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Project Cost Management

Investment Appraisal / Discounted Cash Flow (DCF) - Annual Sinking Fund

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Introduction - Annual Sinking Fund

A fund into which a company sets aside money over time, in order to retire its preferred stock, bonds or debentures.

In the case of bonds, incremental payments into the sinking fund can soften the financial impact at maturity.

Investors prefer bonds and debentures backed by sinking funds because there is less risk of a default.

Definition - Annual Sinking Fund

The sum which must be invested annually at a given rate of interest to accumulate to a target sum of money at the end of a specified period of years.

Formula:

$$C = \frac{r}{(1+r)^n - 1} * A$$

Where:

C = the constant investment pa

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r = the rate of interest over the period

n = the number of years

A = the final amount.

Example 1: Annual Sinking Fund

Example:

If, at the age of 30, I decided that I wanted to have £5,000 available to spend on my 40th birthday. how much should I put away each year to accomplish this? A safe investment medium is available offering 4% pa interest.

Substituting:

$$C = \frac{5,000}{12.006}$$

$$C = 0.08329 * 5,000$$

$$C = 416.45$$

This can be checked back by multiplying £416.45 by the Amount of £1 pa for 10 years @ 4% (12.006).

Example: Annual Sinking Fund (cont.)

- ◆ As with all the formulae, the ASF may be adjusted to take account of sub-annual payments. For instance, had the above example been worked out for monthly contributions to the fund, the result would have been £33.95 per month, or £407.47 per year.
- ◆ The ASF is generally used in situations where it is necessary to calculate how much should be set aside out of current income to meet a known liability which will arise at some point in the future. One instance of this would be in making provision for the replacement of a major item of equipment in a building (say re-wiring it after 20 years). A wiring system has a finite life, and the cost of replacing it should really be paid out of the benefits which its use confers upon the building's users during its life.
- ◆ Similarly, an investor who buys a leasehold interest in a property will be concerned to ensure that the capital tied up in the investment is restored at the end of the period of the lease, and may establish a Sinking Fund (paid out of rents received) to do this.

Example: 2 – Annual Sinking Fund (cont.)

Example:

How much is it worth an investor paying for the right to receive £20,000 pa in rent from a property whose lease expires in 15 years time, assuming that the investor's required rate of return is 10% pa?

Method:

Capitalise the income at the required rate of return (r) adjusted by the amount which needs to be extracted from the income to pay into a sinking fund which will replace the investor's capital at the end of the 15 years.

If the investor pays £200,000 ($1/r * 20,000$), this will deliver the required 10% return over the remaining years of the lease, but will leave the investor with nothing at the end. To restore the capital at the end of 15 years, the investor will have to add the provision for a sinking fund to the required rate of return. Thus the capitalisation factor (Years Purchase) will become:

Example: 2 – Annual Sinking Fund (cont.)

$$\frac{1}{(r_r + sf)}$$

The annual sinking fund to replace £1 of capital at the end of 15 years @ 3% interest is:

$$\frac{0.3}{(1.03)^{15} - 1}$$

$$= 0.05377$$

In this example, therefore, the capitalisation factor (YP) will be:

$$\frac{1}{(0.1 + 0.05377)} = 6.50336$$

Capitalising the rent at this rate gives us a figure of £130,067 as being the sum which the investor could afford to pay for the remaining 15 years of the lease which would deliver a return of 10% on invested money and make provision for the replacement of the capital.

Example: 2 – Annual Sinking Fund (cont.)

We can check the validity of this result by working it back:

Sum Invested 130,067

ASF to replace sum invested over 15 years @ 3% = 6,993

Required rate of return on investment @ 10% = 13,007

Total pa = 20,000

Annuity

The amount of money that can be withdrawn each year, for a specified number of years, from a fund invested at a given rate of interest.

Formula:

$$C = r + \left[\frac{r}{(1+r)^n - 1} \right] * P$$

Where:

C = the constant amount withdrawn pa

P = the amount of the initial investment

r = the rate of interest

n = the number of years.

Example of annuity

If I had £50,000 in savings and wished to use it to finance a five year break from work, how much could I afford to draw out of my fund at the end of each year assuming it could be invested at a secure rate of 7% pa?

Substituting:

$$C = .07 + \left[\frac{.07}{(1+.07)^5 - 1} \right] * 50,000$$

$$C = 0.2439 * 50,000$$

$$C = 12,194.93$$

The Annuity originally developed out of the need to calculate the *annual equivalents* of lump sums, for example in working out pension entitlements. If, for instance, you paid £1,000 pa (monthly) into a pension fund for the whole of a 40 year working life, what sort of pension could you expect to draw at the end of it?

Example of annuity (cont.)

a) Work out the value of the fund at retirement:

Amt of 1 pa, 40 years @ 6% compounded monthly	1991.49
Amt of £83.33 per month, 40 years @ 6%	165,958

b) At this point a rather bleak assumption has to be made. Over how many years should the retirement plan be spread? Let us assume 15 years with monthly pension payments.

Annuity, 15 years @ 6% paid monthly	0.069364
Annual payment from £165,958	£11,511
Monthly Pension	£959

If the pensioner survives for more than 15 years, then the rest is a bonus. If not, then the pension company is the beneficiary.

Annual Equivalent Costs-in-Use

The same approach can be used to evaluate investment options which have dissimilar life spans.

Example:

Two alternative heating systems are being considered for installation into a house.

Alternative A has a life of 9 years, costs £2,000 to install initially, requires maintaining every 3 years at a cost of £200, and has fuel costs of £700 pa.

Alternative B has a life of 12 years, costs £2,300 to install initially, requires maintaining every 3 years at a cost of £220, and has fuel costs of £670 pa.

If we were to bring all of this information about the alternatives back to present value at a discount rate of 10%, we would end up with the following:

Annual Equivalent Costs-in-Use (cont.)

ITEM		System A	System B
Life (Years)		9	12
Initial Cost		2,000.00	2,300.00
Maintenance every 3 years		200.00	220.00
Running cost pa		700.00	670.00
PV at 10% discount rate			
Initial Cost		2,000.00	2,300.00
Maintenance yr	3	150.26	165.29
	6	112.89	124.18
	9	0.00	93.30
Running Costs	9 years	4,031.32	
	12 years		4,565.17
Total		6,297.47	7,247.95

Knowing that Alternative A has a lower PV than Alternative B does not really help us in making the selection because of the different time scales involved. Would it be worth paying the additional cost for Alternative B (PV of which is £953), for the additional 3 years?

Annual Equivalent Costs-in-Use (cont.)

One way of solving this problem would be to run the alternatives over a period (36 years) which contained complete cycles of both. This, however, may be too long a period over which to make realistic comparisons.

Alternatively we could convert the two lump sum PVs into their equivalent annual sums by application of the Annuity formula. Hence:

	System A	System B
PV @ 10%	6,294.47	7,247.95
Annuity, 9 yrs @ 10%	0.1736	
Annuity, 12 yrs @ 10%		0.1468
Annual Equivalent	1,092.98	1,063.73

Effectively what this means is that for the duration of the life of Alternative A it will cost £1,093 pa to buy, service and operate the system, whereas for the duration of the life of Alternative B it will cost £ 1,064 pa. On this information, Alternative B is to be preferred.

Mortgages

The Annuity formula may also be used to calculate mortgage payments. In the pension example above we started off with a lump sum invested at a given rate of interest, and then proceeded to work out how much we could withdraw each month from the investment such that at the end of a fixed term of years the capital in the fund, and accumulated interest on the capital would be entirely depleted.

In essence, a mortgage works in exactly the same way. A loan is made at a given rate of interest and repayments are worked out such that at the end of a fixed term of years the loan is entirely paid back.

Mortgages (cont.)

What would be the monthly repayments on a loan of £50,000 over 25 years at a rate of interest of 8% pa?

a. Convert the annual rate of interest to its monthly equivalent:

$$0.08/12 = .006666$$

b. Substitute into the Annuity formula: $C = \left[r + \left[\frac{r}{(1+r)^n - 1} \right] \right] * P$

$$C = \left[0.00666 + \frac{0.00666}{(1 + .00666)^{(25*12)} - 1} \right] * 50,000$$

$$C = 0.007713 * 50,000$$

$$C = £385.64$$

Mortgages (cont.)

What would be the monthly repayments on a loan of £180,000 over 25 years at a rate of interest of 3.1% pa?

a. Convert the annual rate of interest to its monthly equivalent:

$$0.03/12 = .00258$$

b. Substitute into the Annuity formula:

$$C = \left[r + \frac{r}{(1+r)^n - 1} \right] * P$$
$$C = \left[0.00666 + \frac{0.00258}{(1 + .00258)^{(25*12)} - 1} \right] * 50,000$$
$$C = 0.007713 * 50,000$$
$$C = £385.64$$

Mortgages (cont.)

That is, the monthly payments for such a mortgage would be £385.64.

Having established the monthly payments under the given set of conditions, we can now go on to test its sensitivity to changes in those conditions by simply manipulating the variables in the formula.

Changes in the interest rate:

Rate	Payments PCM
6.00%	320.55
6.50%	335.78
7.00%	351.34
7.50%	367.20
8.00%	383.35
8.50%	399.78
9.00%	416.47
9.50%	433.42
10.00%	450.60

Mortgages (cont.)

Changes in borrowing period:

Years	Payments PCM
20	415.45
21	407.50
22	400.42
23	394.10
24	388.44
25	383.35
26	378.77
27	374.64
28	370.91
29	367.52
30	364.45

Mortgages (cont.)

The above examples relate to what are called *repayment mortgages* in that each instalment contains an element of capital repayment as well as an element of interest payment. This may be illustrated using the simplified example of a £10,000 mortgage repaid through annual instalments over 10 years at 10%:

$$\begin{aligned}\text{Payments pa} &= 0.1 + \left[\frac{0.1}{(1.1)^{10} - 1} \right] * 10,000 \\ &= £1,627.45 \text{ pa}\end{aligned}$$

That the payment of £1,627.45 pa is just sufficient to pay off the debt, and interest on the debt, may be demonstrated as follows:

Mortgages (cont.)

Year	Start of year debt	Interest @ 10%	End of year debt	Payment	Repayment of Capital
1	10,000.00	1,000.00	11,000.00	1,627.45	627.45
2	9,372.55	937.25	10,309.80	1,627.45	690.20
3	8,682.35	868.23	9,550.58	1,627.45	759.22
4	7,923.13	792.31	8,715.44	1,627.45	835.14
5	7,087.99	708.80	7,796.78	1,627.45	918.66
6	6,169.33	616.93	6,786.26	1,627.45	1,010.52
7	5,158.81	515.88	5,674.69	1,627.45	1,111.57
8	4,047.24	404.72	4,451.96	1,627.45	1,222.73
9	2,824.51	282.45	3,106.96	1,627.45	1,345.00
10	1,479.50	147.95	1,627.45	1,627.45	1,479.50
TOTAL		6,274.54		16,274.54	10,000.00

Mortgages (cont.)

Another type of mortgage (known as an interest-only or endowment mortgage) works by having the mortgage paying interest only on the debt whilst at the same time building up a sinking fund to repay the capital. The annual payments under this type of scheme will vary according to the age of the mortgage (which will affect the term and costs of the endowment), and the type of sinking fund which is bought into. Possible figures for the above example would be:

Interest payments pa on £10,000 at 10%	=	1,000
ASF for £10,000 in 10 years at 8%	=	690.30
Total cost pa	=	£1,690.30

Although the cost per year of this type of arrangement is greater than the cost per year of the repayment option (it is bound to be if the SF rate is less than the mortgage rate), it may work out cheaper in the end as more of the payments qualify for tax relief than do for the repayment mortgage.

Mortgages (cont.)

Other tax advantages may accrue to the borrower from the way in which the ASF is structured. It may, for example, include an element of PEP-type saving for which particular tax breaks are currently available.

Total interest payments come to £10,000 (as opposed to £6,274) and traditionally such payments attract tax relief - although it should be noted that the main weight of this benefit is received late on in the repayment period and might reasonably be heavily discounted by a potential borrower.

Mortgages (cont.) - Using a Spreadsheet

All of the above formulae can be entered into any of the standard spreadsheets to produce either tables of compounding/discounting factors or answers to individual calculations. The PV of 1 pa formula on page 12, for example, could be set out in tabular form in the following way:

Formula:
$$P = \frac{1 - \left[\frac{1}{(1+r)^n} \right]}{r} * C$$

	A	B	C	D	E	F	G	H
	Present Value of 1 pa.							
1								
2		Payments pa			100.00			
3		Rate of Interest:			10.00%			
4								
5								
6								
7								
8								
9								
10								
11								
12								
13								
14								
15								

End of Year	Amount
0	0.00
1	90.91
2	173.55
3	248.69
4	316.99
5	379.08
6	435.53

=E\$2*((1-(1/((1+E\$3)^\$B8)))/E\$3)
=E\$2*((1-(1/((1+E\$3)^\$B9)))/E\$3)
=E\$2*((1-(1/((1+E\$3)^\$B10)))/E\$3)
=E\$2*((1-(1/((1+E\$3)^\$B11)))/E\$3)
=E\$2*((1-(1/((1+E\$3)^\$B12)))/E\$3)
=E\$2*((1-(1/((1+E\$3)^\$B13)))/E\$3)
=E\$2*((1-(1/((1+E\$3)^\$B14)))/E\$3)

Mortgages (cont.) - Using a Spreadsheet

Most modern spreadsheets have inbuilt functions which enable these various calculations to be done much more quickly. Care should be exercised with these functions as they do not always do the calculations in the way that you might expect. It is "always worthwhile checking manually the results obtained from using these inbuilt functions to satisfy yourself that they represent exactly the conditions of the problem. For example, older versions of Lotus and Excel work on the" assumption that all payments and receipts arise at the end of each period. Newer versions offer the option of specifying that payments or receipts should be treated as arising at the beginning or the end.

The table below has been constructed using the PV function in Excel (accessed through the Function Wizard)..

`=PV(rate, nper, pmt, tv, type)`

In filling in the variables in the function, the following "translation" of terms from US practice to UK practice may be helpful:

Mortgages (cont.) - Using a Spreadsheet

Term (US)	Description (UK)
PV	Present Value of 1 pa
Rate	Annual rate of interest. If payments are to be made other than annually, then this rate should be adjusted accordingly
Nper	Number of periods. If an adjusted rate of interest has been used, then enter the number of periods (months/quarters etc). If an unadjusted rate has been used, then enter the number of years
Pmt	Payment per period. EXCEL assumes these to be negative, Lotus assumes them to be positive
Fv	Future Value. This is an optional element (square brackets [...]) as used in Lotus indicate options) which enables a lump sum to be inserted at the end of the period. If nothing is entered here, the default is zero
Type	The type of cash flow – ie whether payment/receipts are made at the start of each period (enter 1) or at the end of each period (enter 0). The default value is 0.

Lotus 1-2-3 @functions are similar in their construction, although they use a slightly different terminology and order in which the values are entered. The equivalent function to that explained above is:

Mortgages (cont.) - Using a Spreadsheet

@PVAL(PAYMENTS,INTEREST,TERM,[TYPE, FUTURE_VALUE])

Applying the function (using Excel) produces the following result

	A	B	C	D	E	F	G	H
	Present Value of 1 pa.							
1								
2		Payments pa:			100.00			
3		Rate of Interest:			10.00%			
4								
5								
6								
7								
8								
9								
10								
11								
12								
13								
14								
15								

End of Year	Amount	
0	0.00	=PV(rate,nper,pmt,fv,type)
1	90.91	=PV(\$E\$3,\$B8,-\$E\$2,,0)
2	173.55	=PV(\$E\$3,\$B9,-\$E\$2,,0)
3	248.69	=PV(\$E\$3,\$B10,-\$E\$2,,0)
4	316.99	=PV(\$E\$3,\$B11,-\$E\$2,,0)
5	379.08	=PV(\$E\$3,\$B12,-\$E\$2,,0)
6	435.53	=PV(\$E\$3,\$B13,-\$E\$2,,0)

Mortgages (cont.) - Using a Spreadsheet

Table of equivalent terms:

	EXCEL	Lotus 1-2-3
Amount of 1	none	none
Present Value of 1	=NPV (only up to 30 periods)	@PV
Amount of 1 pa	=FV	@FV or @FVAL
Present Value of 1 pa	=PV	@PVAL
Annuity	=PMT (fv = 0, pv negative)	@PMT
ASF	=PMT (pv = 0, fv negative)	@PAYMT

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