

Q 1

(a)

$$y = 2x^2 + x - 4$$

$$y = x + 3$$

$$2x^2 + x - 4 = x + 3$$

$$2x^2 - 7 = 0$$

$$2x^2 = 7$$

$$x^2 = \frac{7}{2} = 3.5$$

$$\therefore x = \pm \sqrt{3.5}$$

$$x = \pm 1.87 \text{ (2 dec places)}$$

$$\text{when } x = 1.87 \quad y = 1.87 + 3$$

$$y = 4.87$$

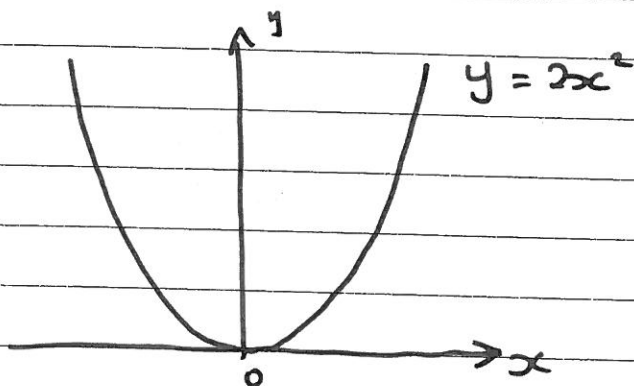
$$P (1.87, 4.87)$$

$$\text{when } x = -1.87 \quad y = -1.87 + 3$$

$$y = 1.13$$

$$Q (-1.87, 1.13)$$

b)



2 for sketch
(1 for correct shape)
(1 for axes labelled)

$$y = 2x^2$$

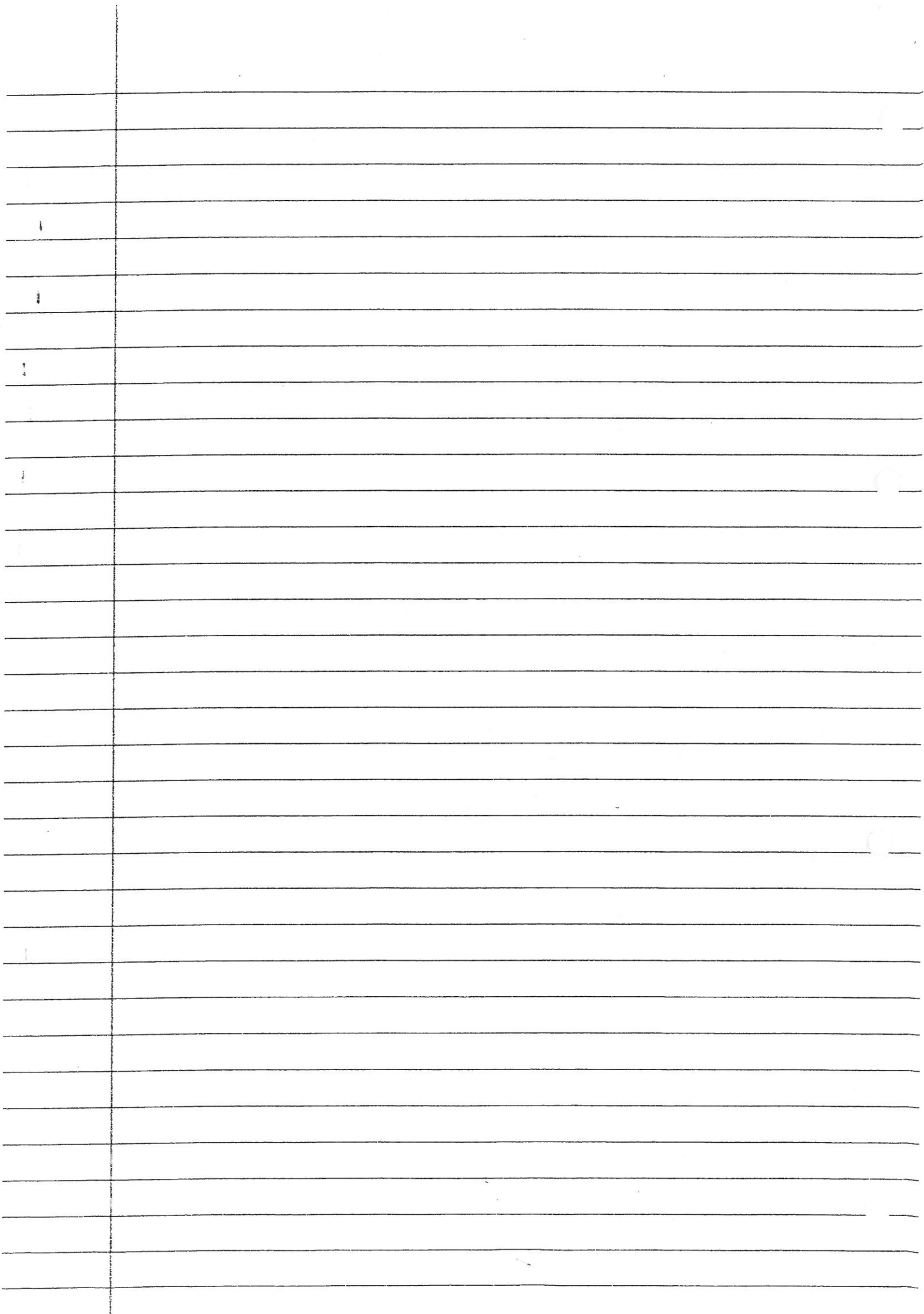
$$\frac{dy}{dx} = 4x$$

$$\text{At } x = 0 \quad \frac{dy}{dx} = 0$$

$$\text{At } x = 4 \quad \frac{dy}{dx} = 16$$

$$\text{At } x = -1 \quad \frac{dy}{dx} = -4$$

(6)



$$(c) \quad v = k(1 + \alpha i + \beta i^2)$$

$$v = k + k\alpha i + k\beta i^2$$

$$\therefore \frac{dv}{di} = 0 + k\alpha + 2k\beta i$$

(4)

$$(d) (i) \log_a x + \log_a y - 1$$

Replacing 1 by $\log_a a$

$$\log_a x + \log_a y - \log_a a$$

$$\log_a xy - \log_a a$$

$$\underline{\log_a \frac{xy}{a}}$$

(4)

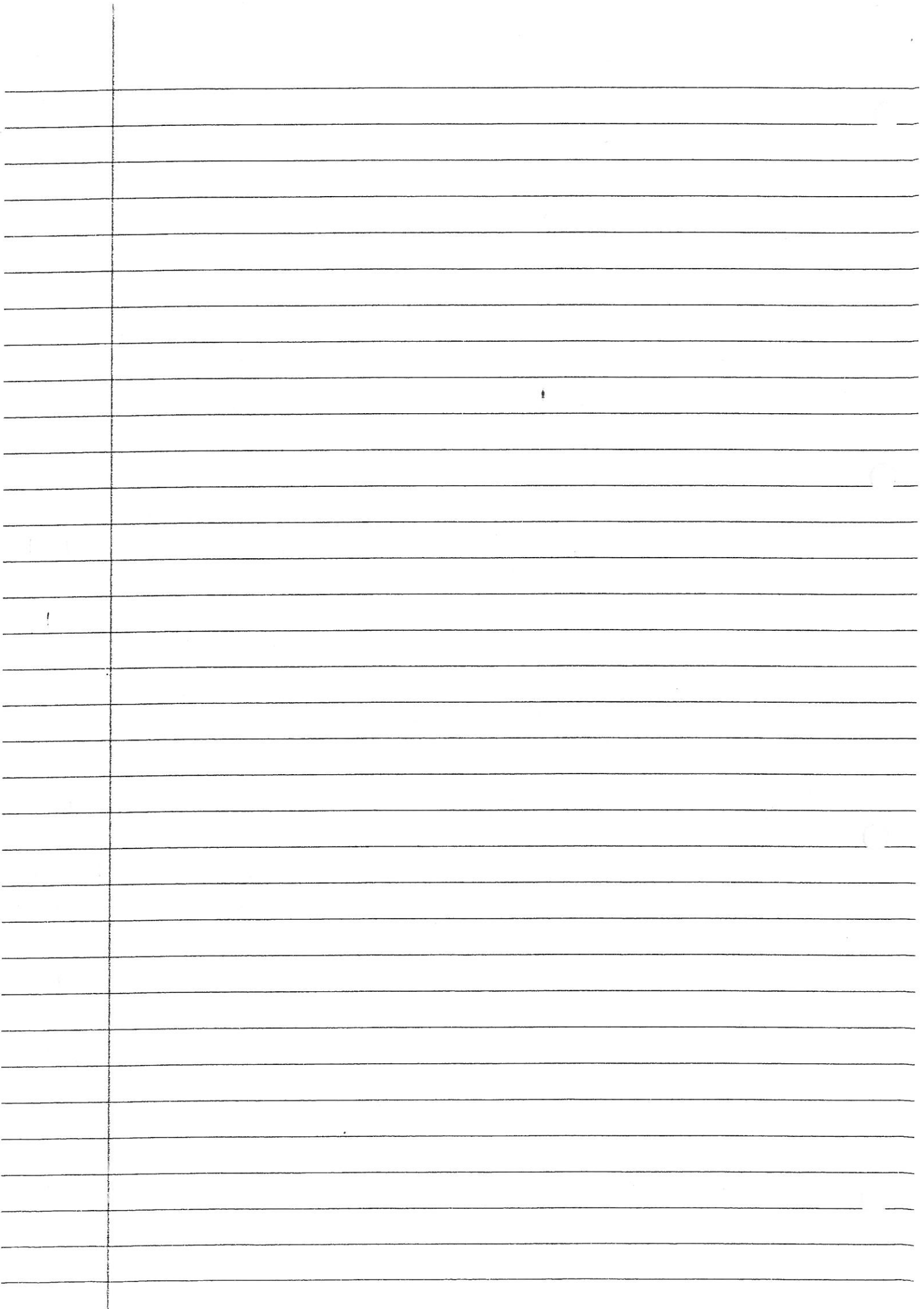
$$(ii) \quad 2\log_a x - \log_a a + \frac{1}{2}\log_a y$$

$$\log_a x^2 - \log_a a + \frac{1}{2}\log_a y$$

$$\log_a \frac{x^2}{a} + \log_a \sqrt{y}$$

$$\underline{\log_a \frac{x^2 \sqrt{y}}{a}}$$

(4)



$$(e) \quad z = 8x^2y^2 + 3x^3y^2 - 6x^3 + 10\sin 2x \cos y$$

$$\frac{\partial z}{\partial x} = 16xy^2 + 9x^2y^2 - 18x^2 + 20\cos 2x \cos y \quad \left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right)$$



$$\frac{\partial^2 z}{\partial x^2} = 16y^2 + 18xy^2 - 36x - 40\sin 2x \cos y \quad \left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right)$$

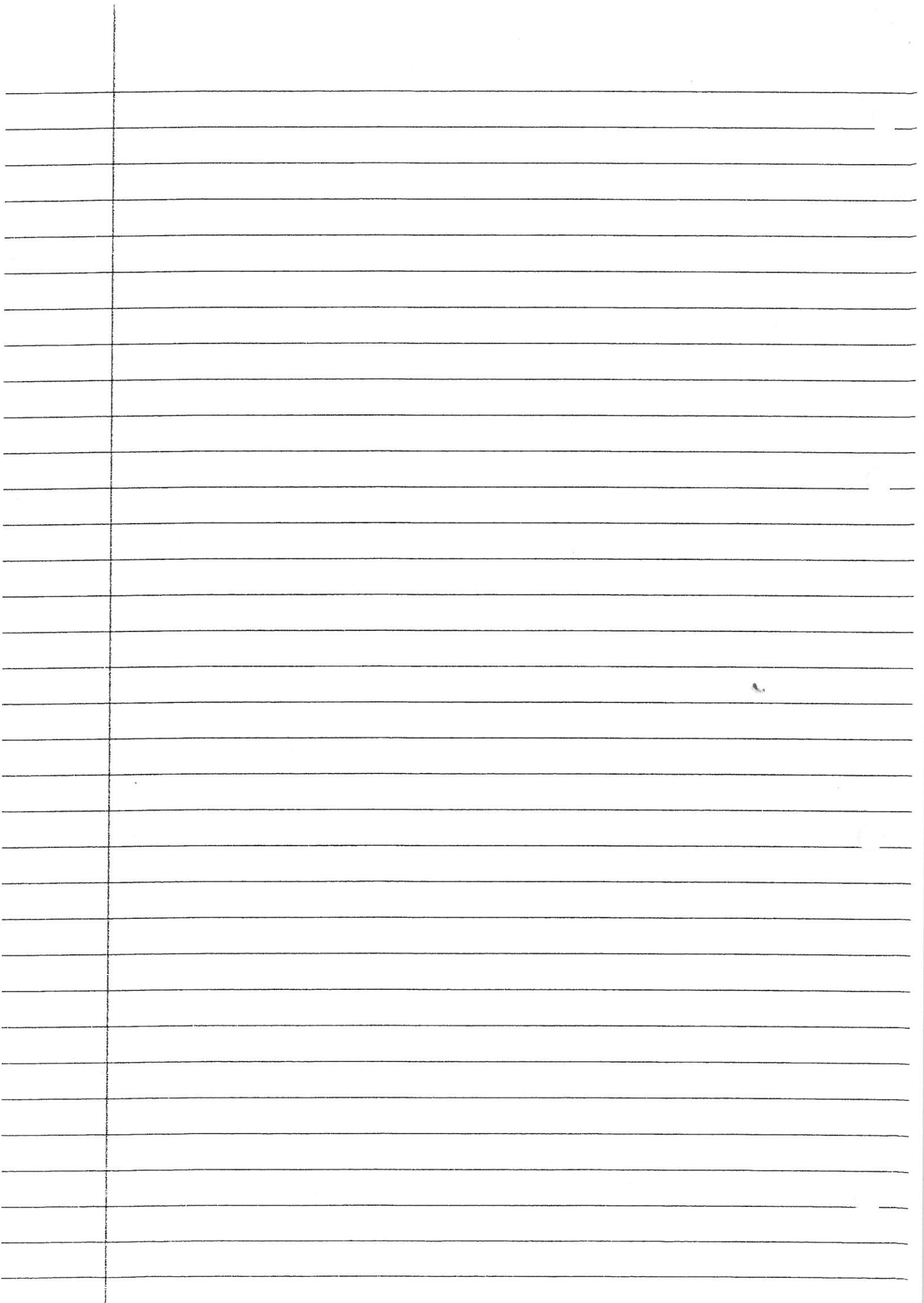
$$\frac{\partial^2 z}{\partial y \partial x} = 32xy + 18x^2y - 0 - 20\cos 2x \sin y \quad \left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right)$$

$$\frac{\partial z}{\partial y} = 16x^2y + 6x^3y - 0 - 10\sin 2x \sin y \quad \left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right)$$

$$\frac{\partial^2 z}{\partial y^2} = 16x^2 + 6x^3 - 10\sin 2x \cos y \quad \left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right)$$

$$\frac{\partial^2 z}{\partial x \partial y} = 32xy + 18x^2y - 20\cos 2x \sin y \quad \left(\frac{1}{2} \text{ for } \frac{\partial^2 z}{\partial x \partial y} \text{ \& } \frac{\partial^2 z}{\partial y \partial x} \text{ the same}\right)$$

(10)



(F) (i) $2 \sin x = -1$

$$\therefore \sin x = -\frac{1}{2}$$

1

$$\therefore x = \sin^{-1}\left(-\frac{1}{2}\right)$$

$$x = 210^\circ \text{ and } 330^\circ$$

1 1

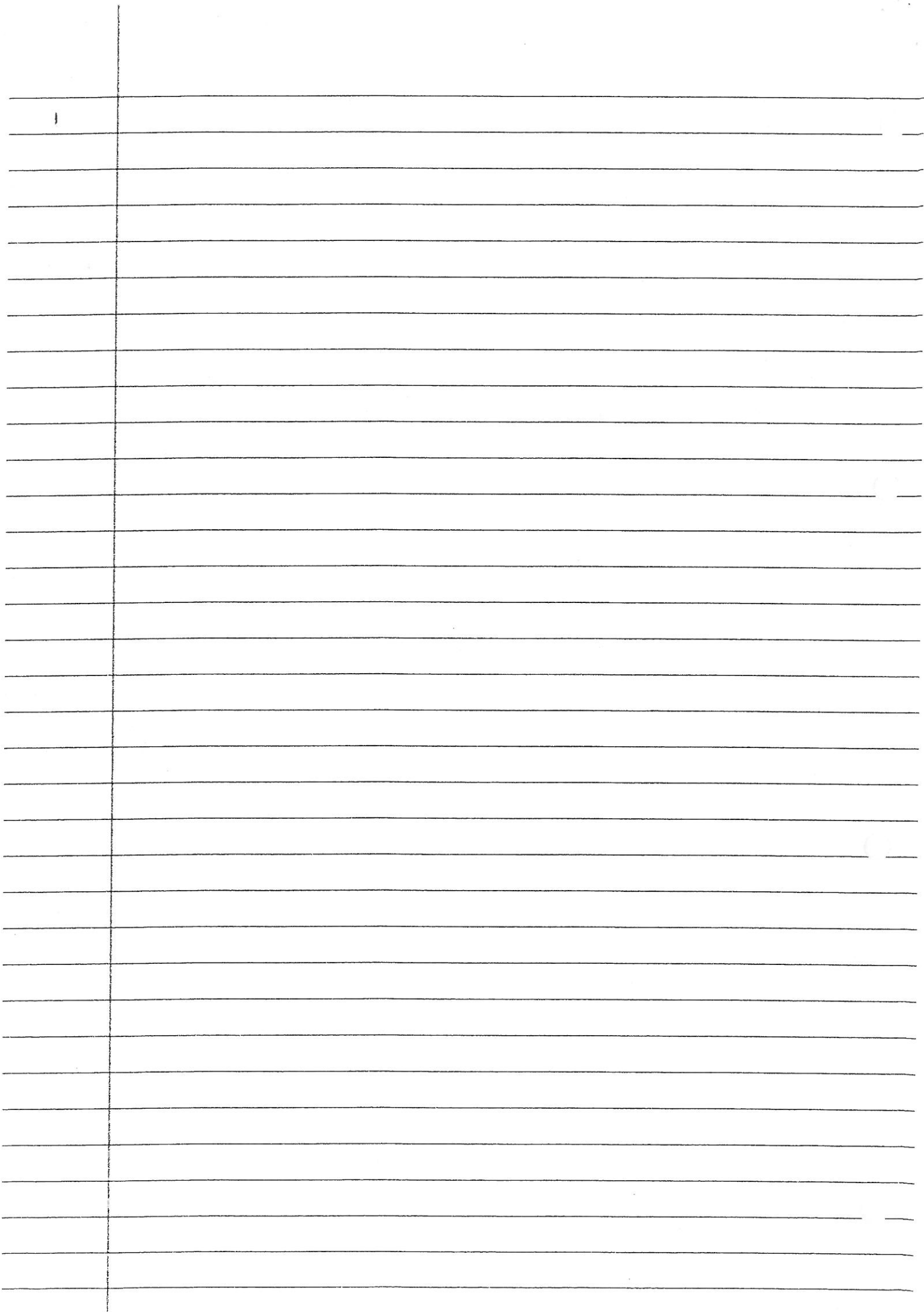
(ii) $\cos x = 0.75$

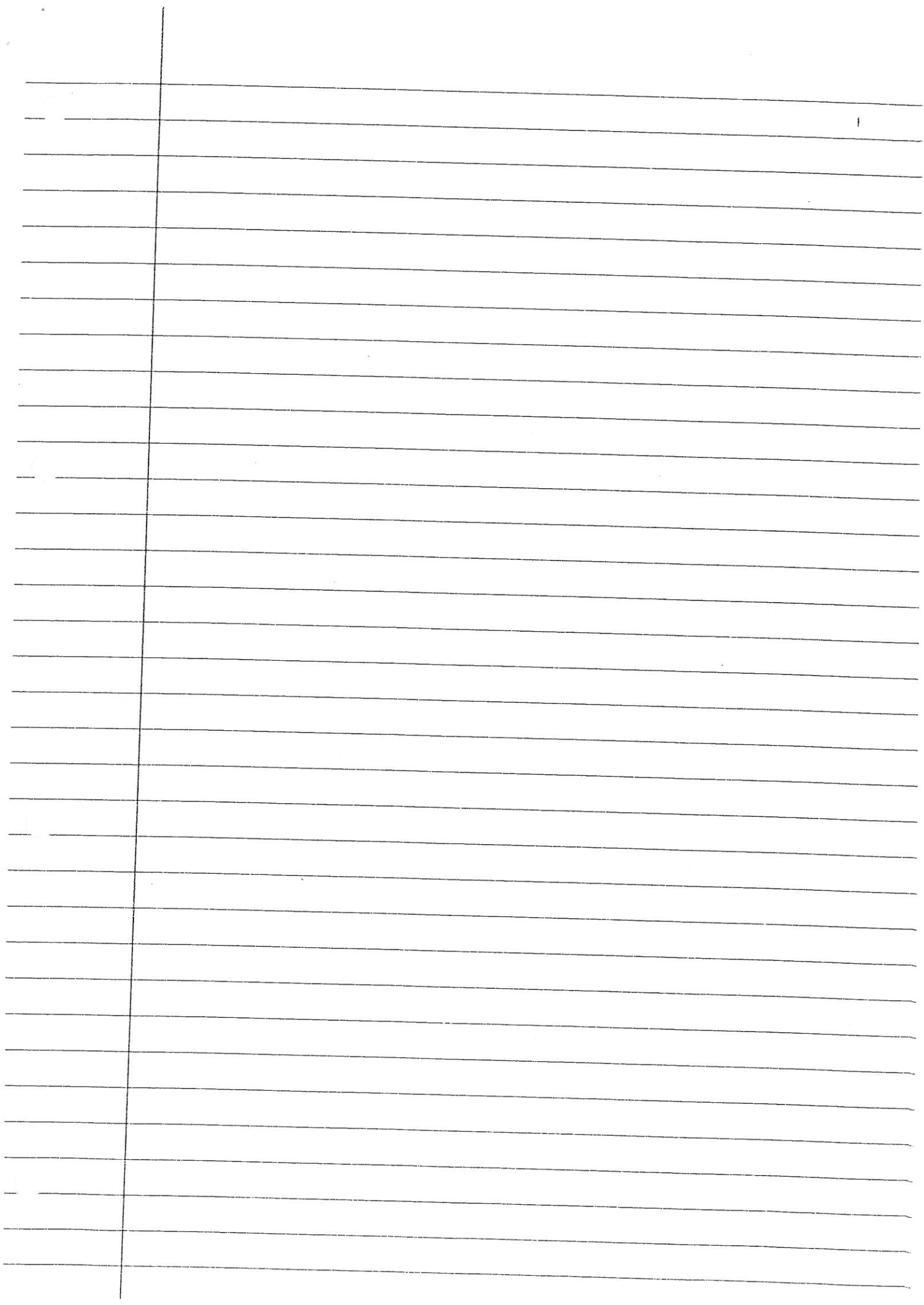
$$\therefore x = \cos^{-1} 0.75$$

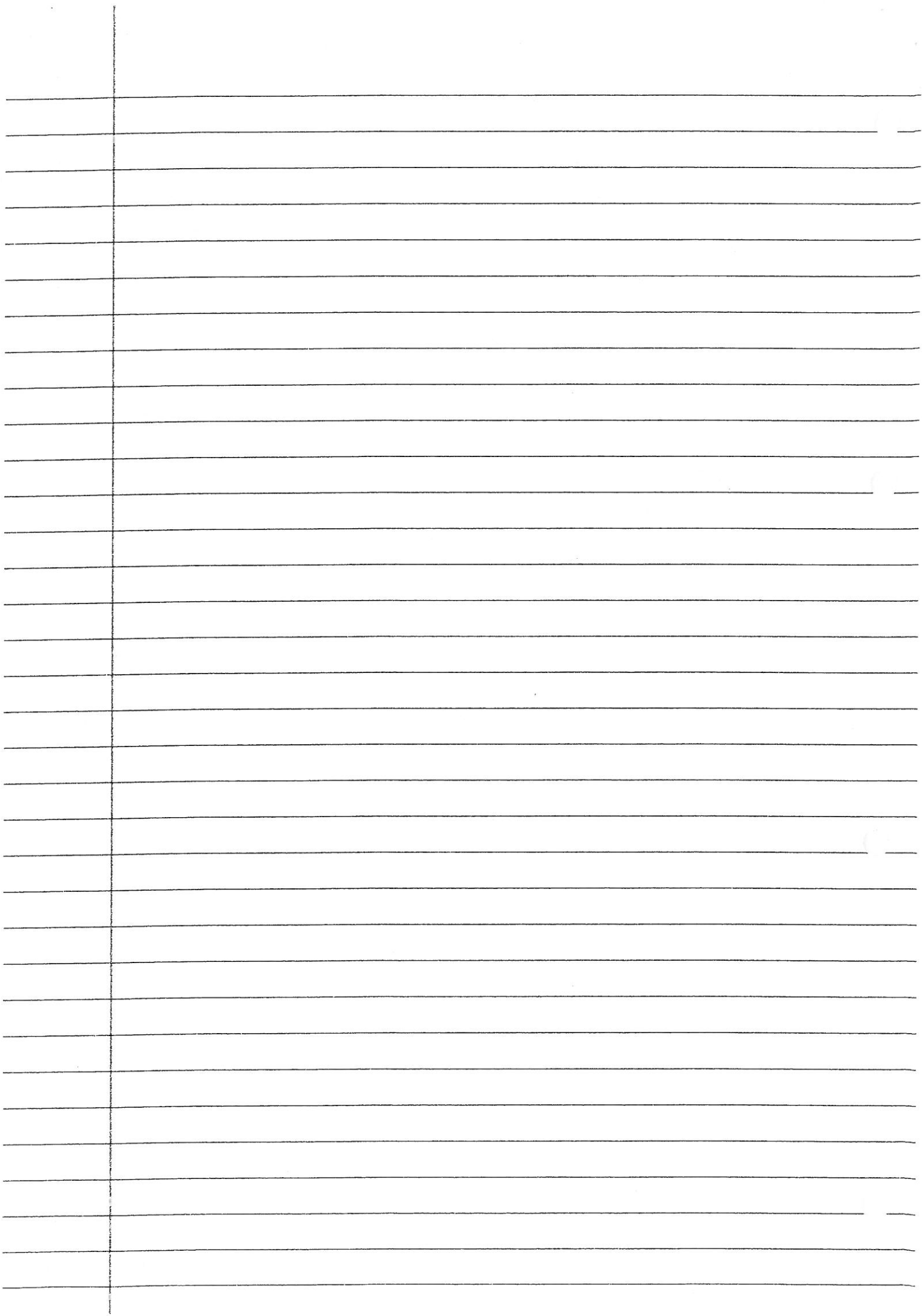
$$x = 41.41^\circ \text{ and } 318.59^\circ$$

1 1

(5)







$$Q2(a) \quad \frac{A}{(x-2)} + \frac{B}{(2x+1)}$$

$$A(2x+1) + B(x-2) = 10x-11$$

$$\text{Put } x = 2 \quad 5A = 9$$

$$A = \frac{9}{5} = 1.8$$

Put $x = 0$ (or any other value)

$$A - 2B = -11$$

$$1.8 - 2B = -11$$

$$1.8 + 11 = 2B$$

$$12.8 = 2B$$

$$\therefore B = 6.4$$

$$\text{guess} \quad \frac{1.8}{(x-2)} + \frac{6.4}{(2x+1)}$$

$$\text{i.e.} \quad 1.8(x-2)^{-1} + 6.4(2x+1)^{-1}$$

$$\text{differentiating} \quad -1.8(x-2)^{-2} - 6.4(2x+1)^{-2} \cdot 2$$

$$\text{i.e.} \quad \frac{-1.8}{(x-2)^2} - \frac{12.8}{(2x+1)^2}$$

(7)

$$(b) \quad p = \frac{W}{W - W_1}$$

$$p(W - W_1) = W$$

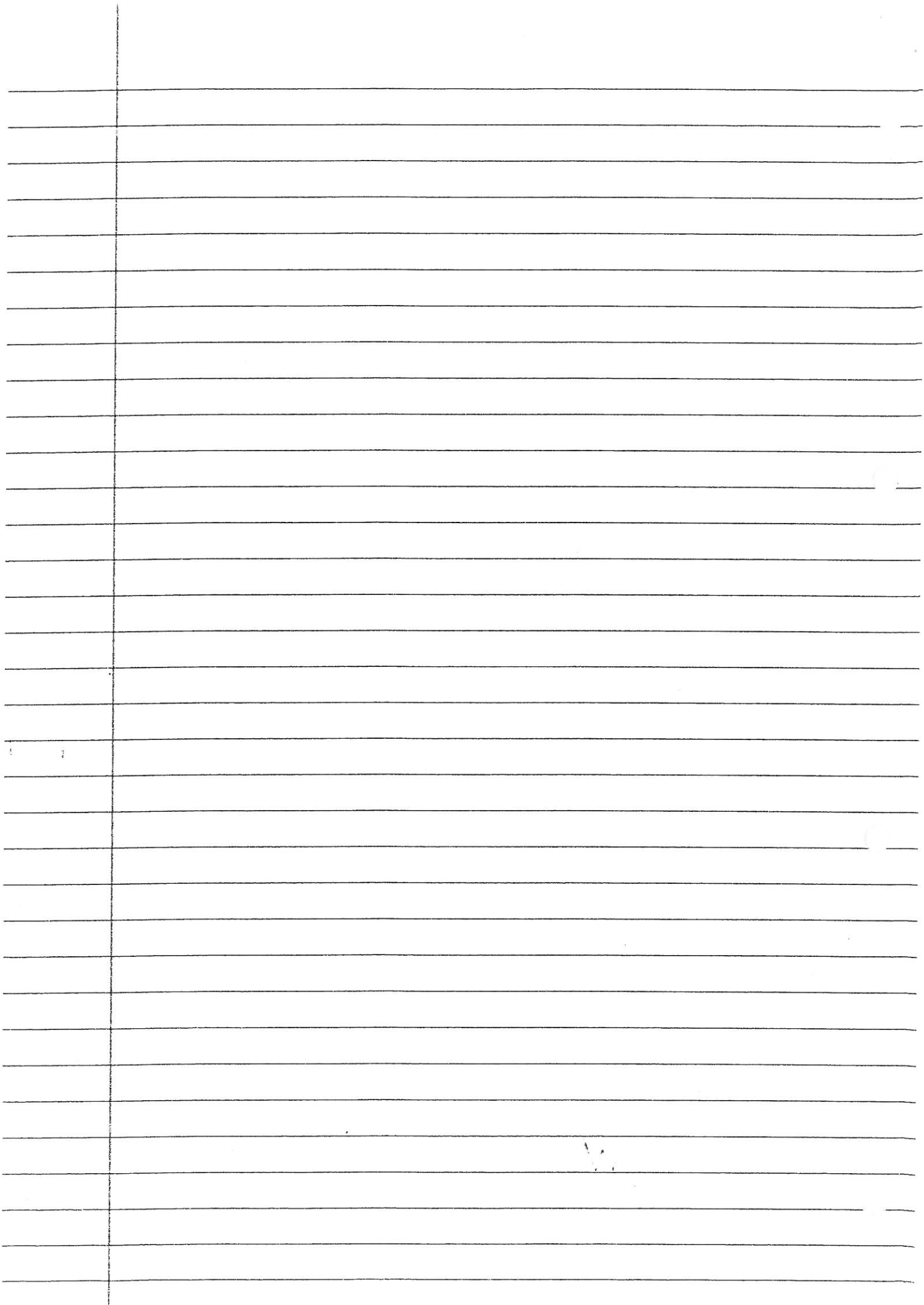
$$pW - pW_1 = W$$

$$pW - W = pW_1$$

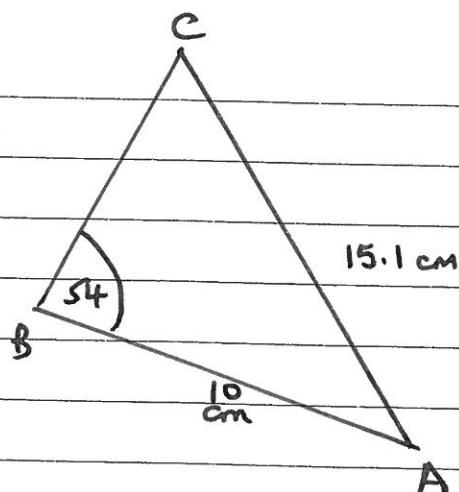
$$W(p-1) = pW_1$$

$$\therefore W = \frac{pW_1}{(p-1)}$$

(5)



(c)



(i) To find $\hat{C}$ Using The Sine Rule: |

$$\frac{10}{\sin C} = \frac{15.1}{\sin 54} \quad |$$

$$\therefore \frac{10 \times \sin 54}{15.1} = \sin C \quad |$$

$$0.5358 = \sin C$$

$$\therefore \hat{C} = 32.4^\circ \quad (1 \text{ decimal}) \quad |$$

(ii) To find $\hat{A}$

$$\hat{A} = 180^\circ - (54 + 32.4) \quad |$$

$$\hat{A} = 93.6^\circ \quad |$$

(or any other correct method)

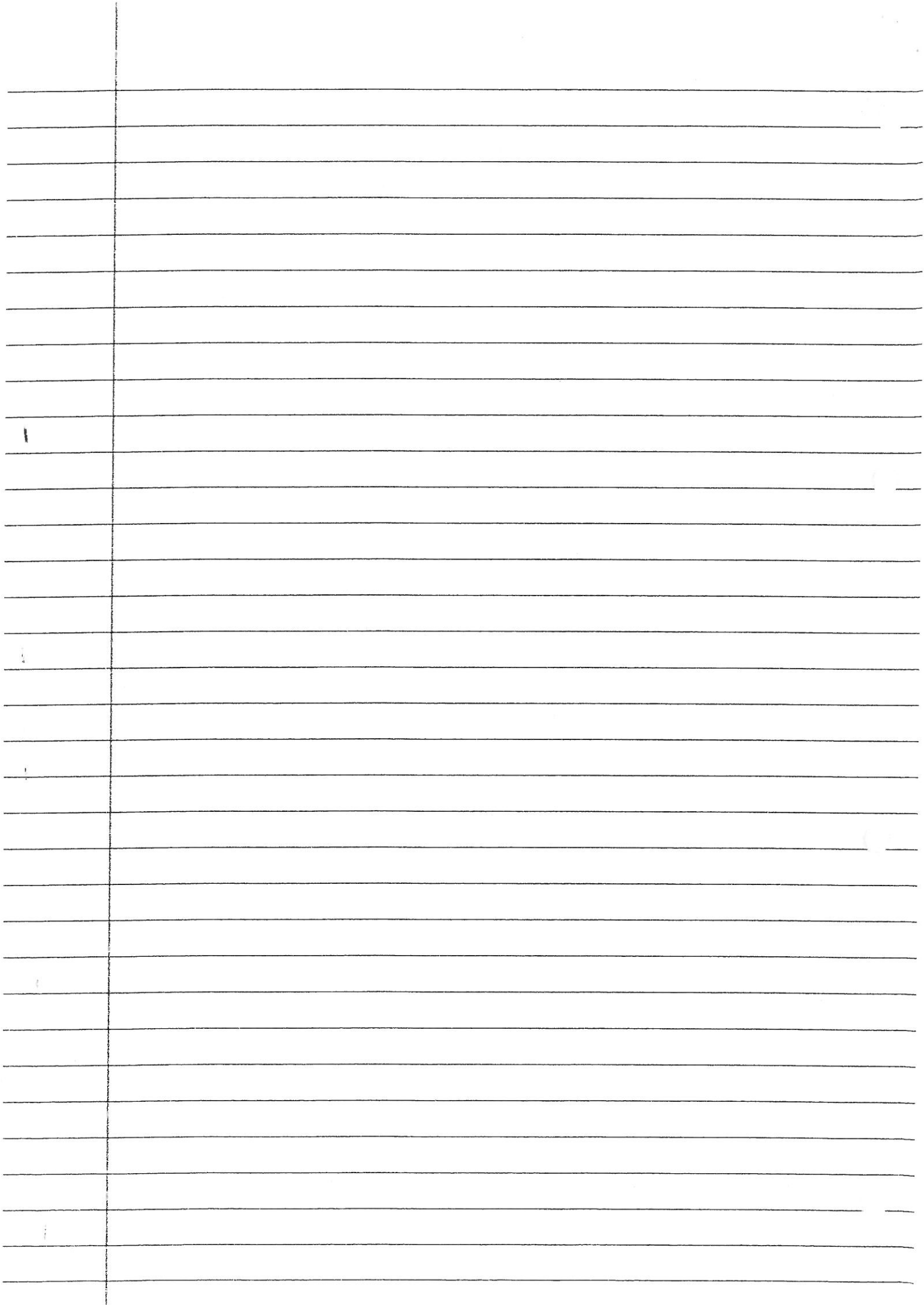
(iii) To find BC Using Sine Rule (or Cosine Rule)

$$\frac{BC}{\sin A} = \frac{15.1}{\sin 54} \quad |$$

$$BC = \frac{15.1 \times \sin 93.6^\circ}{\sin 54}$$

$$BC = \frac{15.1 \times 0.9980}{0.8090}$$

$$BC = 18.63 \text{ cm} \quad |$$



$$(d) (i) y = 10 \ln x + 8x^{1/2} + 10x^{-2}$$

$$\frac{dy}{dx} = \frac{10}{x} + 8 \cdot \frac{1}{2} x^{-1/2} - 20x^{-3}$$

(4)

$$(ii) y = (3x^2 - 5)^{1.5}$$

$$\frac{dy}{dx} = 1.5 (3x^2 - 5)^{0.5} \cdot 6x$$

(2)

$$(iii) y = \frac{2x+1}{x^2+5}$$

$$\text{let } u = 2x+1$$

$$\text{let } v = x^2+5$$

$$\frac{du}{dx} = 2$$

$$\frac{dv}{dx} = 2x$$

Using the Quotient Rule.

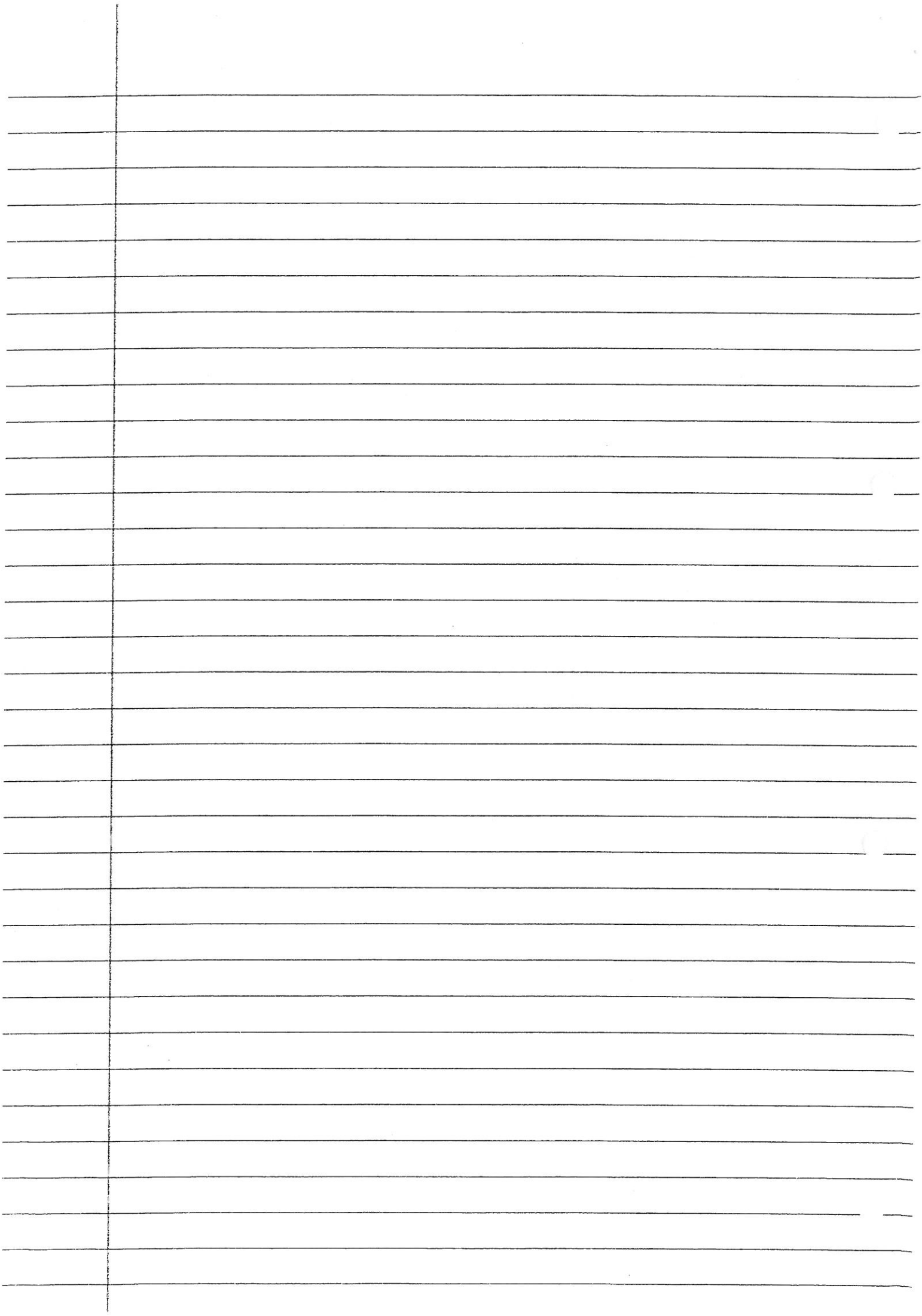
$$\frac{dy}{dx} = \frac{(x^2+5)(2) - (2x+1)(2x)}{(x^2+5)^2}$$

$$\frac{dy}{dx} = \frac{2x^2+10-4x^2-2x}{(x^2+5)^2}$$

$$\frac{-2x^2-2x+10}{(x^2+5)^2}$$

(4)

10.



$$(e) \quad (i) \quad 2x^2 + 7x - 15$$

$$(2x - 3)(x + 5)$$

1 1

$$(ii) \quad m^2 - 144$$

$$(m + 12)(m - 12)$$

1 1

$$(iii) \quad rq - rv + sq - sv$$

$$r(q - v) + s(q - v)$$

$$(r + s)(q - v)$$

1 1

(6)

$$(f) \quad \ln(3x - 5) + \ln(3x + 4) = 2\ln(3x - 4)$$

$$\cancel{\ln(3x - 5)(3x + 4)} = \cancel{\ln(3x - 4)^2}$$

$$\cancel{9x^2 + 12x - 15}x - 20 = \cancel{9x^2 - 12x - 12x}$$

$$+ 16$$

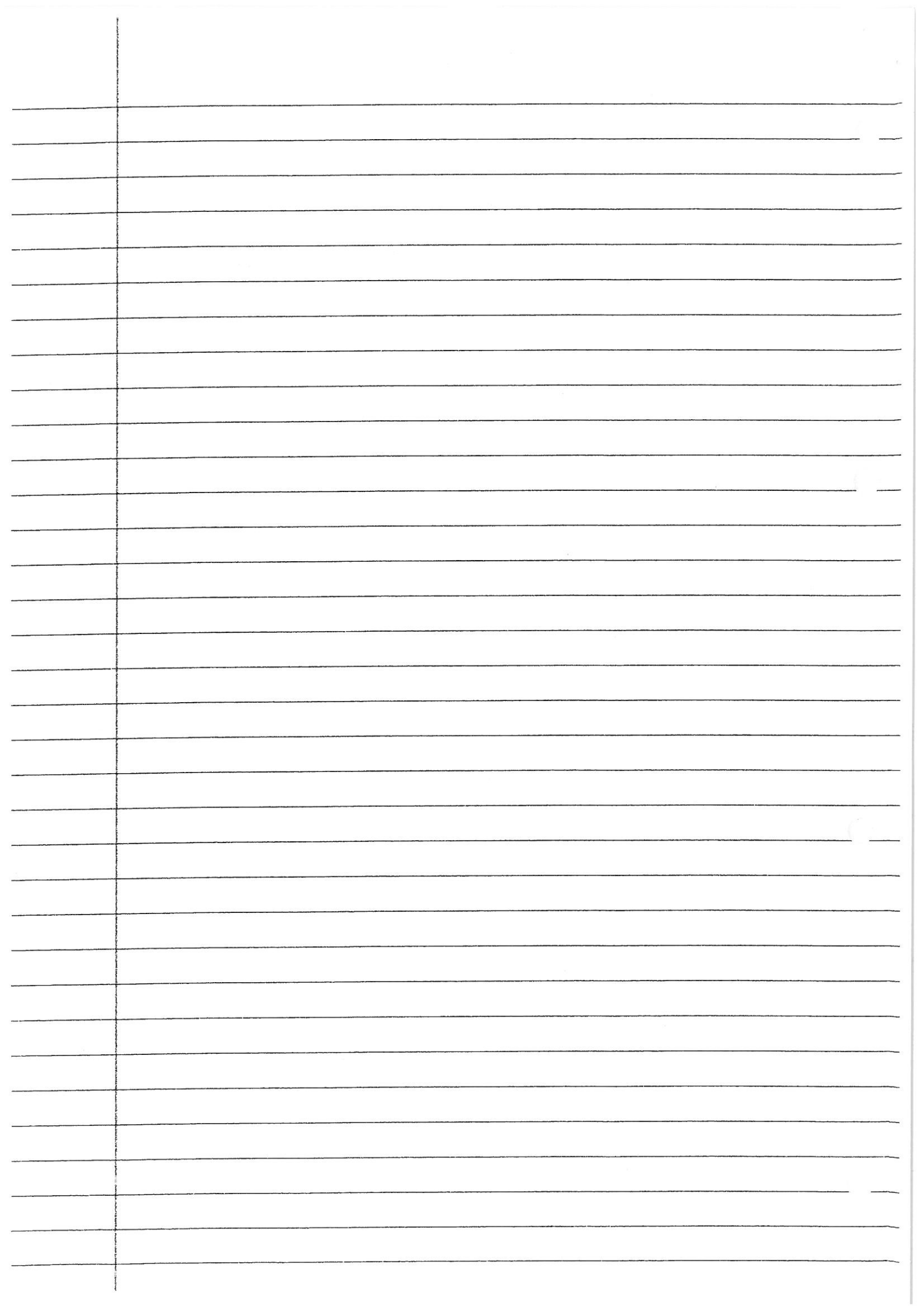
$$-3x - 20 = -24x + 16$$

$$21x = 36$$

$$\therefore x = \frac{36}{21}$$

$$x = 1.714$$

$$\underline{x = 2} \text{ to 1 sig figure.}$$



3 (a)

$$V = - \frac{\mu}{\sqrt{x^2 + y^2}}$$

Squaring both sides

$$V^2 = \frac{\mu^2}{x^2 + y^2}$$

$$V^2(x^2 + y^2) = \mu^2$$

$$V^2 x^2 + V^2 y^2 = \mu^2$$

$$V^2 y^2 = \mu^2 - V^2 x^2$$

$$\therefore y^2 = \frac{\mu^2 - V^2 x^2}{V^2}$$

$$\therefore y = \sqrt{\frac{\mu^2 - V^2 x^2}{V^2}}$$

$$\text{or } y = \frac{\sqrt{\mu^2 - V^2 x^2}}{V}$$

(6)

$$(b) \quad 3^{2+x} \times 2^{3-x} = 5^x$$

$$\log_{10} (3^{2+x} \times 2^{3-x}) = \log_{10} 5^x$$

$$\log_{10} 3^{2+x} + \log_{10} 2^{3-x} = \log_{10} 5^x$$

$$(2+x) \log_{10} 3 + (3-x) \log_{10} 2 = x \log_{10} 5$$

$$(2+x) 0.4771 + (3-x) 0.3010 = x \cdot 0.6990$$

$$0.9542 + 0.4771x + 0.9030 - 0.3010x = 0.6990x$$

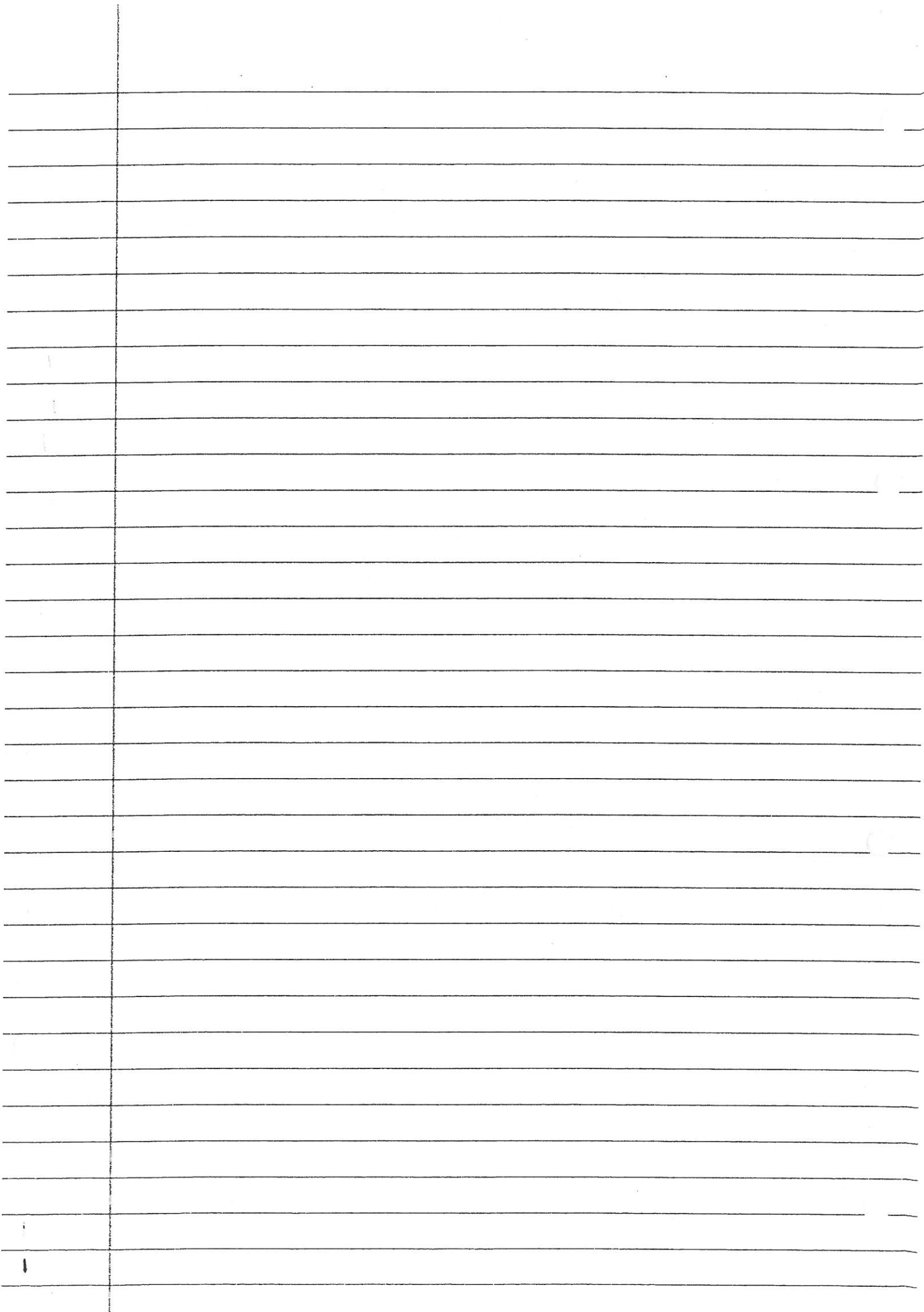
$$0.4771x - 0.3010x - 0.6990x = -0.9542 - 0.9030$$

$$-0.5229x = -1.8572$$

$$\therefore x = +3.55173$$

$$\therefore x = 3.55 \text{ (to 3 sig figs)}$$

(5)



(c)

$$y = e^{2x} \sin 2x$$

By Product Rule.

$$\frac{dy}{dx} = e^{2x} (2 \cos 2x) + \sin 2x (2e^{2x})$$

↓

By Product Rule

$$e^{2x} (-4 \sin 2x) + 2e^{2x} (2 \cos 2x)$$

By Product Rule

$$\sin 2x (4e^{2x}) + 2 \cos 2x (2e^{2x})$$

(9)

(d)

$$3x^2 - 13x + 4$$

$$(3x - 1)(x - 4)$$

so, by factoring

$$(3x - 1)(x - 4) = 0$$

$$\therefore 3x = 1$$

$$x = \frac{1}{3}$$

$$x - 4 = 0$$

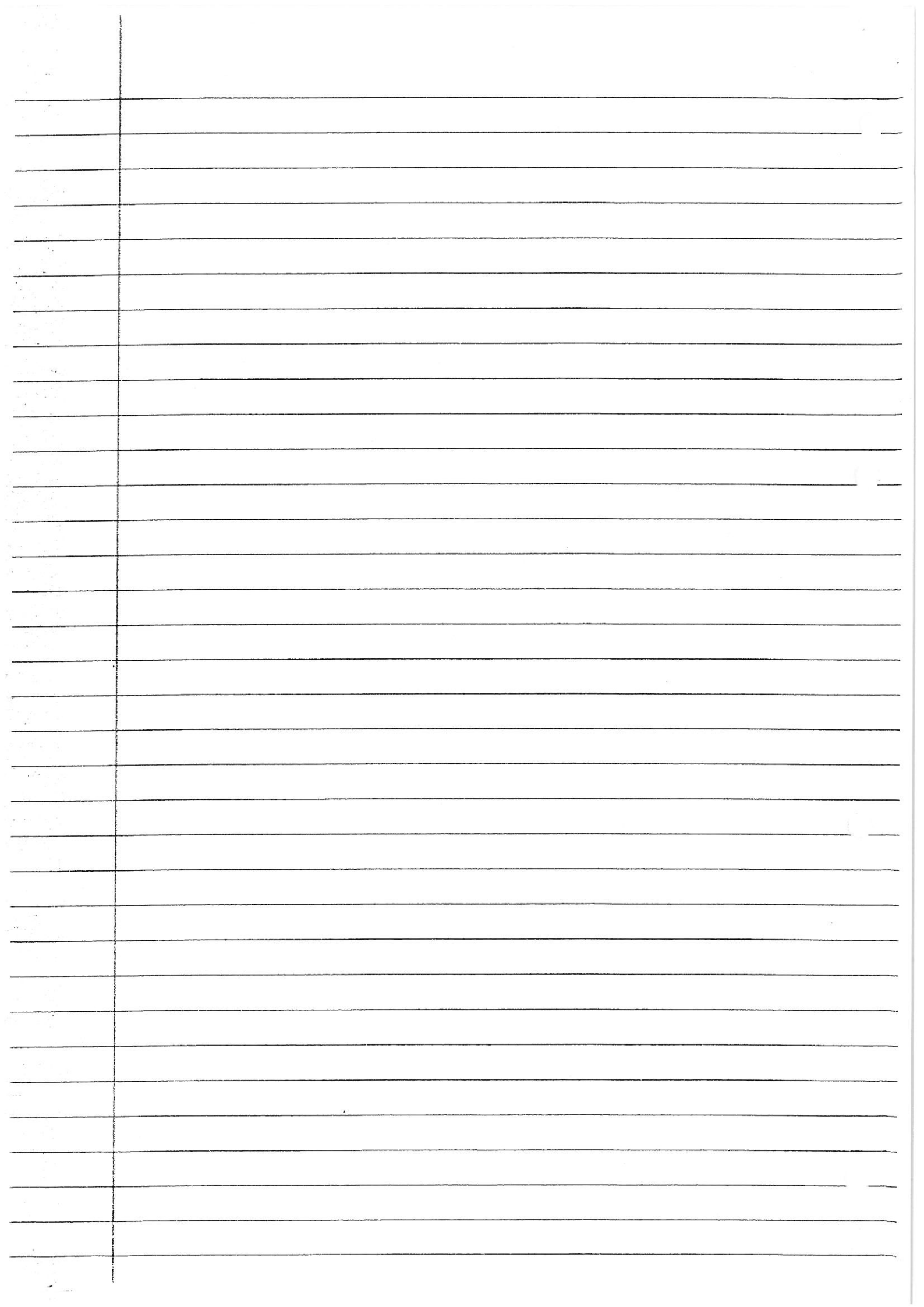
$$\therefore x = 4$$

Using formula.

(5)

$$x = \frac{13 \pm \sqrt{(-13)^2 - 4(3)(4)}}{6}$$

$$x = \frac{13 \pm \sqrt{169 - 48}}{6}$$



$$x = \frac{13 \pm \sqrt{121}}{6}$$

$$x = \frac{13 \pm 11}{6}$$

$$\text{So } x = \frac{13+11}{6} = \frac{24}{6} = 4$$

$$\text{and } x = \frac{13-11}{6} = \frac{2}{6} = \frac{1}{3}$$

(e) (i) $\frac{3x-4}{5x+9} = \frac{2}{3}$

$$3(3x-4) = 2(5x+9)$$

$$9x-12 = 10x+18$$

$$-x = 30$$

$$\therefore x = -30$$

(ii) $\frac{x+4}{3} - \frac{2x-1}{4} = 12$

x by 12 (or equivalent)

$$12 \left(\frac{x+4}{3} \right) - 12 \left(\frac{2x-1}{4} \right) = 144$$

$$4(x+4) - 3(2x-1) = 144$$

$$4x+16 - 6x + 3 = 144$$

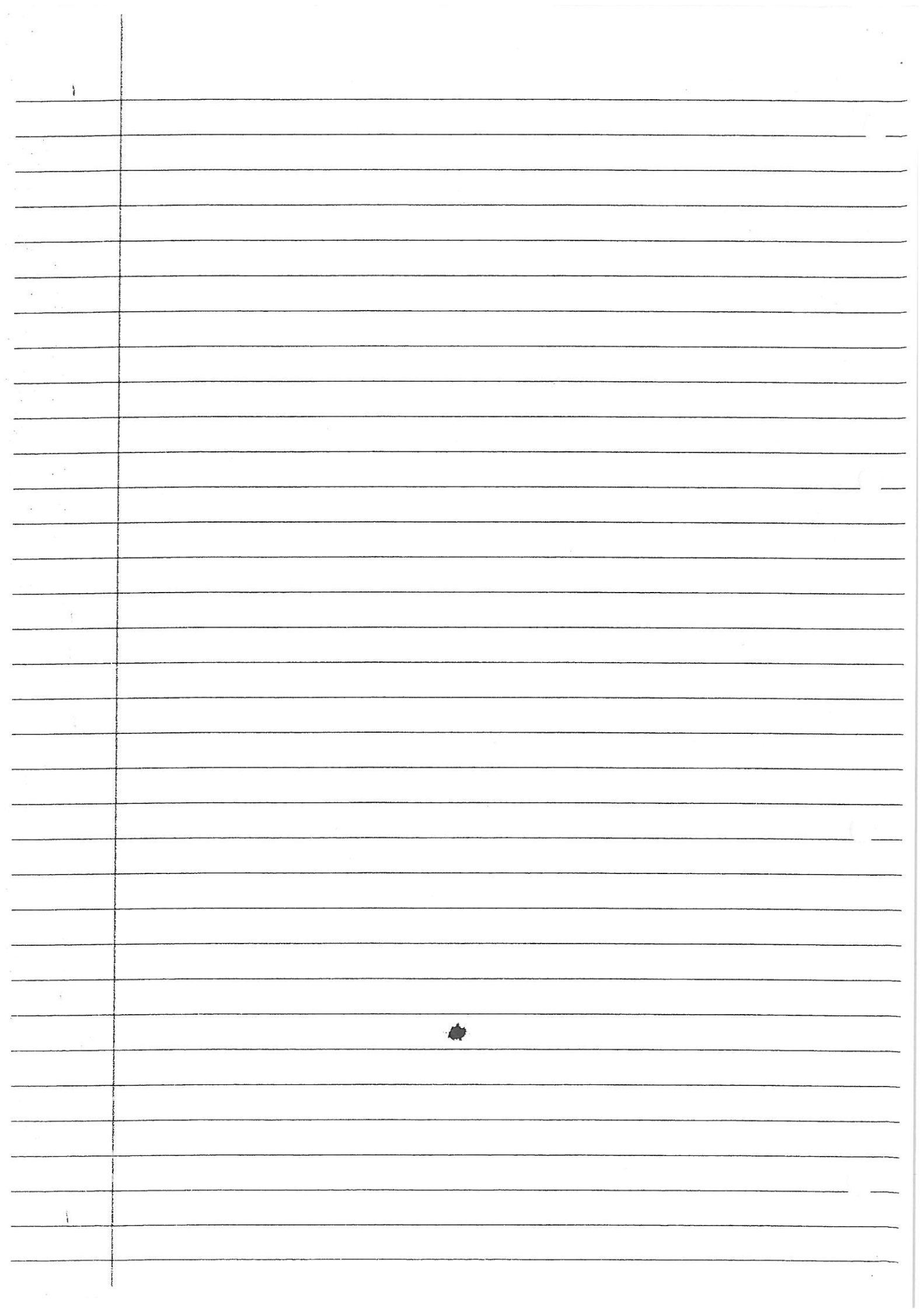
$$-2x = 144 - 16 - 3$$

$$-2x = 125$$

$$x = -\frac{125}{2}$$

$$x = -62.5$$

(5)
(8)



f

$$y = \frac{11}{10} e^x - \frac{1}{10} e^{-9x}$$

Realising they need to find & put $\frac{dy}{dx}$ & $\frac{d^2y}{dx^2}$ into the differential eqn. to show it's a solution.

$$\frac{dy}{dx} = \frac{11}{10} e^x + \frac{9}{10} e^{-9x}$$

$$\frac{d^2y}{dx^2} = \frac{11}{10} e^x - \frac{81}{10} e^{-9x}$$

Substituting their values into the eqn.

$$\frac{11}{10} e^x - \frac{81}{10} e^{-9x} + 8 \left(\frac{11}{10} e^x + \frac{9}{10} e^{-9x} \right) - 9 \left(\frac{11}{10} e^x - \frac{1}{10} e^{-9x} \right) = 0$$

Simplifying to give $0 = 0$

(7)

