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Question 2: integration

a) (i) $\int (x^2 - 6x)(3x^2 - x) dx$

$$\therefore \int 3x^4 - 18x^3 - 5x^3 + 6x^2 dx$$

$$\therefore \int 3x^4 + 5x^2 - 18x^3 dx$$

$$\therefore = \frac{3x^5}{5} + \frac{5x^3}{3} - \frac{18x^4}{4} + c$$

(ii) $\int (x^2 + 3x)^2 + e^{0.5x} - \sin 2x dx$

$$\therefore \int (x^2 + 3x)(x^2 + 3x) + e^{0.5x} - \sin 2x dx$$

$$\therefore \int x^4 + 3x^3 + 3x^3 + 9x^2 + e^{0.5x} - \sin 2x dx$$

$$\therefore \int x^4 + 6x^3 + 9x^2 + e^{0.5x} - \sin 2x dx$$

$$\therefore = \frac{x^5}{5} + \frac{6x^4}{4} + \frac{9x^3}{3} + 2e^{0.5x} - \frac{1}{2} \cos 2x + c$$

(iii) $\int_0^3 (2x^2 + 3)^3 dx$

$$\therefore \int_0^3 (2x^2 + 3)(2x^2 + 3)(2x^2 + 3) dx$$

$$\therefore \int_0^3 (4x^4 + 6x^2 + 6x^2 + 9)(2x^2 + 3) dx$$

$$\therefore \int_0^3 (4x^4 + 12x^2 + 9)(2x^2 + 3) dx$$

$$\therefore \int_0^3 (8x^6 + 12x^4 + 24x^4 + 36x^2 + 18x^2 + 27) dx$$

$$\therefore \int_0^3 (8x^6 + 36x^4 + 54x^2 + 27) dx$$

$$\therefore \int (8(3)^6 + 36(3)^4 + 54(3)^2 + 27) - (0)$$

$$= \frac{8x^7}{7} + \frac{36x^5}{5} + \frac{54x^3}{3} + \frac{27x}{1}$$

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$$\therefore \left[\frac{8(3)^7}{7} + \frac{36(3)^5}{5} + \frac{54(3)^3}{3} + 27(3) \right] - [0]$$

$$\therefore = [2499 \cdot 43 + 1749 \cdot 6 + 486 + 81] - [0]$$

$$\therefore = 4816.03$$

$$(iv) \int_2^5 x^3 + 2x^2 - x + 5 \, dx$$

$$\therefore \int_2^5 \frac{x^4}{4} + \frac{2x^3}{3} - \frac{x^2}{2} + 5x \, dx$$

$$\therefore \left[\frac{x^4}{4} + \frac{2x^3}{3} - \frac{x^2}{2} + 5x \right]_2^5$$

$$\therefore \left[\frac{(5)^4}{4} + \frac{2(5)^3}{3} - \frac{(5)^2}{2} + 5(5) \right] - \left[\frac{(2)^4}{4} + \frac{2(2)^3}{3} - \frac{(2)^2}{2} + 5(2) \right]$$

$$\therefore = [156.25 + 83.33 - 12.5 + 25] - [4 + 5.33 - 2 + 10]$$

$$\therefore = [252.08] - [17.33]$$

$$\therefore = 234.75$$

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(b).

(i). $\int 3x \sin 5x \, dx$

$\uparrow \quad \quad \uparrow$
 $u \quad \quad \frac{dv}{dx}$

$$\frac{du}{dx} = 3$$

$$v = -\frac{1}{5} \cos 5x$$

$$\text{equation} = uv - \int v \frac{du}{dx} \, dx$$

$$\therefore \underline{\underline{-\frac{3}{5} \cos 5x - (3)x(-)}}$$

$$\therefore -\frac{3}{5} x \cos 5x + \frac{3}{5} \cos 5x \, dx$$

$$\therefore -\frac{3}{5} x \cos 5x + \frac{3}{5} \sin 5x \, dx$$

(ii). $\int x \cos \frac{x}{2} \, dx$

$\uparrow \quad \quad \uparrow$
 $u \quad \quad \frac{dv}{dx}$

$$\frac{du}{dx} = 1$$

$$v = 2 \sin \frac{x}{2}$$

$$\text{equation} = uv - \int v \frac{du}{dx} \, dx$$

$$\therefore \underline{\underline{x 2 \sin \frac{x}{2} + 4 \cos \frac{x}{2} + C}}$$

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~~C) (i) (iii)~~

$$\int \frac{15x^2 - 4x}{5x^3 + 2x^2 + 5} dx$$

$$= \ln(5x^3 + 2x^2 + 5) + c$$

$$(ii) \int \frac{7}{4x-15} dx$$

$$\therefore 7 \int \frac{1}{4x-15} dx$$

$$\therefore = \frac{7}{4} \ln(4x-15) + c$$

$$(iii) \int \tan^2 x dx$$

$$= \tan(x) - x + c$$

$$(d) \int \frac{x^3}{\sqrt[3]{2+x^4}}$$

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Question 1: Complex numbers

Q1a i). $Z_1 = 7 + 5j$ $Z_2 = 3 - 7j$ $Z_3 = -2 + 6j$

$$\therefore Z_1 + Z_3 = (7 + 5j) + (-2 + 6j)$$

$$\therefore 7 + 5j + -2 + 6j =$$
$$= 5 + 11j$$

(ii) $Z_1 - Z_3 = (7 + 5j) - (-2 + 6j)$

$$7 + 5j - -2 + 6j$$
$$= 9 - j$$

(iii) $\frac{Z_1}{Z_2} = \frac{(7 + 5j)}{(3 - 7j)}$

$$\therefore = \frac{(7 + 5j)(3 + 7j)}{(3 - 7j)(3 + 7j)} = \frac{21 + 49j + 15j + 35j^2}{9 + 21j - 21j + -49j^2}$$

$$\therefore = \frac{21 + 64j - 35}{9 + 49} = \frac{-14 + 64j}{58}$$

$$\therefore \frac{-14}{58} + \frac{64j}{58}$$

(iv) $\bar{Z}_1 - \bar{Z}_2$

$$\therefore (7 - 5j) - (3 + 7j)$$

$$\therefore = 4 - 12j$$

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(v). $(z_1)^2 - z_2 - z_3$

$$(7+5j)(7+5j) - (3-7j) - (-2+6j)$$

$$\therefore = (49 + 35j + 35j + 25j^2) - (3-7j) - (-2+6j)$$

$$\therefore = (24 + 70j) - (3-7j) - (-2+6j)$$

$$\therefore = 21 + 71j$$

$$= 23 + 71j$$

(b) $12x^2 + 5x = -13$

$$\underset{a}{12}x^2 + \underset{b}{5}x + \underset{c}{13} = 0$$

$$\text{Formula} = x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\therefore x = \frac{-5 \pm \sqrt{5^2 - (4)(12)(13)}}{(2)(12)}$$

$$\therefore = \frac{-5 \pm \sqrt{25 - 624}}{24}$$

$$\therefore x = \frac{-5 + \sqrt{25 - 624}}{24} \quad y = \frac{-5 + \sqrt{599}}{24} = 0.811$$

$$r = \sqrt{x^2 + y^2}$$

$$\therefore x = \frac{-5 - \sqrt{599}}{24} = -1.23$$

$$\theta = \tan^{-1} \left(\frac{y}{x} \right)$$

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$$r = \sqrt{1.23^2 + 0.811^2} = 1.47$$

$$\theta = \tan^{-1}\left(\frac{y}{x}\right) \therefore \theta = \tan^{-1}\left(\frac{0.811}{1.23}\right) = 33.4$$

$$\text{Polar expression} = r(\cos\theta + j\sin\theta)$$

$$\therefore = 1.47 \times (\cos(33.4) + j\sin(33.4))$$

a.

$$r(\cos\theta - j\sin\theta)$$

$$\theta = 360 - 33.4 = 326.6$$

$$\therefore \text{expression} = 1.47 \times (\cos(326.6) - j\sin(326.6))$$

(c)

(i).

$$(4 + 9j)^4$$

$\uparrow \quad \uparrow$
x real imaginary y

$$\text{DMT} = r^n (\cos n\theta + j\sin n\theta)$$

$$r = \sqrt{x^2 + y^2}$$
$$\therefore = \sqrt{4^2 + 9^2} = 9.85$$

$$\theta = \tan^{-1}\left(\frac{y}{x}\right) \therefore \tan^{-1}\left(\frac{9}{4}\right) = 66.03$$

$$\therefore 9.85^4 \times (\cos(4 \times 66.03) + j\sin(4 \times 66.03)) =$$

$$\therefore 9413.37 \times (\cos(264.08) + j\sin(264.08))$$

$$\therefore 9413.37 \cos 264.08 = -970.89$$

$$\therefore 9413.37 \sin 264.08 = -9363.167$$

$$= -970.89 - 9363.167j$$

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(ii)

$$(-5 - 2j)^5$$

 $\uparrow$
(x) real $\uparrow$
imaginary (y)

$$D_{MT} = r^n (\cos n\theta + j \sin n\theta)$$

$$r = \sqrt{x^2 + y^2} = \sqrt{5^2 + 2^2} = 5.36$$

$$\theta = \tan^{-1} \left(\frac{y}{x} \right) \therefore \tan^{-1} \left(\frac{2}{5} \right) = 21.80$$

$$\therefore 5.36^5 \times (\cos(5 \times 21.80) + j \sin(5 \times 21.80))$$

$$\therefore 4424.01 \times (\cos(109) + j \sin(109))$$

$$\therefore 4424.01 \cos 109 = -1440.32$$

$$\therefore 4424.01 \sin 109 = 4182.98$$

$$\therefore -1440.32 + 4182.98j$$

(d)

$$j^4 = j^2 \times j^2 = (-1) \times (-1) = 1$$