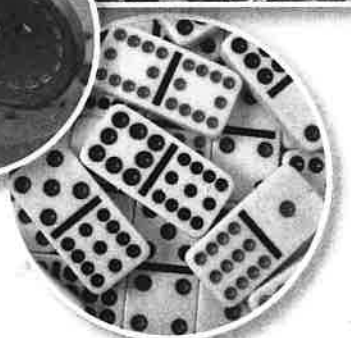


Improving Primary Mathematics Teaching and Learning



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2 Using children's mathematical misconceptions to support teaching and learning

Alice Hansen

Introduction

This chapter aims to enhance teachers' understanding of children's misconceptions and to help teachers to see them as a tool for developing children's mathematical understanding and improving attainment. Taking a social constructivist approach, the chapter explains how children construct their own concepts, which include 'misconceptions' or 'naive conceptions'. By taking this approach, the chapter emphasizes how misconceptions are a naturally occurring aspect of children's (and adults') mathematical conceptual development and therefore should be an expected outcome of children constructing their own knowledge. Once this has been established, the chapter considers how teachers can learn more about the misconceptions their pupils have by offering practical ideas for exploring misconceptions with children such as assessment for learning, analysing children's written work and summative assessment. Finally, it will identify broad strategies for how teachers might support children to address their misconceptions, such as creating a classroom culture that encourages talk and learning from each other as well as more practical ideas, such as preparing examples of possible misconceptions to present to children than can be addressed.

Chapter aims

This chapter aims to enhance your understanding of children's misconceptions by:

- Identifying what misconceptions are
- Discussing why misconceptions happen

- Considering how we can learn more about the misconceptions our pupils have
- Identifying what we might do to support children to address their misconceptions.

Conceptual and theoretical perspectives

Errors and misconceptions: what are they and how are they different?

Because errors and misconceptions exist regardless of a specific curriculum, this section will start by inviting you to reflect on an example of the work of two children when undertaking a simple addition task. Having done this, and further explored a case study from practice, I then consider how revealing and sharing misconceptions supports the development of effective pedagogy within the curriculum.

Reflection

Analyse Ishmal's and Frankie's work in Table 2.1. Think about which child seems to have made an accidental error, and which child may have a misconception that can be identified in their written work. Can you identify what the misconception is?

Table 2.1 Ishmal's and Frankie's sums

Ishmal	Frankie
1. $12 + 4 = 16$	1. $12 + 4 = 15$
2. $23 + 5 = 28$	2. $23 + 5 = 27$
3. $54 + 2 = 57$	3. $54 + 2 = 55$
4. $72 + 6 = 78$	4. $72 + 6 = 77$
5. $87 + 3 = 90$	5. $87 + 3 = 89$

Errors

Errors are commonplace in classrooms and are normally easy to spot. There are two reasons why errors happen. One is that the error was accidental. For example, from his written work above, it is possible to conclude that Ishmal appears to have made an accidental error in question 3 because he has correctly answered the other sums. When Ishmal was

questioned about his work, he responded (pointing to the addend, the number 2): 'Oh, oops, I thought that was a "3"'.

Alternatively, errors can be driven by a misconception.

Misconceptions

Frankie answered all five sums incorrectly. Analysis of Frankie's written work shows she provided an answer that is one less than the correct answer each time. A consistent incorrect pattern of answers such as this is one way of seeing that a misconception exists. Indeed, on talking with Frankie about her work it was possible to understand how the error was occurring. To calculate $12 + 4$, Frankie counted on four from *twelve*, instead of correctly from the next number, *thirteen*. She said, 'I put twelve in my head and then counted on four. 12, 13, 14, 15'.

Misconceptions occur in children, teenagers and adults of all ages regardless of their level of attainment, and not just in mathematics. Although errors are straightforward to identify, not all misconceptions will be evident to you or even the person who holds the misconception. It may be many years before a misconception comes to light and for everyone, many misconceptions will never be identified and/or addressed.

The relationship between errors and misconceptions

Figure 2.1 shows one representation of the relationship between errors and misconceptions. The Venn Diagram (Figure 2.1) shows an intersection of the sets {misconceptions} and {errors}, signalled by the arrow. These are the errors that occur as a result of misconceptions, or the misconceptions that are reinforced or created due to making the same error repeatedly.

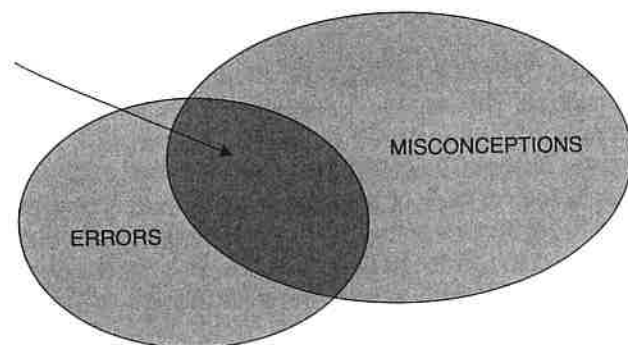


Figure 2.1 Venn Diagram showing the relationship between errors and misconceptions

	Error	Error
Misconception	An error is caused by a misconception. This is the same subset as the intersection in Figure 2.1	When a misconception exists but there is no error made or evident, it is not possible to address the misconception
Misconception	An error occurs, but is due to a mistake of some sort. This is easily rectified by the child	When a child is operating most effectively, they are not making errors and have no misconceptions about the specific study that negatively effect their learning

Figure 2.2 Carroll Diagram showing the relationship between errors and misconceptions

Another way to think about the relationship is using a Carroll Diagram, as in Figure 2.2. This representation provides four outcomes. The outcome that is the equivalent to the intersection in Figure 2.1 is the shaded set.

In both cases the errors made due to misconceptions are most interesting to teachers because the errors point to a misconception that can be addressed. This idea is revisited later in the chapter.

Why do mathematical misconceptions exist?

Now that we have broadly identified what errors and misconceptions are and how they are related, this section looks at why we all develop mathematical misconceptions.

Constructivist theories (Piaget 1953; Vygotsky 1978; Bruner 2006a, 2006b) explain the psychological view of learning as a learner constructing (rather than transmitting and receiving) their knowledge internally. Similarly, Noss and Hoyles (1996: 105) put forward the case for learning as the construction of a web of connections 'between classes of problems, mathematical objects and relationships, "real" entities and personal situation-specific experience'. They explain how learners use formal and informal resources to construct mathematical meaning by making and reinforcing links between these resources. However, a key extension of

Noss and Hoyles' (1996) notion of webbing and constructivist theory is that in webbing the resources can be internal (as for constructivist theory) or external (physical or virtual). Within any given situation, a learner will act within the situation using the range of resources they have access to. Noss and Hoyles (1996: 122) refer to this as *situated abstraction*. Of course, when the resources (internal or external) are naive or limited in any way, misconceptions will naturally emerge.

Let us take an example to illustrate this. A short time ago I met Harry, who was 9 years old. He was using a calculator to explore questions such as $5 \times 0.2 = \square$. He knew that

- $2 + 2 + 2 + 2 + 2 = 10$
- multiplication can be thought of as repeated addition
- $5 \times 2 = 10$
- the answer to 2×5 is equal to the answer to 5×2
- an array can be used to represent multiplication:



He could also recall all the basic facts to 10×10 at speed.

However, when Harry found the calculator informing him that $5 \times 0.2 = 1$, he was confused and asked his neighbour if his calculator was working properly. He had developed the misconception that *multiplication will always make the answer bigger*.

Reflection

Does this misconception make Harry's mathematics understanding deficient in any way? Are there any other experiences, or ideas about multiplication, you would have introduced Harry to prior to asking him to multiply using decimal numbers? Why / why not?

From the example of Harry it would not be accurate to describe his mathematical understanding as deficient in any way. After all, Harry's misconception is unsurprising when his experiences of multiplication to date have used positive whole integers and have indeed produced a bigger answer. Although it is likely that no one has instructed Harry that the answer to a multiplication question is always bigger, he has constructed that notion for himself from the plethora of examples he has seen and used. Indeed, Harry's response was typical in the class that day and the teacher had carefully planned the use of calculators in the lesson to elicit these kinds of responses from the children.

Simpson (2009) describes another example from her class:

Shaun had successfully measured a number of pencils and had been able to record their lengths in order; he knew that 21.4cm was longer than 21.25cm. However, in another lesson later in the same week, he was ordering a list of numbers and stated that 13.65 was bigger than 13.7.

(Simpson 2009: 15)

From the two examples, it is possible to see that learning is messy. At any moment, a child will move from thinking they understand a mathematical idea to finding that they need to know more. This can often be motivating for children as they desire to learn more about the world around them, and the role of the teacher is to effectively support their mathematical development by harnessing children's curiosity.

Can misconceptions be avoided?

The previous section explained how misconceptions are created as a by-product of children's conceptual development and are therefore an inevitable part of learning mathematics. However, in this section we will consider how the things you say and do as a teacher can inadvertently lead children to formulate misconceptions. So, although misconceptions are par for the course, you should be mindful not to teach additional misconceptions: there are enough potential difficulties or barriers to learning without you adding to the list!

What the teacher says

Case Study 2.1 provides an illustration of how one teacher unwittingly fuels some misconceptions in her classroom. Read the case study and consider how Simone approaches her teaching of primary mathematics.

Case Study 2.1: Simone's story

Simone has been a teacher for three years. She has worked with Key Stage 2 children during that time. At her request, the mathematics subject leader, Mike, has been given the opportunity to work with Simone over one term to support her mathematics planning, teaching and assessment. Through working with Simone, Mike identifies a number of examples where she has made comments to the children that could potentially reinforce misconceptions.

Mike produced the following list of examples in preparation for his discussion with Simone:

- 'Five take seven. You can't do that'
- 'Fives into three don't go'
- 'Add a zero when multiplying by ten'
- 'Look at these triangles ...' 
- 'You can remember a rhombus by thinking of it as a square that has been sat on'

Commentary

Through the case study examples you may be able to see that Simone's mathematics pedagogical subject knowledge is poor. She also lacks confidence when it comes to teaching mathematics. As a result, her approach to teaching children is procedural and she has learnt over her three years of teaching that the children 'get the answer right' if they follow the rules and apply the tricks she teaches them.

Reflection

Revisit the statements Simone has made. Reflect on what future difficulties the children may meet if they learn the rules and tricks that Simone has modelled or encouraged them to apply. Use Table 2.2 to help you.

Table 2.2 Statements that cause potential later difficulties

Simone's statement	Future potential difficulty created
'Five take seven. You can't do that'	
'Fives into three don't go'	
'Add a zero when multiplying by ten'	
'Look at these triangles ...'	
	
'You can remember a rhombus by thinking of it as a square that has been sat on'	

The first three statements are inaccurate. It is possible to take 7 from 5: the result is negative 2 (-2). So because the calculation can be done (although it may have been undesirable in the strategy Simone was teaching the class), it is misleading to suggest that it cannot be carried out. Similarly, 5 can be divided into 3, and the answer is $3/5$, or 0.6. Finally, if one 'adds a zero' when multiplying a decimal number by 10 without paying attention to the place value of the digits, then the answer will remain the same (e.g. $2.34 \times 10 = 2.340$).

While the fourth statement about triangles is not incorrect, Simone is limiting the examples she is using with the children. Although she has produced right-angled, isosceles and equilateral examples, they are still all prototypical because the base of each triangle sits horizontally. It would be desirable for Simone to also show other orientations of the triangles.

Simone's final statement asks the children to think about shapes at only a visual level (van Hiele 1986) (i.e. 'a square that has been sat on'). By doing this, she is not encouraging them to consider the figure's properties or how rhombuses are defined. This potentially limits their ability to think about geometric definition. It also unwittingly makes children think of squares and rhombuses as two shapes that are unrelated to each other and this can inhibit later geometric reasoning (Hansen 2008). Undertaking the next reader reflection will help you to develop this idea.

Reflection

Visualize the diagonals of a square. What do you notice? You will probably notice that the diagonals are the same length, they are perpendicular and that they bisect. Now visualize the square turning into a different rhombus, a rhombus that is not a square. What happens to the diagonals? Which properties stay the same? Which change? Why? Now visualize the rhombus returning to a square.

You can repeat this visualization activity with other properties such as order of rotation or reflective symmetry.

Did the reflection activity help you to think of a square as a particular type of rhombus? That is, did you think about how a rhombus has two perpendicular bisectors, and that to be a particular instantiation of rhombus – a square – the perpendicular bisectors also need to be of equal length?

You may have found the task you were asked to complete in the reflection above challenging. Did you already know that there was a relationship between squares and rhombuses that could be explored through their diagonals? Or was this new to you? Perhaps you thought that there

was no relationship between the two shapes except for the length of their sides remaining the same. This task may have revealed something that you didn't know, or perhaps even a misconception you hold yourself. If so, the reflection may have been a good illustration of how the tasks that children complete in the classroom can hide or reveal a number of misconceptions. It is to this very issue that the discussion now turns.

Tasks the children complete

In order for children to widen their 'fields of experience' (Ackermann 1991) or broaden their 'contextual neighbourhood' (Pratt and Noss 2002), it is important for teachers to think carefully about the tasks they present to children. The following illustration presents two tasks that share the same objective: to identify squares. Read through the tasks in Figures 2.3 and 2.4 and carry out the reader reflection that follows.

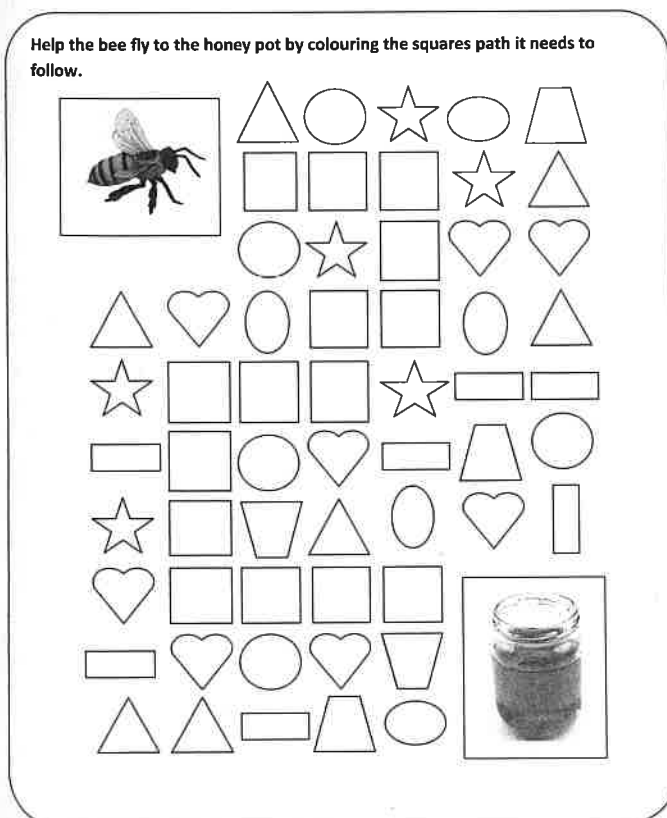


Figure 2.3 Sample worksheet

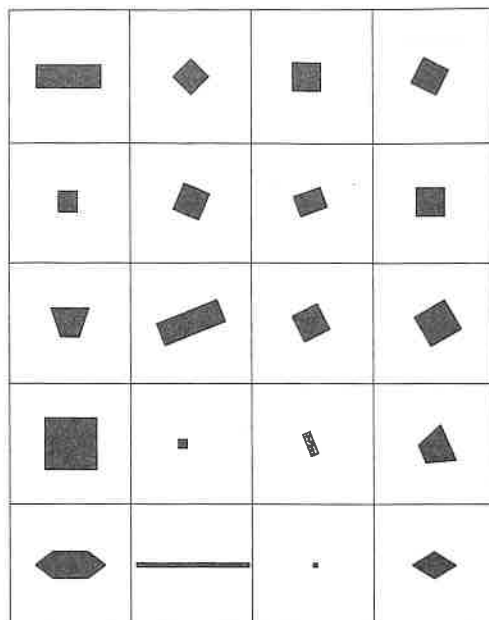


Figure 2.4 Sorting cards

The task in Figure 2.3 is similar to a worksheet that I recently saw.

The task in Figure 2.4 shares the same learning outcome. It asks the children to work in a small group (of two or three) to sort the shapes on the cards (see Figure 2.4) into two sets: Set A contains squares, Set B contains shapes that are not squares. The children need to feel confident that all the children in the group are convinced that each card is in the correct set by talking with each other about the reasons for their decisions.

Reflection

Think about what skills, knowledge and understanding a child needs to demonstrate in order to correctly complete the worksheet and the sorting activity. How are the tasks different?

Mason and Johnston-Wilder (2006) offer a most useful explanation of what to look for in a task:

A relevant type of task involves situations that appear to give rise to contradictions or surprises. In these tasks, learners need to sort out what is happening, resolve differences of opinion or conflicting

explanations, and find some way to account for what is going on. Learners are called upon to explain things to each other and to locate differences and agreements in their explanations.

(Mason and Johnston-Wilder 2006: 64)

To what extent do the two tasks offer a context for children to explore, discuss and explain contradictions and surprises?

Reflect more broadly now. Think about the types of tasks you expect your own pupils to carry out. What do you notice about them? How do those tasks reflect your approach to learning and teaching primary mathematics?

In the reader reflection box you were asked to think about how the types of tasks you ask learners to engage with reflect your own pedagogy. This theme is developed in the following section as the discussion turns to addressing misconceptions.

The mathematics curriculum and children's learning: how can misconceptions be addressed?

Earlier in this chapter I discussed how misconceptions are an inevitable by-product of learning and as such they should not be seen as a negative aspect of learning. Instead they should be positively used to support children's cognitive development.

There are many ways to address children's misconceptions. The following discussion offers three principles that you can apply within whatever classroom context you find yourself in. What remains consistent throughout is the use of high quality tasks (refer to discussion above). The principles are:

- Developing a classroom (and school) ethos that celebrates the complexity of learning.
- Using misconceptions to support children's cognitive development.
- Planning an effective curriculum that allows for misconceptions to be revealed.

Developing a classroom (and school) ethos that celebrates the complexity of learning

In the previous reader reflection you were asked to think about the types of tasks you plan for your learners. The tasks you plan for your learners

provide a window into your mathematics pedagogy. One key aspect of your pedagogy is how you foster a mathematical ethos and what happens in your classroom and school. After all, 'the quality of an education system cannot exceed the quality of its teachers and principals, since student learning is ultimately the product of what goes on in classrooms' (OECD 2010: 4).

This is an aspect that has been a focus of Ofsted research also: 'The most effective teachers ... cultivate an ethos where children do not mind making mistakes because errors are seen as part of the learning. In these cases, children are prepared to take risks with their answers' (Ofsted 2003: 18).

Yackel and Cobb (1996) identify how the teacher is instrumental in developing the class ethos. They refer to the class ethos in mathematics lessons as sociomathematical norms.

Sociomathematical norms

Sociomathematical norms are particular to mathematics lessons. They are 'normative understandings of what counts as mathematically different, mathematically sophisticated, mathematically efficient, and mathematically elegant in a classroom ... Similarly, what counts as an acceptable mathematical explanation and justification is a sociomathematical norm' (Yackel and Cobb 1996: 461). These are constrained by the 'current goals, beliefs, suppositions and assumptions' of the classroom participants (Yackel and Cobb 1996: 460).

The process of developing sociomathematical norms is *reflexive*: as you gain more insight into your pupils' mathematical development, you can adjust the sociomathematical norms. Sociomathematical norms are imperative within classroom discussion because they create the backdrop for the argumentation of mathematical ideas. As argumentation evolves, *taken-as-shared communication* is subtly adjusted because it continues to form the backdrop for discussion. This in turn brings about children's cognitive development.

Reflection

Research shows us that as children progress through school they learn not to ask questions, even when they do not understand (Boaler 2009).

How often do you encourage children to ask questions? Who do they ask questions to? Questioning and arguing about mathematics helps to develop children's ability to reason about mathematics. Think about your planning and teaching. When do the children use reasoning to justify their answers and defend their methods?

High quality talk in mathematics

Talking about mathematics has been highlighted and discussed for many years. The seminal report *Mathematics Counts* by the Committee of Inquiry into the Teaching of Mathematics in Schools (Cockcroft 1982) referred to the significant role of talk. Paragraph 252 provides one example:

It is necessary to realise that much of the value of an investigation can be lost unless the outcome of the investigation is discussed. Such discussion should include consideration not only of the method which has been used and the results which have been obtained but also of false trails which have been followed and mistakes which may have been made in the course of the investigation.

(Cockcroft 1982: para 252)

Note the reference to errors and misconceptions in this paragraph. Using misconceptions to support cognitive development will be discussed further in the next section.

Crucially, talk has remained at the forefront of policy-makers', researchers' and teachers' minds for many decades, yet regardless of this ongoing focus, talk in mathematics still remains an issue. For example, Ofsted's review of the National Literacy and Numeracy Strategies and their effects on the primary curriculum (Ofsted 2003) included a key finding that 'in too many lessons, teachers' talk dominates and there are too few opportunities for pupils to talk and collaborate to enhance their learning' (Ofsted 2003: 3).

Furthermore, in the first independent review of mathematics teaching in Early Years settings and primary schools since the Cockcroft Report, Sir Peter Williams identified that talk

should extend to high-quality discussion that develops children's logic, reasoning and deduction skills, and underpins all mathematical learning activity. The ultimate goal is to develop mathematical understanding – comprehension of mathematical ideas and applications.

(Williams 2008: 165)

Reflection

Think about the tasks presented earlier in Figures 2.3 and 2.4. Focusing on children's potential misconceptions related to squares, which task would be more effective in exposing these? How and why? Which gives rise to more opportunity for children to talk about what they know and don't know about squares?

Think about what sociomathematical norms might be evident in a classroom where the type of task presented in Figure 2.3 is typical compared to a classroom where the task in Figure 2.4 is typical.

Using misconceptions to support children's cognitive development

Earlier in this chapter I discussed the constructivist approach to learning and explained how making links between related ideas helps children build up a conceptual picture. Of course, this process also develops misconceptions which are an inevitable part of learning mathematics. This way of thinking about learning is advantageous to teachers. Seeing value in using misconceptions as a pedagogical tool to support cognitive development is widely espoused in the research literature. By teachers adopting 'a constructive attitude to their children's mistakes' (Koshy 2000: 173), children develop a coherent mathematical knowledge (Barmby et al. 2009). Drews (2011) explains how talking about misconceptions can help children to see themselves as successful learners who attempt more challenging work. The assumption behind promoting this approach is that teachers have good subject knowledge. This was illustrated in Case Study 2.1 of Simone.

Case Study 2.2 shows how Sheila, a Year 6 (10–11 year olds) teacher, helped her children improve their mathematical disposition as well as their confidence and readiness for the statutory tests at the end of their primary schooling by systematically introducing talking about errors.

Case Study 2.2: Sheila's story

Like most schools we have a bank of test questions from previous test papers. From Year 4 all the teachers include the odd question related to the topic the children are learning about into lessons. We agreed that this drip-feeding of questions in regular lessons would improve the children's confidence for the tests at the end of Year 6 because they have met many before the actual test, which most of them have been successful in working out.

For most children that works well, but I was really mindful of the children who would still continue to get incorrect answers. I thought that for some in my own class it was having the opposite effect to the desired one. So, over the first term I started to keep a record of inaccurate comments that children made or incorrect answers that they gave in written pieces. Then in the terms that followed, when we revisited the topics, I used some of

the comments or wrong answers as a talking point to start off the main part of the lesson when I thought it was useful. I even made some up, thinking of what the children might do wrong. It was really successful. The key to its success was that the children were talking about the errors and possible misconceptions behind the errors. They were happy to talk about their mistakes. Over the year I saw a real shift in how we talked about maths. The children volunteered their own mistakes for discussion, wanting help from their peers about how to improve their understanding. There was also an improvement in the children's desire to want to check their independent work more fully, and also their stickability at a maths problem ... they were willing to try more avenues if the first ones they tried didn't work.

With the children's permission, I used their work in the first term the following year with the new class so I had something to use from the start. The maths attainment of the children in that class over that year improved at a greater rate than any class I'd had before. Now all the teachers from Year 2 and above use this approach in some maths lessons.

Commentary

Sheila's findings reflect the work carried out by Mike Spooner. Spooner (2002) found that studying incorrect answers in past test papers provided an opportunity for greater openness on the part of children to explore and discuss their misconceptions.

Planning an effective curriculum that allows for misconceptions to be revealed

This final principle encourages you to think about how you can provide an effective class and school curriculum for misconceptions to be revealed. In this section we discuss curriculum content (using and applying mathematics) and curriculum assessment (assessment for learning).

Using and applying mathematics

The 2000 National Curriculum (Department for Education and Employment (DfEE) 1999) for mathematics contained a Programme of Study (PoS) called 'Using and Applying Mathematics'. This attainment target is embedded within the other three attainment targets – Number; Shape, Space and Measures; Handling Data – showing its importance within mathematics learning. Within using and applying mathematics, children should be taught problem solving, communicating and reasoning. Table 2.3 identifies the skills, knowledge and understanding required for using and applying mathematics.

Table 2.3 What 'Using and Applying Mathematics' involves

Problem solving involves	Communicating involves	Reasoning involves
• making connections • discussion • enquiry • planning • deciding • organizing • interpreting • reasoning • imagination, inventiveness, diverse strategies • investigation • justification • persistence • representations • trial and error	• accepting and giving feedback (including peer coaching) • ad hoc as well as more structured representation • calculations • demonstrating reasoning • diagrams • explaining methods and solutions • ICT • lists • making choices and decisions • pictures • resources • tables • using mental images and models • written skills	• creating • deduction • applying • exploring • predicting • hypothesizing • testing • classifying • explaining • finding patterns • inference • logic • making connections • mapping • organizing information • proof • reflection • trial and error

Source: Adapted from Hansen and Vaukins (2011: 13–25)

Reflection

Problem solving, communicating and reasoning are core to learning mathematics. Reflect on a mathematics lesson you recently taught. Use Table 2.3 to help you to consider what problem solving, communicating and reasoning you were explicitly engaging your children in. How could you focus on these aspects further with the children? How would a focus on these aspects support revealing and addressing errors and misconceptions?

Use and application, when systematically planned for and effectively facilitated, is a conduit for children to discuss and address their errors and misconceptions. What remain central tenets throughout are effective task choice and ongoing development of sociomathematical norms.

A crucial component of systematic planning involves clear and effective assessment. Therefore, in addition to thinking about the curriculum content that is on offer in your classroom and school, also consider how you use assessment for learning within the curriculum on offer.

Assessment for learning

Koshy (2000: 173) reminds us that 'as in real life, mistakes made in mathematical lessons can also be very useful. They provide us with very useful insights into children's thinking and mathematical understanding. Mistakes that are persistent can often highlight gaps in the child's knowledge'. Lee (2006) explains how using mathematical misconceptions can lead to creative approaches in assessment for learning strategies. In light of this, what role can children's errors and misconceptions play in assessment for learning, and how can assessment for learning address errors and misconceptions?

The Cambridge Primary Review (Alexander 2010) warns that although assessment for learning helps teachers to promote children's learning, 'it is difficult to shift teachers from reliance on specific techniques (the letter of AfL) to practices based on deep principles integrated into the flow of lessons (the spirit of AfL)' (Alexander 2010: 290). Indeed, Hargreaves (2005) identifies six different conceptions teachers hold of assessment for learning. These are:

1. *Monitoring pupils' performance against targets or objectives.* This focuses on a process the teacher follows.
2. *Using assessment to inform next steps in teaching and learning.* This focuses on what action the teacher needs to take next.
3. *Giving feedback for improvement.* This focuses on the teacher informing children how to take the next steps towards a given standard.
4. *Teachers learning about children's learning.* This focuses on children improving their future learning.
5. *Children taking some control of their own learning and assessment.* This focuses on children's learning processes.
6. *Turning assessment into a learning event.* This focuses on children's involvement in assessment.

Reflection

Write down your own definition of 'assessment for learning', then compare your definition to those conceptions Hargreaves (2005) found that teachers hold.

Which of Hargreaves' definitions might value children's errors and misconceptions most?

While it would be possible to answer that each of the six conceptions of assessment for learning could value errors and misconceptions, it is the final three that reflect the journey this chapter has taken you on. *Teachers learning about children's learning* is part of the reflexive process of developing sociomathematical norms in your classroom; *children taking some control of their own learning and assessment* and *turning assessment into a learning event* firmly centre children's talk within mathematics learning.

Summary

This chapter has focused on how you can use children's misconceptions to support learning and teaching. In summary, the chapter identified the difference between errors and misconceptions and how they relate to each other. It demonstrated that misconceptions exist as a by-product of learning and can be found in children and adults (even teachers!) alike.

Although misconceptions cannot be avoided, the teacher's role includes ensuring additional misconceptions are not created through the way they talk about mathematics. Teachers are also responsible for choosing and designing the tasks children undertake. Planning for tasks that encourage children's misconceptions to come to the fore is crucial for conceptual development. Underpinning everything is the ethos teachers foster in their classrooms and schools. Central to this should be high quality talk in mathematics and valuing the errors children make and misconceptions they exhibit as a core component of children's learning.

Further reading

Cockburn, A. and Littler, G. (2008) *Mathematical Misconceptions: A Guide for Primary Teachers*. London: Sage.

This thought-provoking book brings together a selection of writing about children's misconceptions and big ideas in mathematics such as zero,

place value and equality. It also looks at how children learn mathematical ideas and aims to develop your own mathematical understanding along the way.

Hansen, A. (ed.) (2011) *Children's Errors in Mathematics: Understanding Common Misconceptions in Primary Schools* (2nd edn). Exeter: Learning Matters.

This practical guide to children's common errors and misconceptions in mathematics provides a bank of common errors children 3–11 years of age make. It discusses learning theory and mathematical development in each attainment target of the mathematics curriculum to explain why the errors may have occurred. The teacher's role in understanding and addressing common misconceptions is also covered.

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