

Mock Exam Paper

- 1.) Using the discrete Fourier Transform (DFT), Determine the $k=0, 1, 2, 3, 4, 5$ bin frequency absolute magnitude and phase values for the following sampled data set.

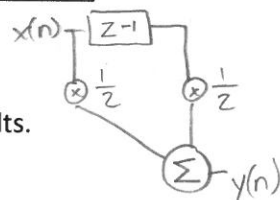
n	0	1	2	3	4	5
x	0	2	7	-6	-2	4

- 2.) If the data was acquired with sample time equal to $179\mu\text{s}$ determine the resolution of the transformed data in the frequency domain.

- 3.) Provide a sketch illustrating how a 7^{th} order Moving average filter can be realised.

- 4.) Determine the output of a 2^{nd} order moving average filter for the following input values.

n	0	1	2	3	4	5	6	7	8	9	10
x	55	58	57	50	48	57	53	51	59	60	47



- 5.) A signal is sampled using an 8-bit analogue to digital convertor. The voltage range is ± 2 volts. Calculate the minimum change in voltage that can be measured.

8 bits = 256 steps $\frac{12}{256} = 46.875\text{mV}$

- 6.) Using the discrete Fourier transform determine the $K=0$ and $K=1$ bin frequency real, imaginary and absolute magnitudes and phase for the following sampled data set.

n	0	1	2	3	4	5	6	7	8
x	0.7	1	0.7	0.4	0.7	1	0.7	0.4	0.7

Mock Exam Paper

Using the discrete Fourier transform (DFT), determine the $k=0, 1, 2, 3, 4, 5$ bin frequency absolute magnitude and phase values for the following sample data set

n	0	1	2	3	4	5
x	0	2	7	-6	-2	4

using
$$X(k) = \sum_{n=0}^{N-1} x(n) e^{-\frac{j2\pi kn}{N}}$$

$$K=0 \quad X(0) = \sum_{n=0}^{N-1} x(n) e^0$$

$$X(0) = \sum_{n=0}^{N-1} x(n)$$

$$X(0) = \sum_{n=0}^5 x(n)$$

$$X(0) = 0 + 2 + 7 + (-6) + (-2) + 4$$

$$X(0) = 5$$

$$K=1 \quad X(1) = \sum_{n=0}^{N-1} x(n) e^{-\frac{j2\pi n}{6}}$$

$$\text{real} \quad X(1) = \sum_{n=0}^5 x(n) \cos\left(\frac{2\pi n}{6}\right)$$

$$X(1) = 0\cos(0) + 2\cos\left(\frac{2\pi \times 1}{6}\right) + 7\cos\left(\frac{2\pi \times 2}{6}\right) - 6\cos\left(\frac{2\pi \times 3}{6}\right) - 2\cos\left(\frac{2\pi \times 4}{6}\right) + 4\cos\left(\frac{2\pi \times 5}{6}\right)$$

$$X(1) = 0 + 1 - 3.5 + 6 + 1 + 2$$

$$X(1) = 6.5$$

$$\text{Imag} \quad X(1) = -\sum_{n=0}^5 x(n) \sin\left(\frac{2\pi n}{6}\right)$$

$$X(1) = 0\sin\left(\frac{2\pi \times 0}{6}\right) + 2\sin\left(\frac{2\pi \times 1}{6}\right) + 7\sin\left(\frac{2\pi \times 2}{6}\right) - 6\sin\left(\frac{2\pi \times 3}{6}\right) - 2\sin\left(\frac{2\pi \times 4}{6}\right) + 4\sin\left(\frac{2\pi \times 5}{6}\right)$$

$$X(1) = 0 + 1.73 + 6.06 + 0 + 1.73 - 3.46$$

$$X(1) = 6.06$$

$$X(1) = 6.5 + j6.065$$

$$K=2 \quad X(z) = \sum_{n=0}^{N-1} x(n) e^{-j\frac{4\pi}{6}n}$$

$$\text{real} \quad X(z) = \sum_{n=0}^5 x(n) \cos\left(\frac{4\pi}{6}n\right)$$

$$X(z) = 0 \cos(0) + 2 \cos\left(\frac{4\pi}{6} \times 1\right) + 7 \cos\left(\frac{4\pi}{6} \times 2\right) - 6 \cos\left(\frac{4\pi}{6} \times 3\right) - 2 \cos\left(\frac{4\pi}{6} \times 4\right) + 4 \cos\left(\frac{4\pi}{6} \times 5\right)$$

$$X(z) = 0 - 1 - 3.5 - 6 + 1 - 2$$

$$X(z) = -11.5$$

$$\text{imag} \quad X(z) = \sum_{n=0}^5 x(n) \sin\left(\frac{4\pi}{6}n\right)$$

$$X(z) = 0 \sin(0) + 2 \sin\left(\frac{4\pi}{6} \times 1\right) + 7 \sin\left(\frac{4\pi}{6} \times 2\right) - 6 \sin\left(\frac{4\pi}{6} \times 3\right) - 2 \sin\left(\frac{4\pi}{6} \times 4\right) + 4 \sin\left(\frac{4\pi}{6} \times 5\right)$$

$$X(z) = 0 + 1.73 - 6.06 + 0 - 1.73 - 3.66$$

$$X(z) = -9.52$$

$$X(z) = -11.5 - 9.525$$

$$K=3 \quad X(3) = \sum_{n=0}^{N-1} x(n) e^{-j \frac{6\pi}{6} n}$$

$$\text{real} \quad X(3) = \sum_{n=0}^5 x(n) \cos\left(\frac{6\pi}{6} n\right)$$

$$X(3) = 0 \cos(0) + 2 \cos\left(\frac{6\pi}{6} \times 1\right) + 7 \cos\left(\frac{6\pi}{6} \times 2\right) \\ - 6 \cos\left(\frac{6\pi}{6} \times 3\right) - 2 \cos\left(\frac{6\pi}{6} \times 4\right) + 4 \cos\left(\frac{6\pi}{6} \times 5\right)$$

$$X(3) = 0 - 2 + 7 + 6 - 2 - 4$$

$$X(3) = 5$$

$$\text{imag} \quad X(3) = \sum_{n=0}^5 x(n) \sin\left(\frac{6\pi}{6} n\right)$$

$$X(3) = \sin(0) + 2 \sin\left(\frac{6\pi}{6} \times 1\right) + 7 \sin\left(\frac{6\pi}{6} \times 2\right) \\ - 6 \sin\left(\frac{6\pi}{6} \times 3\right) - 2 \sin\left(\frac{6\pi}{6} \times 4\right) + 4 \sin\left(\frac{6\pi}{6} \times 5\right)$$

$$X(3) = 0 + 0 + 0 + 0 + 0 + 0$$

$$X(3) = 0$$

$$X(3) = 5 + 0j$$

$$k=4 \quad X(4) = \sum_{n=0}^{N-1} x(n) e^{-j \frac{8\pi}{6} n}$$

$$\text{real} \quad X(4) = \sum_{n=0}^5 x(n) \cos\left(\frac{8\pi}{6} n\right)$$

$$X(4) = 0 \cos(0) + 2 \cos\left(\frac{8\pi}{6} \times 1\right) + 7 \cos\left(\frac{8\pi}{6} \times 2\right) - 6 \cos\left(\frac{8\pi}{6} \times 3\right) - 2 \cos\left(\frac{8\pi}{6} \times 4\right) + 6 \cos\left(\frac{8\pi}{6} \times 5\right)$$

$$X(4) = 0 - 1 - 3.5 - 6 + 1 - 2$$

$$X(4) = -11.5$$

$$\text{Imag} \quad X(4) = \sum_{n=0}^{N-1} x(n) \sin\left(\frac{8\pi}{6} n\right)$$

$$X(4) = \sin(0) + 2 \sin\left(\frac{8\pi}{6} \times 1\right) + 7 \sin\left(\frac{8\pi}{6} \times 2\right) - 6 \sin\left(\frac{8\pi}{6} \times 3\right) - 2 \sin\left(\frac{8\pi}{6} \times 4\right) + 6 \sin\left(\frac{8\pi}{6} \times 5\right)$$

$$X(4) = 0 = 1.73 + 6.06 + 0 + 1.73 + 3.46$$

$$X(4) = 9.525$$

$$X(4) = -11.5 + 9.525$$

K=5

$$X(5) = \sum_{n=0}^{N-1} x(n) e^{-j \frac{10\pi}{6} n}$$

real

$$X(5) = \sum_{n=0}^5 x(n) \cos\left(\frac{10\pi}{6} n\right)$$

$$X(5) = 0 \cos(0) + 2 \cos\left(\frac{10\pi}{6} \times 1\right) + 7 \cos\left(\frac{10\pi}{6} \times 2\right) - 6 \cos\left(\frac{10\pi}{6} \times 3\right) - 2 \cos\left(\frac{10\pi}{6} \times 4\right) + 6 \cos\left(\frac{10\pi}{6} \times 5\right)$$

$$X(5) = 0 + 1 - 3.5 + 6 + 1 + 2$$

$$X(5) = 6.5$$

imag

$$X(5) = \sum_{n=0}^5 x(n) \sin\left(\frac{10\pi}{6} n\right)$$

$$X(5) = \sin(0) + 2 \sin\left(\frac{10\pi}{6} \times 1\right) + 7 \sin\left(\frac{10\pi}{6} \times 2\right) - 6 \sin\left(\frac{10\pi}{6} \times 3\right) - 2 \sin\left(\frac{10\pi}{6} \times 4\right) + 6 \sin\left(\frac{10\pi}{6} \times 5\right)$$

$$X(5) = 0 - 1.73 - 6.06 + 0 - 1.73 + 3.46$$

$$X(5) = 6.06$$

$$X(5) = 6.5 - 6.06j$$

$$X(0) = 5$$

$$X(1) = 6.5 + 6.06j$$

$$X(2) = -11.5 + 9.52j$$

$$X(3) = 5$$

$$X(4) = -11.5 + 9.52j$$

$$X(5) = 6.5 - 6.06j$$

If the data was acquired with sample time equal to $179\mu\text{s}$ determine the resolution of the transformed data in the frequency domain?

$$T_s = 179\mu\text{s} \quad \text{and} \quad f_s = \frac{1}{T_s}$$

$$f_s = 1$$

$$179\mu\text{s}$$

$$f_s = 1$$

$$179 \times 10^{-6}$$

$$f_s = 5586.6$$

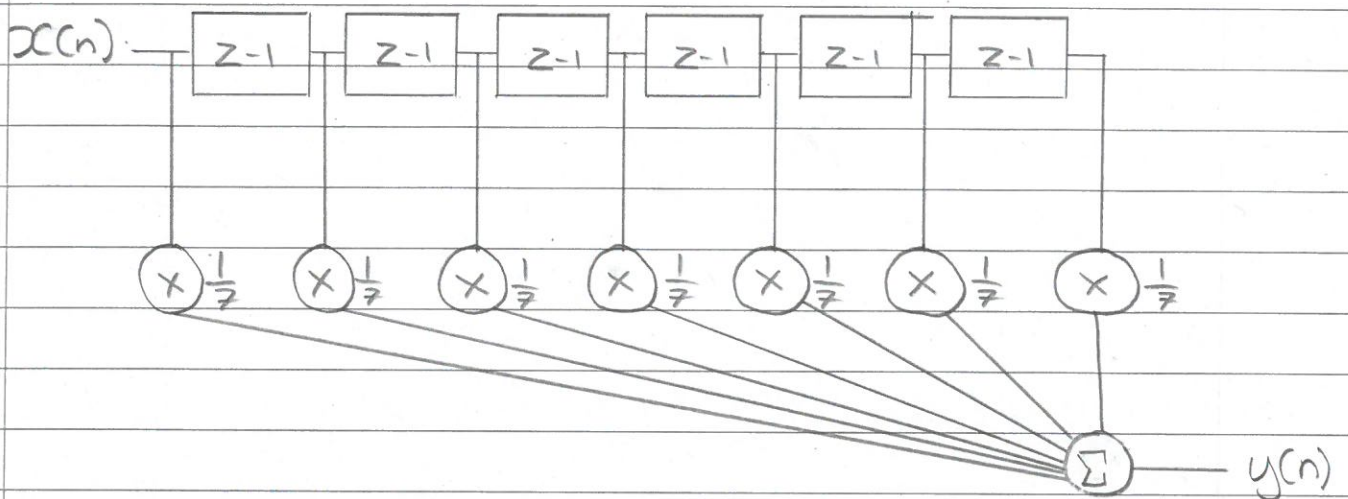
$$f_s = 5.58\text{kHz}$$

$$f_{\text{resolution}} = \frac{f_s}{N}$$

$$f_{\text{resolution}} = 5586.6$$

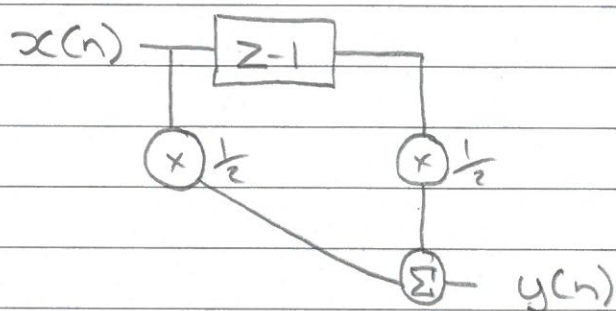
$$f_{\text{resolution}} = 931\text{ Hz}$$

Provide a sketch illustrating how a 7th order moving average filter can be realised?



Determine the output of a 2nd order moving average filter for the following input values

n	0	1	2	3	4	5	6	7	8	9	10
x	55	58	57	50	48	57	53	51	59	60	47



n	x	y
0	55	27.5
1	58	$27.5 + 29 = 56.5$
2	57	$29 + 28.5 = 57.5$
3	50	$28.5 + 25 = 53.5$
4	48	$25 + 24 = 49$
5	57	$24 + 28.5 = 52.5$
6	53	$28.5 + 26.5 = 55$
7	51	$26.5 + 25.5 = 52$
8	59	$25.5 + 29.5 = 55$
9	60	$29.5 + 30 = 59.5$
10	47	$30 + 23.5 = 53.5$
11	0	23.5

A signal is sampled using an 8-bit analogue to digital converter. The voltage range is 12 volts. Calculate the minimum change in voltage that can be measured?

$$8 \text{ bits} = 256 \text{ steps}$$

$$\frac{12 \text{ Volts}}{256} = 46.9 \text{ mV}$$

Using the discrete fourier transform determine the $k=0$ and $k=1$ bin frequency real, imaginary and absolute magnitudes and phase for the following sampled data set:

n	0	1	2	3	4	5	6	7	8
x	0.7	1	0.7	0.6	0.7	1	0.7	0.6	0.7

using

$$X(k) = \sum_{n=0}^{N-1} x(n) e^{-j2\pi \frac{kn}{N}}$$

$$K=0 \quad X(0) = \sum_{n=0}^{N-1} x(n) e^0$$

$$X(0) = \sum_{n=0}^8 x(n)$$

$$X(0) = 0.7 + 1 + 0.7 + 0.6 + 0.7 + 1 + 0.7 + 0.6 + 0.7$$

$$X(0) = 6.3$$

$$k=1 \quad X(1) = \sum_{n=0}^{N-1} x(n) e^{(-j \frac{2\pi}{9} n)}$$

$$\text{real} \quad X(1) = \sum_{n=0}^8 x(n) \cos\left(\frac{2\pi}{9} n\right)$$

$$\begin{aligned} X(1) &= \cos(0) + \cos\left(\frac{2\pi}{9} \times 1\right) + 0.7 \cos\left(\frac{2\pi}{9} \times 2\right) + 0.6 \cos\left(\frac{2\pi}{9} \times 3\right) \\ &\quad + 0.7 \cos\left(\frac{2\pi}{9} \times 4\right) + \cos\left(\frac{2\pi}{9} \times 5\right) + 0.7 \cos\left(\frac{2\pi}{9} \times 6\right) \\ &\quad + 0.6 \cos\left(\frac{2\pi}{9} \times 7\right) + 0.7 \cos\left(\frac{2\pi}{9} \times 8\right) \end{aligned}$$

$$\begin{aligned} X(1) &= 1 + 0.77 + 0.12 - 0.2 + 0.56 - 0.96 \\ &\quad - 0.35 + 0.07 + 0.56 \\ X(1) &= 1.25 \end{aligned}$$

$$\text{imag} \quad X(1) = \sum_{n=0}^8 x(n) \sin\left(\frac{2\pi}{9} n\right)$$

$$\begin{aligned} X(1) &= \sin(0) + \sin\left(\frac{2\pi}{9} \times 1\right) + 0.7 \sin\left(\frac{2\pi}{9} \times 2\right) + 0.6 \sin\left(\frac{2\pi}{9} \times 3\right) \\ &\quad + 0.7 \sin\left(\frac{2\pi}{9} \times 4\right) + \sin\left(\frac{2\pi}{9} \times 5\right) + 0.7 \sin\left(\frac{2\pi}{9} \times 6\right) \\ &\quad + 0.6 \sin\left(\frac{2\pi}{9} \times 7\right) + 0.7 \sin\left(\frac{2\pi}{9} \times 8\right) \end{aligned}$$

$$\begin{aligned} X(1) &= 0 + 0.66 + 0.69 + 0.35 + 0.26 - 0.36 \\ &\quad - 0.61 - 0.39 - 0.65 \end{aligned}$$

$$X(1) = 1.25 + j 0.13$$

$$X(0) = 6.3$$

$$X(1) = 1.25 + j 0.13$$