12t - 26$ when $a = 10$ $10 = 12t - 26$ $36 = 12t$ $t = 3.$	1		
		2		
		5		
		1		
		2	11	25

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2a)	(i) $y = 4x^4 - 5x^{-2} + 3$ $\frac{dy}{dx} = 24x^3 + 10x^{-3}$ (ii) $y = x^3 \cos x$ Let $u = x^3$ $v = \cos x$ $\frac{du}{dx} = 3x^2$ $\frac{dv}{dx} = -\sin x$ $\frac{dy}{dx} = (x^3)(-\sin x) + (\cos x)(3x^2)$ $= -x^3 \sin x + 3x^2 \cos x$ (iii) $y = \frac{6x+2}{2x+1}$ Let $u = 6x+2$ $v = 2x+1$ $\frac{du}{dx} = 6$ $\frac{dv}{dx} = 2$ $\frac{dy}{dx} = \frac{(2x+1)(6) - (6x+2)(2)}{(2x+1)^2}$ $= \frac{12x+6 - 12x - 4}{(2x+1)^2}$ $= \frac{2}{(2x+1)^2}$ (iv) $y = (2+5x)^5$ Let $u = 2+5x$ so $y = u^5$ $\frac{du}{dx} = 5$ $\frac{dy}{du} = 5u^4$ $\frac{dy}{dx} = 5 \times 5u^4$ $= 25(2+5x)^4$	3		
		4		
		4		
		3	14	

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b.	(i) $\frac{dy}{dx} = 6x^2 - 4x - 1$ for stat pt $\frac{dy}{dx} = 0$ $0 = 6x^2 - 4x - 1$ $x = \frac{4 \pm \sqrt{(-4)^2 - 4(6)(-1)}}{2(6)}$ $= \frac{4 \pm \sqrt{16 + 24}}{12}$ $= \frac{4 \pm \sqrt{40}}{12}$ $= 0.860 \text{ or } -0.194$ If $x = 0.860$ $y = -2.4 \times 10^{-3}$ If $x = -0.194$ $y = 1.816 \times 10^{-3}$ (ii) $\frac{d^2y}{dx^2} = 12x - 4$ If $x = 0.860$ $\frac{d^2y}{dx^2} = 6.32$ +ve so Min If $x = -0.194$ $\frac{d^2y}{dx^2} = -6.33$ -ve so max (iii) Sketch	1		
		3		
		1		
		1	6	
		1		
		1		
		1	3	
		2	2	25

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				Qu total
3a)	(i) $\int (4e^{2x} + 2\sin x) dx$ $= 2e^{2x} - 2\cos x + c$ (ii) $\int_3^4 (2+4x+3x^2) dx = \left[2x + 2x^2 + x^3 \right]_3^4$ $= [2(4) + 2(4)^2 + 4^3] - [2(3) + 2(3)^2 + 3^3]$ $= 104 - 33 = 71$ (iii) $\int_0^{\pi/3} (6\cos 2x + \sin 3x) dx$ $= \left[3\sin 2x - \frac{1}{3}\cos 3x \right]_0^{\pi/3}$ $= \left[3\sin(2\pi/3) - \frac{1}{3}\cos(\pi) \right] - \left[3\sin 0 - \frac{1}{3}\cos 0 \right]$ $= 2.9314 + 1/3 = 3.265$	3		
		6		
		6	15	
b)	$\int \frac{1}{(2x+3)^3} dx$ Let $u = 2x+3$ $\frac{du}{dx} = 2 \rightarrow dx = \frac{1}{2} du$ $\therefore \int \frac{1}{(2x+3)^3} dx = \int \frac{1}{u^3} \cdot \frac{1}{2} du$ $= \frac{1}{2} \int u^{-3} du$ $= \frac{1}{2} \cdot \frac{u^{-2}}{-2}$ $= -\frac{1}{4} u^{-2}$ $= -\frac{1}{4} (2x+3)^{-2}$			
			10	25

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4a)	$\int (x+2) \cos x \, dx$ Let $u = x+2$ $\frac{dv}{dx} = \cos x$ $\frac{du}{dx} = 1$ $v = \sin x$ $\int (x+2) \cos x \, dx = (x+2)(\sin x) - \int \sin x (1) \, dx$ $= (x+2)(\sin x) + \cos x + C$			8
b) (i)	$\int (3 \cos 2x - 5 \sin 3x) \, dx$ $= \frac{3}{2} \sin 2x + \frac{5}{3} \cos 3x + C$	3		
(ii)	$\int_1^5 (3 - 2x + 6x^2) \, dx$ $= [3x - x^2 + 2x^3]_1^5$ $= [3(5) - 5^2 + 2(5)^3] - [3(1) - 1^2 + 2(1)^3]$ $= 240 - 4 = 236$	6		
(iii)	$\int_0^3 (2e^{4x} + 2x^{-3}) \, dx$ $= \left[\frac{1}{2} e^{4x} - \frac{1}{2} x^{-2} \right]_0^3$ $= \left[\frac{1}{2} e^{12} - \frac{1}{2} (3)^{-2} \right] - \left[\frac{1}{2} e^0 - 0 \right]$ $= 81377.4 - 0.5$ $= 81376.9$	8	17	25

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5a)	$\frac{6}{(s-2)(s+1)} = \frac{A}{s-2} + \frac{B}{s+1}$ $6 = A(s+1) + B(s-2)$ Let $s = -1$ $6 = A(0) + B(-3)$ $6 = -3B \rightarrow B = -2$ Let $s = 2$ $6 = A(3) + B(0)$ $6 = 3A \rightarrow A = 2$ $\therefore \frac{6}{(s-2)(s+1)} = \frac{2}{(s-2)} - \frac{2}{(s+1)}$ $\int_4^5 \left(\frac{2}{(s-2)} - \frac{2}{(s+1)} \right) ds$ $= \left[2 \ln(s-2) - 2 \ln(s+1) \right]_4^5$ $= [2 \ln 3 - 2 \ln 6] - [2 \ln 2 - 2 \ln 5]$ $= -1.3863 - (-1.8326)$ $= 0.447$	6														
b)	$h = \frac{5-4}{4} = 0.25$ <table border="1"> <tr> <td>x</td> <td>4</td> <td>4.25</td> <td>4.5</td> <td>4.75</td> <td>5</td> </tr> <tr> <td>y</td> <td>0.6</td> <td>0.5079</td> <td>0.4364</td> <td>0.3794</td> <td>0.3333</td> </tr> </table> $\therefore \int \approx h(0.25) \left[\frac{1}{2}(0.6 + 0.3333) + 0.5079 + 0.4364 + 0.3794 \right]$ $\approx 0.25 \times 1.79035 = 0.448$ $\% \text{ age error} = \frac{0.448 - 0.447}{0.447} \times 100$ $= 0.22\%$	x	4	4.25	4.5	4.75	5	y	0.6	0.5079	0.4364	0.3794	0.3333	9 1 3 2 2 2	15	
x	4	4.25	4.5	4.75	5											
y	0.6	0.5079	0.4364	0.3794	0.3333											
			10	25												