



LOCATION: SWANSEA

LLEOLIAD: ABERTAWE

LEVEL 5

LEFEL 5

EXAMINATION PERIOD
MAY 2026

CYFNOD ARHOLIADAU
MAI 2026

MODULE CODE: GOEE5000

ELECTRICAL POWER

SOLUTION

TIME: 2 HOURS

ANSWER ALL QUESTIONS

SHOW ALL CALCULATIONS AND WORKINGS CLEARLY

Equipment or materials required – Calculator & Formula booklet

Question One (25 Marks)

- a. The single-phase a.c. power system shown in Figure 1 operates at 50Hz with a supply voltage of 240V (RMS). The circuit consists of a load supplied through an internal source reactance and an overhead transmission line, with component values as shown. Calculate the power factor of the circuit and state whether it is leading or lagging.

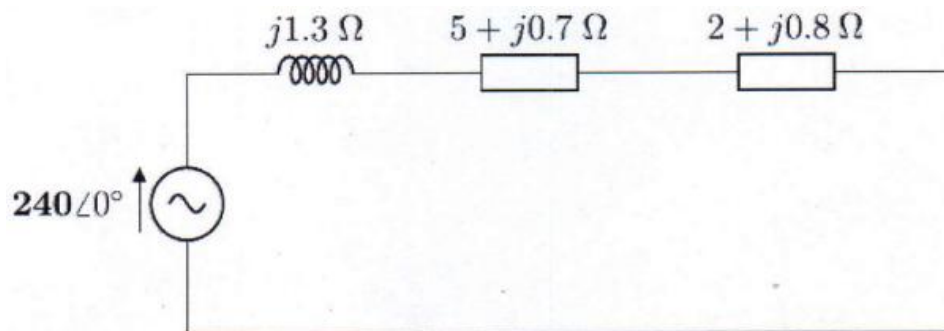


Figure 1

[5]

Solution

$$Z_T = (j1.3\ \Omega) + (5 + j0.7\ \Omega) + (2 + j0.8\ \Omega)$$

$$Z_T = 7 + j2.8\ \Omega$$

$$|Z_T| = \sqrt{7^2 + 2.8^2}$$

$$|Z_T| = 7.54\ \Omega$$

$$\phi = \tan^{-1}\left(\frac{2.8}{7}\right)$$

$$\phi = \tan^{-1}(0.4)$$

$$\phi = 21.8^\circ$$

$$Z_T = 7.54 \angle 21.8^\circ\ \Omega \quad \text{in polar form}$$

$$pf = \text{power factor of the circuit} = \cos \phi$$

$$pf = \cos(21.8^\circ)$$

$$pf = 0.93 \text{ Lagging}$$

[5]

- b. In Figure 2 below, calculate the complex apparent power (S) delivered by the source, as well as the total real power (P) and the total reactive power (Q).

[20]

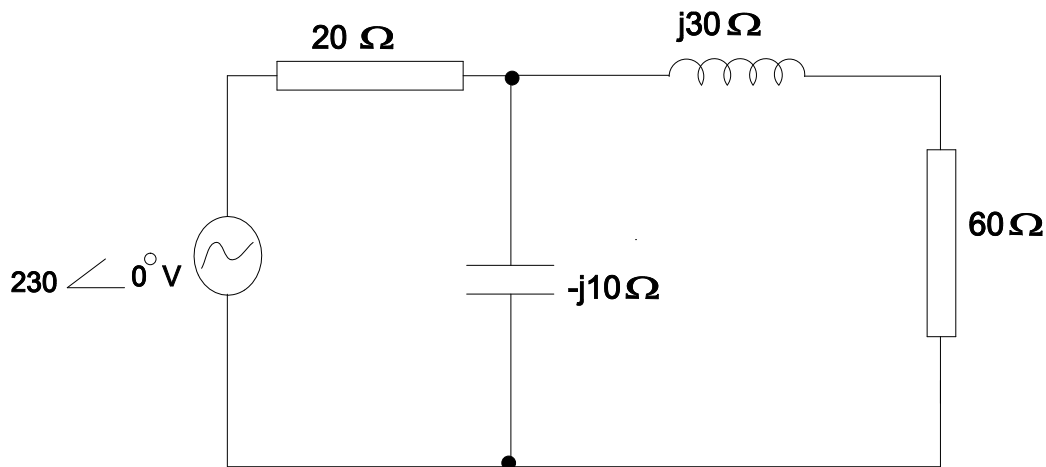


Figure 2

$$Z_C = 0 - j10 \, \Omega \quad Z_L = 60 + j30 \, \Omega \quad Z_R = 20 + j0 \, \Omega$$

$$Z_T = Z_R + \frac{Z_C Z_L}{Z_C + Z_L}$$

$$Z_T = 20 + \frac{(0 - j10)(60 + j30)}{(0 - j10) + (60 + j30)}$$

$$Z_T = 20 + \frac{-j10(60 + j30)}{60 + j20} = 20 + \frac{-j600 + 300}{60 + j20}$$

$$Z_T = 20 + \frac{300 - j600}{60 + j20} \quad \text{OR} \quad Z_T = 20 + \frac{670.8 \angle -63.4^\circ}{63.2 \angle 18.4^\circ}$$

$$Z_T = 20 + (10.6 \angle -81.8^\circ) \, \Omega \quad \text{OR} \quad Z_T = 20 + 1.5 - j10.5 \, \Omega$$

$$Z_T = 21.5 - j10.5$$

$$Z_T = 21.5 - j10.5 \, \Omega \quad \text{OR} \quad Z_T = 23.9 \angle -26.02^\circ \, \Omega$$

[10]

$$I = \frac{V}{Z_T} = \frac{230 \angle 0^\circ}{23.9 \angle -26.02^\circ} = 9.6 \angle 26^\circ \, \text{A}$$

$$I^* = 9.6 \angle -26^\circ$$

[2]

$$S = VI^* = (230\angle 0^\circ)(9.6\angle -26^\circ) = 2208\angle -26^\circ \text{ VA}$$

$$S = 2208\angle -26^\circ \text{ VA}$$

$$S = 1984 - j968 \text{ VA}$$

[4]

$$S = P - jQ$$

$$\therefore P = 1984 \text{ W}$$

[2]

$$\therefore Q = -968 \text{ VAR}$$

[2]

Question Two (25 marks)

- a. A balanced Delta (Δ) - connected load with an impedance of $20 - j15 \Omega$ is supplied by a delta (Δ) connected, positive sequence generator with a line voltage $V_{ab} = 330\angle 0^\circ \text{ V}$ as shown in Figure 3 below. Calculate:

1. The phase voltages V_{AB} , V_{BC} and V_{CA} [2]
2. The phase currents I_{AB} , I_{BC} and I_{CA} [6]
3. The line currents I_a , I_b and I_c [2]
4. The per phase real power dissipated in the load [3]

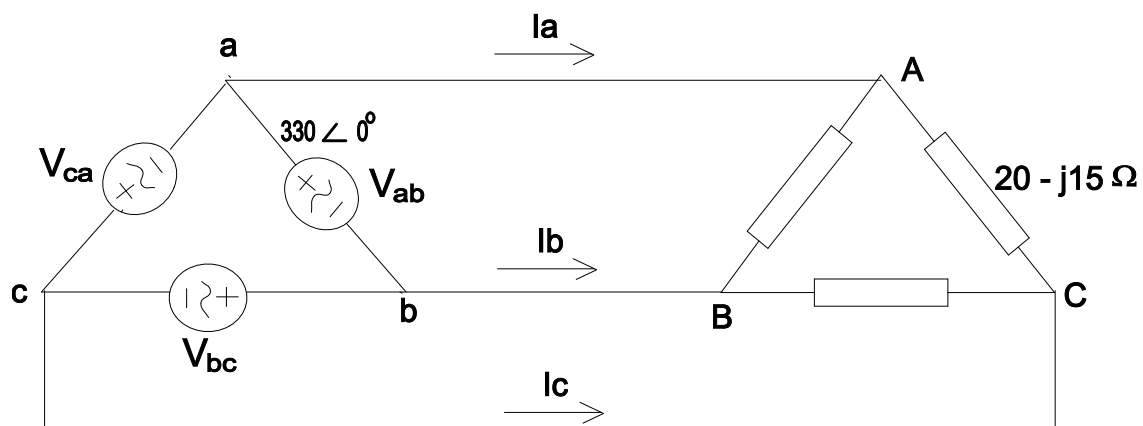


Figure 3

Solutions

1. The phase voltages V_{AB}, V_{BC}, V_{CA}

$$V_{AB} = V_{ab} = 330 \angle 0^\circ \text{ Volt}$$

$$V_{AB} = 330 \angle 0^\circ \text{ Volt}$$

$$V_{BC} = 330 \angle -120^\circ \text{ Volt}$$

$$V_{CA} = 330 \angle 120^\circ \text{ Volt} \quad [2]$$

2. The phase currents I_{AB}, I_{BC}, I_{CA}

$$V_{AB} = V_{ab} = 330 \angle 0^\circ \text{ Volt}$$

$$Z_{\Delta} = 20 - j15 \Omega = 25 \angle -36.9^\circ \Omega$$

$$I_{AB} = \frac{V_{AB}}{Z_{\Delta}} = \frac{V_{ab}}{Z_{\Delta}} = \frac{330 \angle 0^\circ}{25 \angle -36.9^\circ} = 13.2 \angle 36.9^\circ \text{ A}$$

$$I_{BC} = \frac{V_{BC}}{Z_{\Delta}} = \frac{V_{bc}}{Z_{\Delta}} = \frac{330 \angle -120^\circ}{25 \angle -36.9^\circ} = 13.2 \angle -83.1^\circ \text{ A}$$

$$I_{CA} = \frac{V_{CB}}{Z_{\Delta}} = \frac{V_{ca}}{Z_{\Delta}} = \frac{330 \angle 120^\circ}{25 \angle -36.9^\circ} = 13.2 \angle 156.9^\circ \text{ A} \quad [6]$$

3. The line currents I_a, I_b, I_c

$$I_a = I_{AB} \sqrt{3} \angle -30^\circ$$

$$I_a = 13.2 \times \sqrt{3} \angle 36.9^\circ - 30^\circ$$

$$I_a = 22.86 \angle 6.9^\circ \text{ A}$$

$$I_b = I_{BC} \sqrt{3} \angle -30^\circ$$

$$I_b = 13.2 \times \sqrt{3} \angle -83.1^\circ - 30^\circ$$

$$I_b = 22.86 \angle -113.1^\circ \text{ A}$$

$$I_c = I_{CA} \sqrt{3} \angle -30^\circ$$

$$I_c = 13.2 \times \sqrt{3} \angle 156.9^\circ - 30^\circ$$

$$I_c = 22.86 \angle 126.9^\circ \text{ A} \quad [2]$$

- 4.

$$P_{\text{phase}} = V_{\text{Phase}} I_{\text{phase}} \cos \phi = (330)(13.2) \cos (36.9^\circ)$$

$$P_{\text{phase}} = 3484.8 \text{ W} \quad [3]$$

- b. The balanced three-phase circuit shown in Figure 4 consists of a three-phase Y-connected source supplying a balanced load through identical line impedances. The phase voltage of the source is $V_{an} = 220 \angle 0^\circ$ and each line has an impedance of

$Z_{line} = 6 - j2 \Omega$. The load is balanced, with each phase having an impedance of $Z_{load} = 12 + j8 \Omega$. Calculate:

1. The phase voltages V_{bn} and V_{cn} [2]
2. The line voltages V_{AB} , V_{BC} and V_{CA} [2]
3. The phase currents I_{AN} , I_{BN} and I_{CN} [3]
4. The line currents I_a , I_b and I_c [2]
5. Calculate the complex power absorbed by total three-phase load and state the total real and reactive power. [3]

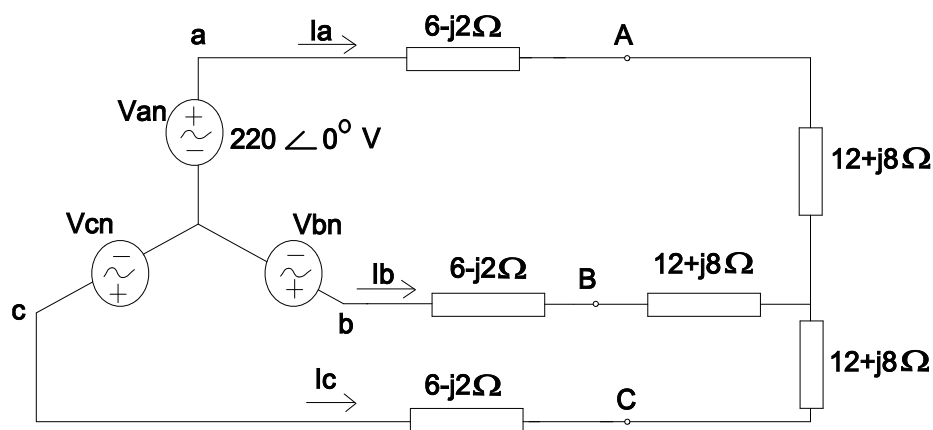


Figure 4

Solutions

1. The phase voltages

$$\begin{aligned} V_{an} &= 220 \angle 0^\circ \text{ Volt} \\ V_{bn} &= 220 \angle -120^\circ \text{ Volt} \\ V_{cn} &= 220 \angle 120^\circ \text{ Volt} \end{aligned}$$

2. The line voltages

$$\begin{aligned} V_{AB} &= \sqrt{3} V_{an} \angle 30^\circ \\ V_{AB} &= 220\sqrt{3} \angle 30^\circ \\ V_{AB} &= 381 \angle 30^\circ \text{ volt} \end{aligned}$$

$$\begin{aligned} V_{BC} &= V_{AB} \angle (\phi - 120^\circ) \\ V_{BC} &= 381 \angle (30^\circ - 120^\circ) \end{aligned}$$

$$\mathbf{V_{BC} = 381 \angle - 90^{\circ} \text{ volt}}$$

$$V_{CA} = V_{AB} \angle (\emptyset + 120^{\circ})$$

$$V_{CA} = 381 \angle (30 + 120^{\circ})$$

$$\mathbf{V_{CA} = 381 \angle 150^{\circ} \text{ volt}}$$

3.

$$I_{AN} = \frac{V_{an}}{Z_{line} + Z_{load}}$$

$$I_{AN} = \frac{\mathbf{220 \angle 0^{\circ}}}{18 + j6}$$

$$I_{AN} = \frac{\mathbf{220 \angle 0^{\circ}}}{18.97 \angle 18.43^{\circ}}$$

$$\mathbf{I_{AN} = 11.59 \angle - 18.43^{\circ} \text{ A}}$$

$$I_{BN} = I_{AN} \angle (\emptyset - 120^{\circ})$$

$$I_{BN} = 11.59 \angle (-18.43^{\circ} - 120^{\circ})$$

$$\mathbf{I_{BN} = 11.59 \angle - 138.43^{\circ} \text{ A}}$$

$$I_{CN} = I_{AN} \angle (\emptyset + 120^{\circ})$$

$$I_{CN} = 11.59 \angle (-18.43^{\circ} + 120^{\circ})$$

$$\mathbf{I_{CN} = 11.59 \angle 101.57^{\circ} \text{ A}}$$

4.

$$\mathbf{I_a = I_{AN} , \quad I_b = I_{AB} \text{ and } I_c = I_{AC}}$$

$$\mathbf{I_a = 11.59 \angle - 18.43^{\circ} \text{ A}}$$

$$\mathbf{I_b = 11.59 \angle - 138.43^{\circ} \text{ A}}$$

$$\mathbf{I_c = 11.59 \angle 101.57^{\circ} \text{ A}}$$

5.

$$S = 3 V I^* = 3 (V_{an})(11.59 \angle - 18.43^{\circ})^*$$

$$S = 3 (220 \angle 0^{\circ})(11.59 \angle - 18.43^{\circ})^*$$

$$S = 7649.4 \angle 18.43^{\circ} \text{ VA}$$

$$S = 7257 + j2413.3 \text{ VA}$$

$$\mathbf{P = 7257 \text{ W} \quad \text{and} \quad Q = 2413.3 \text{ VAR}}$$

Question Three (25 marks)

- a. A single-phase inductive load draws 120 kVA at a lagging power factor of 0.65 from a 240 V, 50 Hz supply. A capacitor is connected in parallel with the load to improve the overall power factor to 0.9 lagging. Calculate the capacitance value (in mF) required to achieve the desired power factor.

[20]

$$pf_1 = 0.65$$

$$pf_1 = \cos \phi_1$$

$$\cos \phi_1 = 0.65$$

$$\phi_1 = \cos^{-1} 0.65 = 49.46^\circ$$

$$S_1 = 120 \text{ kVA}$$

$$Q_1 = S_1 \sin \phi_1$$

$$Q_1 = 120 \sin(49.46^\circ)$$

$$Q_1 = 120 \times 0.760 = 91.2 \text{ kVAR}$$

$$\mathbf{Q_1 = 91.2 \text{ kVAR} \quad [5]}$$

$$P = S_1 \cos(\phi_1)$$

$$P = 120 \times \cos 49.46^\circ = 78 \text{ kW}$$

$$\mathbf{P = 78 \text{ kW} \quad [3]}$$

$$\text{the new power factor } pf_2 = 0.9$$

$$pf_2 = \cos \phi_2$$

$$\cos \phi_2 = 0.9$$

$$\phi_2 = \cos^{-1} 0.9 = 25.84^\circ$$

$$pf_2 = \frac{P}{S_2}$$

$$S_2 = \frac{P}{pf_2} = \frac{78}{0.9}$$

$$\mathbf{S_2 = 86.7 \text{ kVA} \quad [4]}$$

$$Q_2 = S_2 \sin \phi_2$$

$$Q_2 = 86.7 \sin \phi_2 = 86.7 \times \sin 25.84^\circ$$

$$Q_2 = 37.8 \text{ kVAR} \quad [2]$$

$$Q = Q_1 - Q_2$$

$$Q = 91.2 - 37.8$$

$$Q = 53.4 \text{ kVAR} \quad [2]$$

$$Q_{\text{capacitor}} = \frac{(V_{\text{capacitor}})^2}{X_C}$$

$$53.4 \times 10^3 = \frac{(240)^2}{X_C}$$

$$X_C = 1.080 \, \Omega \quad [2]$$

$$X_C = \frac{1}{2\pi f C}$$

$$C = \frac{1}{2\pi f X_C} = \frac{1}{2\pi \times 50 \times 1.08}$$

$$C = 2.95 \text{ mF} \quad [2]$$

- b. Explain the significance of maintaining a high power factor in industrial electrical installations. In your answer, discuss the causes of low power factor and describe its effects on system losses, equipment loading, and operating costs.

[5]

1. Power factor is the ratio of real power (kW) to apparent power (kVA). A high-power factor ensures efficient use of electrical power in industrial systems.
2. Low power factor is mainly caused by inductive loads such as induction motors, transformers, and fluorescent lighting.
3. A low power factor increases line current, resulting in higher I^2R losses in cables and equipment.
4. It causes overloading of generators, transformers, and conductors, reducing their effective capacity and lifespan.
5. Low power factor leads to higher operating costs due to utility penalties and the need for larger-rated electrical equipment.

Question Four (25 marks)

- a. Consider a single-phase power system comprising a generator, transmission line and load as shown in Figure 5. The system operates with a base apparent power of $S_B = 20\text{kVA}$, and an applied voltage at the generator end of 400V . The system includes two transformers: the first transformer has a turns ratio of $a_1 = 0.2$ and the second transformer has a turns ratio of $a_2 = 10$. The load impedance is given by $Z_{\text{Load}} = 10\angle 30^\circ \Omega$ and the transmission line impedance is $Z_{\text{Line}} = 18 + j36 \Omega$.

Given this information, perform the following tasks:

1. Using the specified system base apparent power, determine the base quantities in each region of the network (generator side, transmission line side, and load side). This should include the base voltage, base current, base impedance, and base apparent power. [8]
2. Express all system components in per-unit form and draw the corresponding per-unit equivalent circuit referred to a common base. [4]
3. Using the per-unit equivalent circuit, determine the real power absorbed by the load under the given operating conditions. [3]

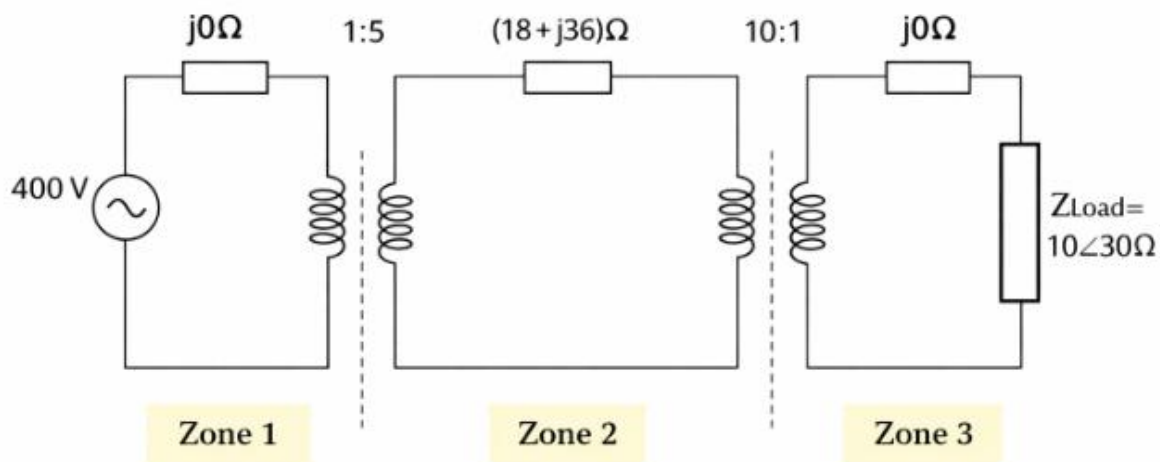


Figure 5

Solution

Generator Region (Zone 1)

$$\begin{aligned}
 V_{base1} &= 400 \text{ V}, S_B = 20 \text{ kVA} \\
 I_{base1} &= \frac{S_B}{V_{base1}} = \frac{20000}{400} = 50 \text{ A} \\
 Z_{base1} &= \frac{V_{base1}}{I_{base1}} = \frac{400}{50} = 8 \Omega
 \end{aligned}$$

Transmission Line Region (Zone 2)

The turns ratio of the first transformer is

$$a_1 = 0.20$$

Therefore, the base voltage in the transmission line region is

$$V_{base2} = \frac{V_{base1}}{a_1} = \frac{400}{0.20} = 2000 \text{ V}$$

The other base quantities are

$$I_{base2} = \frac{S_B}{V_{base2}} = \frac{20000}{2000} = 10 \text{ A}$$
$$Z_{base2} = \frac{V_{base2}}{I_{base2}} = \frac{2000}{10} = 200 \Omega$$

Load Region (Zone 3)

The turns ratio of the second transformer is

$$a_2 = 10$$

Therefore, the base voltage in the load region is

$$V_{base3} = \frac{V_{base2}}{a_2} = \frac{2000}{10} = 200 \text{ V}$$

The other base quantities are

$$I_{base3} = \frac{S_B}{V_{base3}} = \frac{20000}{200} = 100 \text{ A}$$
$$Z_{base3} = \frac{V_{base3}}{I_{base3}} = \frac{200}{100} = 2 \Omega$$

[8]

Per-Unit Conversion

To convert a power system to a per-unit system, each component must be divided by its base value in its region.

Generator per-unit voltage

$$V_{G,pu} = \frac{400 \angle 0^\circ}{400} = 1 \angle 0^\circ \text{ pu}$$

Transmission line per-unit impedance

$$Z_{line,pu} = \frac{18 + j36}{200} = 0.09 + j0.18 \text{ pu}$$

Load per-unit impedance

$$Z_{load,pu} = \frac{10 \angle 30^\circ}{2}$$
$$Z_{load,pu} = 5 \angle 30^\circ = 4.33 + j2.50 \text{ pu}$$

Per-Unit Equivalent Circuit

The per-unit equivalent circuit consists of:

- A source of $1 \angle 0^\circ \text{ pu}$
- Series impedances:

$$Z_{total,pu} = (0.09 + j0.18) + (4.33 + j2.50)$$

$$Z_{total,pu} = 4.42 + j2.68 \text{ pu}$$

Per-Unit Current

$$I_{pu} = \frac{V_{pu}}{Z_{total,pu}} = \frac{1 \angle 0^\circ}{4.42 + j2.68}$$

$$|Z_{total}| = \sqrt{4.42^2 + 2.68^2} = 5.17$$

$$\angle Z_{total} = \tan^{-1} \left(\frac{2.68}{4.42} \right) = 31.2^\circ$$

$$I_{pu} = 0.193 \angle (-31.2^\circ) \text{ pu}$$

[4]

Per-Unit Power at the Load

$$P_{load,pu} = |I_{pu}|^2 R_{load,pu}$$

$$P_{load,pu} = (0.193)^2 \times 4.33 = 0.161 \text{ pu}$$

Actual load power

$$P_{load} = P_{load,pu} \times S_B$$

$$P_{load} = 0.161 \times 20000 = 3220 \text{ W}$$

[3]

b. A battery-powered electric winch used for maintenance work on the UK rail network is driven by a DC series motor supplied from a 72 V DC source.

The motor has an armature resistance of 0.12Ω and a series field resistance of 0.05Ω .

When lifting equipment, the motor draws a current of 30 A and operates at a speed of 1200 revolutions per minute (rpm). The motor's equivalent circuit is shown in Figure 6 below.

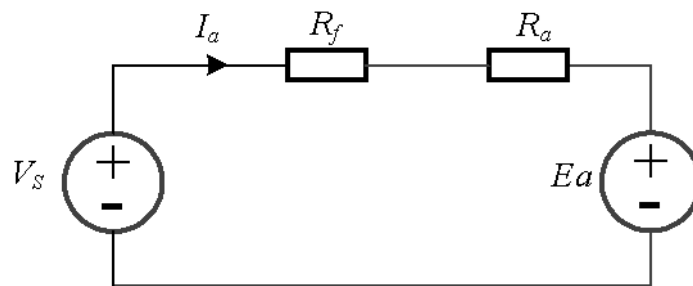


Figure 6

1. Calculate the back EMF E_a of the motor. [2]
2. Calculate the angular speed ω of the motor in radians per second. [2]
3. Calculate the mechanical power developed by the motor. [2]
4. Calculate the torque developed by the motor. [4]

Solution:

$V_S = 72 \text{ V}; R_a = 0.12 \text{ } \Omega; R_f = 0.05 \text{ } \Omega; I_a = 30 \text{ A}; \text{ speed: } 1200 \text{ rpm}$

Induced emf

$$E_a = V_S - RI_a = V_S - (R_a + R_f)I_a = 72 - (0.12 + 0.05) \times 30 = 66.9 \text{ V}$$

Angular speed:

$$\omega = \frac{n}{60} \times 2\pi = \frac{1200}{60} \times 2\pi = 125.66 \text{ rad/s}$$

Mechanic power losses are neglected, so the output power equals to the converter power:

output power

$$P_{out} = P_a = E_a I_a$$

$$P_{out} = 66.9 \times 30 = 2007 \text{ W}$$

Torque:

$$T_a = \frac{P_a}{\omega} = \frac{2007}{125.66}$$

$$\boxed{T_a = 15.97 \text{ Nm}}$$

OR:

$$E_a = K\phi \omega$$

$$K\phi = \frac{E_a}{\omega}$$

$$K\phi = \frac{66.9}{125.66}$$

$$K\phi = 0.532$$

$$T_a = K\phi I_a$$

$$T_a = 0.532 \times 30$$

$$T_a = 15.96 \text{ Nm}$$