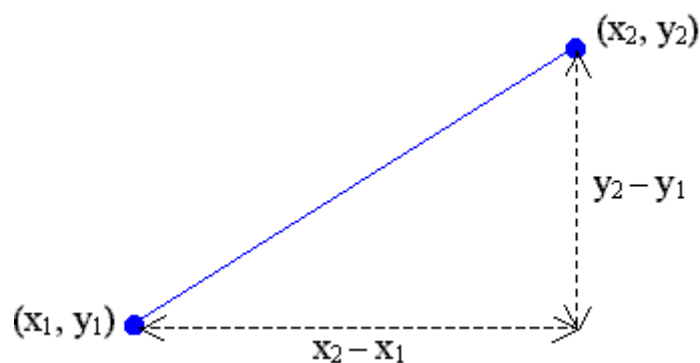


Gradients

Remember a gradient is defined as:

the change in 'y' = $y_2 - y_1$ divided by change in 'x' = $x_2 - x_1$

These quantities are shown on the following diagram:



The gradient 'm' of the line segment is defined as follows:

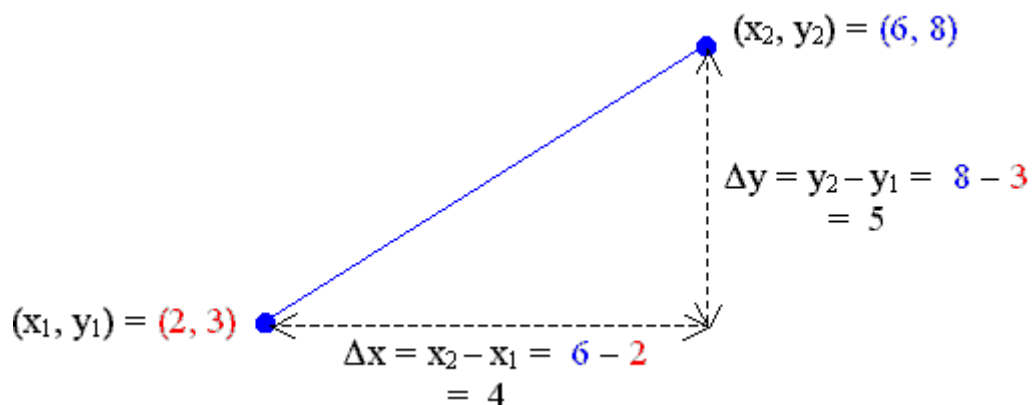
$$\text{gradient 'm'} = \frac{\text{change in 'y'}}{\text{change in 'x'}} = \frac{y_2 - y_1}{x_2 - x_1}$$

The Greek capital letter 'Delta': Δ is often used in mathematics to mean a change in some quantity. So the gradient definition can be written like this:

$$m = \frac{\Delta y}{\Delta x}$$

To measure the change in 'x' and 'y' between the two points, we draw a right triangle beneath the graph. The length of the vertical leg is: $(y_2 - y_1)$, and the horizontal leg has length $(x_2 - x_1)$.

In the diagram below, $(x_1, y_1) = (2, 3)$ and $(x_2, y_2) = (6, 8)$...

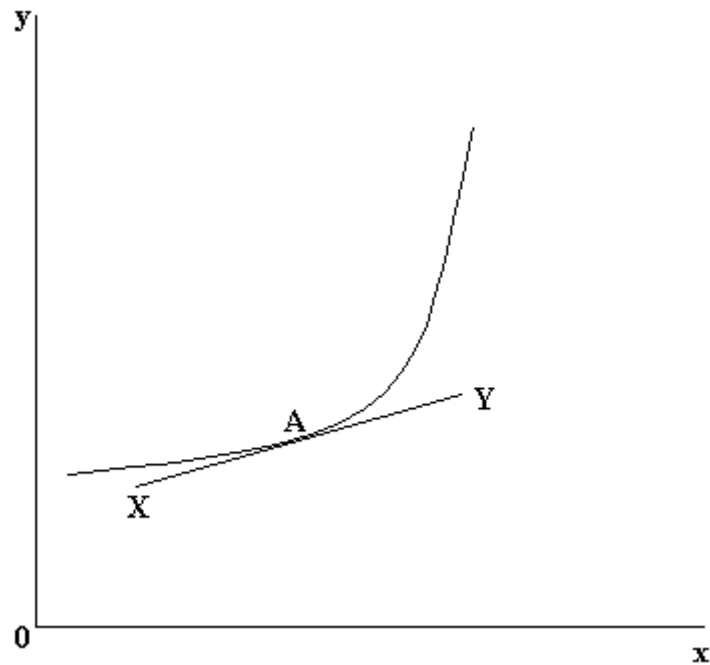


So the gradient of this line segment is:

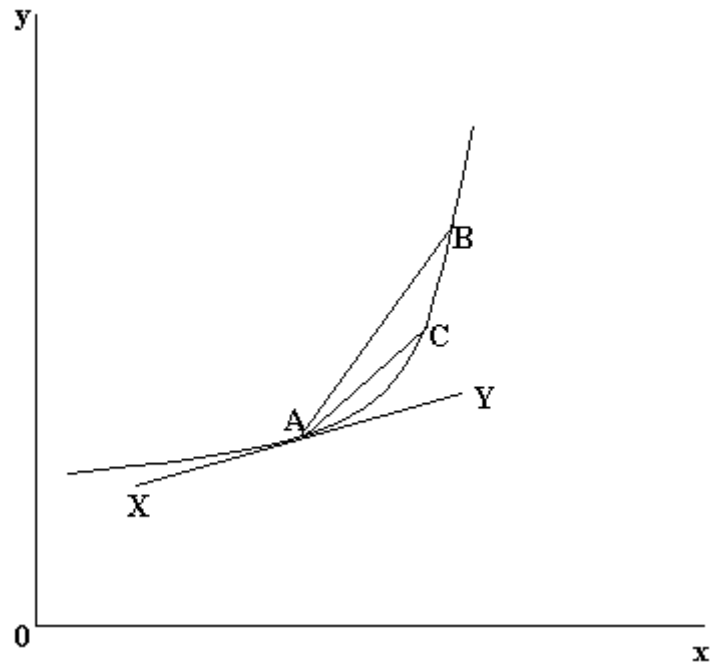
$$\text{gradient 'm'} = \frac{\Delta y}{\Delta x} = \frac{5}{4} = 1.25$$

Gradient of a curve

A curve does not have a constant gradient. At any point on a curve, the gradient is equal to the gradient of the tangent at that point (a tangent to a curve is a line touching the curve at one point only). For example, the gradient of the below curve at A is equal to the gradient of the tangent at A, which is XY.



An approximation to the gradient at any point can be found by drawing a chord. A chord joins together two points on a curve. The closer together these two points are, the closer one gets to the actual gradient of the graph at the point in question.

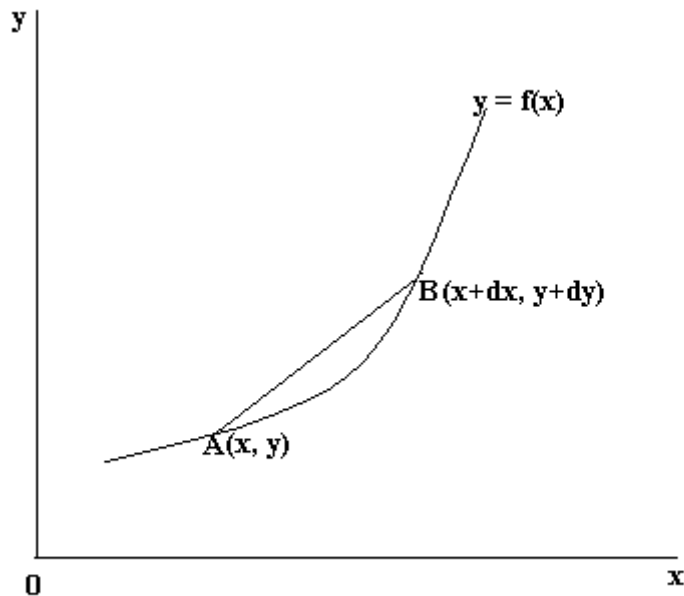


Therefore in the above diagram, AB and AC are chords. The gradient at A is closer to the gradient of AC than AB, since the chord AC is shorter. Every time one makes the chord shorter, the gradient of the chord gets closer and closer to the gradient of the curve at A. Eventually, when the chord becomes so short that it is a tangent, the gradient of the graph will equal the gradient of this tangent.

Calculus and Differentiation

Don't forget that a gradient of a curve at a point P is equal to the gradient of the tangent to the curve at that point. It turns out that we can use algebra to find out what the gradient of this tangent will be. This is called differentiation and the gradient at this point is referred to as the derivative. This gradient represents the amount by which one quantity is changing (increasing) with respect to another at a particular point.

For a curve represented by the function $y = f(x)$, the derivative of y with respect to x is written as dy/dx and is the rate at which y is changing with respect to x . We often write the derivative as $f'(x)$.



A is any point, (x, y) . To find the gradient at A, we need to find the gradient of the tangent at A. Let B be a point which is just a little further along the graph. The gradient of the chord AB is approximately the gradient of A. If the horizontal distance between A and B is called dx ('delta' x) and the vertical distance between A and B is called dy , the coordinates of B are $(x + dx, y + dy)$.

From the coordinate geometry section, we know that the gradient of a straight line joining two points is:

$$\frac{y_2 - y_1}{x_2 - x_1}, \text{ where the two points are } (x_1, y_1) \text{ and } (x_2, y_2)$$

In this case, the two points are (x, y) and $(x + dx, y + dy)$. So substituting these values into the formula, the gradient of the chord is:

$$\frac{y + dy - y}{x + dx - x} = \frac{dy}{dx} \text{ (pronounced 'delta } y \text{ by delta } x')$$

This is the gradient of the chord. The gradient of the curve is the gradient of the chord when the chord has no length- ie when it is a tangent. This will happen when $dx = 0$.

The gradient of the curve is therefore:

$$\lim_{dx \rightarrow 0} \left(\frac{dy}{dx} \right)$$

This basically means that the gradient is dy/dx as dx approaches or 'tends to' ($\rightarrow$) zero.

We can rewrite the coordinates of (x, y) as $(x, f(x))$ and the coordinates of $(x + dx, y + dy)$ as $(x + dx, f(x + dx))$, since y is a function of x ($y = f(x)$).

So the gradient of the curve is:

$$\lim_{dx \rightarrow 0} \frac{(y + dy - y)}{(x + dx - x)}$$

since $y = f(x)$ and $y + dy = f(x + dx)$:

Gradient is:

$$\lim_{dx \rightarrow 0} \frac{f(x + dx) - f(x)}{dx}$$

This is denoted by dy/dx ('dee y by dee x'). dy/dx is known as the derivative of y with respect to x .

So, in summary,

$$\frac{dy}{dx} = \lim_{dx \rightarrow 0} \frac{f(x + dx) - f(x)}{dx}$$

Example:

Find the formula for the gradient of the graph $y = x^2$.

$$\frac{dy}{dx} = \lim_{dx \rightarrow 0} \frac{(x + dx)^2 - x^2}{dx}$$

$$= \lim_{dx \rightarrow 0} \frac{x^2 + 2x dx + (dx)^2 - x^2}{dx}$$

$$= \lim_{dx \rightarrow 0} \frac{2x dx + (dx)^2}{dx}$$

The dx on the denominator cancels with those on the numerator.

Therefore $dy/dx =$

$$\lim_{dx \rightarrow 0} 2x + dx$$

When dx becomes zero, $dy/dx = 2x$.

Therefore the gradient of $y = x^2$ is $2x$.

For example, at the point $(2, 4)$, the gradient is $2x = 4$.

Using methods such as this it is possible to create a table of standard derivatives.