

a.

1.) $y = \frac{x^2}{2x+1}$

$y = \frac{u}{v}$

$y' = \frac{u'v - v'u}{v^2}$

$u = x^2 \quad u' = 2x$

$v = 2x+1 \quad v' = 2$

$y' = \frac{[2x(2x+1) - (2x^2)]}{(2x+1)(2x+1)}$

$y' = \frac{(4x^2 + 2x) - (2x^2)}{4x^2 + 4x + 1}$

$y' = \frac{2x^2 + 2x}{4x^2 + 4x + 1}$ or

$\frac{2x(x+1)}{(2x+1)^2}$

1.) $y = \sqrt{x^2+1}$

$y = (x^2+1)^{\frac{1}{2}}$

$y' = \frac{1}{2}(x^2+1)^{-\frac{1}{2}}$

$\frac{x^2+1}{2\sqrt{x^2+1}}$

$\frac{2x+0}{2\sqrt{x^2+1}} \cdot \frac{x}{\sqrt{x^2+1}}$

$\frac{3}{x^2} = 3 \cdot \frac{d}{dx} \left[\frac{1}{x^2} \right]$
 $3 \cdot (-2) x^{-3}$
 $= -\frac{6}{x^3}$

1. 3a) $y = 3x^{\frac{3}{2}} + \frac{3}{x^2} - 6x$

$y = 3x^{\frac{3}{2}} + 3x^{-2} - 6x$

$y' = \frac{9}{2}x^{\frac{1}{2}} - \frac{6}{x^3} - 6$

$y' = \frac{9\sqrt{x}}{2} - \frac{6}{x^3} - 6$

1.) $y = \sec^3 4x$

$3 \tan^2 4x$
 $12 \tan^2 x$

$3 \cdot \frac{d}{dx} \sec 4x \sec^2$

$3 \cdot \sec 4x \tan \cdot \sec^2 4x$

$y' = 12 \sec^3 4x \cdot \tan 4x$

$12 \sec^3 (4x) \tan (4x)$

$\therefore$

$y' = 12 \sec^3 4x \cdot \tan 4x$

1.) $y = (x + \sin 2x)$

$y' = 1 + \sin 2x$

$y' = 1 + 2 \cos 2x$

$y' = 2 \cos (2x) + 1$

$$1.) \text{ B) } y = \frac{1-10x}{(2x-1)^2}$$

$$y = \frac{[1-10x] \times (2x-1) - (1-10x) \times [2x+1]}{(2x-1)^2}$$

$$[(1-10) \times (2x-1) - (1-10) \times (2x+1)]$$

$$\frac{[0-10] \times (2x-1) - (1-10x) \times [2]}{(2x-1)^2}$$

$$\frac{-10 \times (2x-1) - 2(1-10x)}{(2x-1)^2}$$

$$\frac{-10(2x-1) - 2(1-10x)}{(2x-1)^2} \therefore$$

$$\frac{-20x + 10 - 2 + 20x}{(2x-1)^2}$$

$$1.) \text{ B) } y = \frac{1}{x} + \cos 2x \quad \text{find } \frac{d^5 y}{dx^5} = -32 \sin(2x) - \frac{120}{x^5}$$

$$y = \frac{1}{x} + \cos 2x$$

$$y' = \frac{dy}{dx} \left[\cos(2x) \right] + \frac{dy}{dx} \left[\frac{1}{x} \right]$$

$$y' = (-\sin(2x)) \cdot \frac{dy}{dx}(2x) - \frac{[x]}{x^2}$$

$$y' = -2 \sin 2x - \frac{1}{x^2}$$

$$\frac{d^2 y}{dx^2} = [-2 \sin(2x) - \frac{1}{x^2}]$$

$$= -2 \cdot \frac{d}{dx} [\sin 2x] - \frac{d}{dx} \left[\frac{1}{x^2} \right]$$

$$= -2 \cos(2x) \cdot x^2(2x) - (-2)x^{-3}$$

$$= -2 \cos 2x \cdot 2 \cdot 1 + \frac{2}{x^3}$$

$$= \frac{2}{x^3} - 4 \cos 2x$$

$$\frac{d^3 y}{dx^3} = \left[\frac{2}{x^3} - 4 \cos(2x) \right]$$

$$2 \cdot \frac{1}{x^4} - 4 \cdot \cos(2x)$$

$$2 \cdot (-3) \frac{1}{x^5} - 4(-\sin(2x)) \cdot [2x]$$

$$4 \times 2 \times \frac{d}{dx} [x] \times \sin(2x) - \frac{6}{x^4}$$

$$8 \times 1 \sin(2x) - \frac{6}{x^4}$$

$$8 \sin 2x - \frac{6}{x^4}$$

$$\frac{d^4 y}{dx^4} = 8 \sin 2x - \frac{6}{x^4}$$

$$= 8 \cdot [\sin 2x] - 6 \cdot \left(\frac{1}{x^4} \right)$$

$$= 8 \cos 2x \cdot \frac{d}{dx} [2x] - 6(-4)x^{-5}$$

$$= 8 \cos(2x) \cdot 2[1] + \frac{24}{x^5}$$

$$= 16 \cos(2x) + \frac{24}{x^5}$$

$$\frac{d^5 y}{dx^5} = 16 \cos(2x) + \frac{24}{x^5}$$

$$= 16(\cos(2x)) + 24 \cdot \frac{1}{x^5}$$

$$= 16(-\sin(2x)) \cdot [2x] + 24x(-5)x^{-6}$$

$$= -32 \sin(2x) - \frac{120}{x^5} \therefore$$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

1.) c) $\ln(\operatorname{cosec} x - \cot x)$ show $f'(x) = \operatorname{cosec} x$

$$\frac{dy}{dx} = \frac{1}{u} \cdot \frac{d(u)}{dx} \quad \text{where } u = \operatorname{cosec} x - \cot x$$

$$\frac{dy}{dx} = \frac{\operatorname{cosec} x \cdot \cot x + \operatorname{cosec}^2 x}{\operatorname{cosec} x - \cot x}$$

$$\frac{dy}{dx} = \frac{\operatorname{cosec} x (\cot x + \operatorname{cosec} x)}{\operatorname{cosec} x - \cot x}$$

$$\frac{dy}{dx} = \operatorname{cosec} x \quad \therefore$$

2.) $\int \frac{2x}{(4+3x^2)} dx$ $u = 3x^2 + 4$

$$\int \frac{2x}{u} dx$$

$$\int 2x u^{-1} dx = -2x u^0 + C = -2x(3x^2 + 4) + C$$

$\frac{d}{dx}(3x^2 + 4) = 6x$
 $dx = \frac{1}{6x}$

2.) B) $\int_0^4 e^{\frac{x}{2}} dx$

$$2 \int_0^4 e^u du$$

$$2e^u \text{ where } u$$

$$2e^{\frac{x}{2}} + C$$

$$e^u = e^u$$

$$u = \frac{x}{2} = \frac{1}{2}$$

$$\frac{du}{dx} = dx = 2du$$

In Radians Answer =

$$12.7781122$$

$$12.78(2dp)$$

2.) B) $\int_0^4 \frac{5x+13}{(2x+)(x+4)} dx$

$$\int \frac{1}{ax+b} dx = \frac{1}{a} \ln |ax+b| + C$$

Answer in Radians =

$$3.98984087$$

$$3.99(2dp)$$

$$2) b) \int \frac{dx}{x(1+4\ln x)^2} = \frac{4}{x(1+4\ln x)^2} dx$$

$$\int \frac{1}{u^2} du > \text{power rule 2 comes out} \quad u = 4\ln(x) + 1$$

$$\frac{du}{dx} = \frac{4}{x}$$

$$dx = \frac{x}{4} du$$

$$-\frac{1}{u} + C$$

$$-\frac{1}{4\ln(x)+1} + C$$

2 (c)

i)

$$\int f g' = f g - \int f' g$$

$$f = \ln(x), g' = 1$$

$$f' = \frac{1}{x}, g = x$$

$$ii) \int_1^e \ln x dx = x \ln(x) - \int 1 dx$$

$$\int 1 dx = x$$

$$x \ln(x) - (x) + C$$

$$Q3 (a) \left(\frac{1+i}{1-i} \right)^3 = i^3$$

$$= -i$$

$$i^1 = i$$

$$i^2 = -1$$

$$i^3 = -i$$

$$(b) \frac{3}{(1+i)(2+i)}$$

(c)

(d)