

Question 1

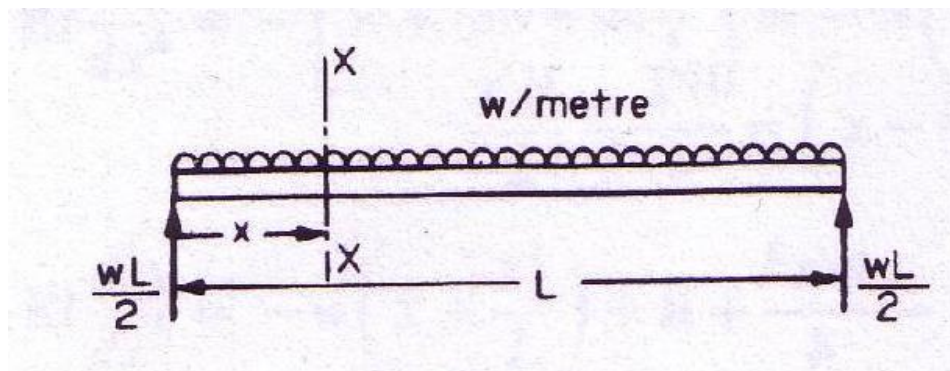
- a. The below figure shows a simply supported beam with a uniformly distributed load.

i. Prove that the maximum deflection, $y_{\max}$ is given by $y_{\max} = \frac{-5WL^4}{384EI}$

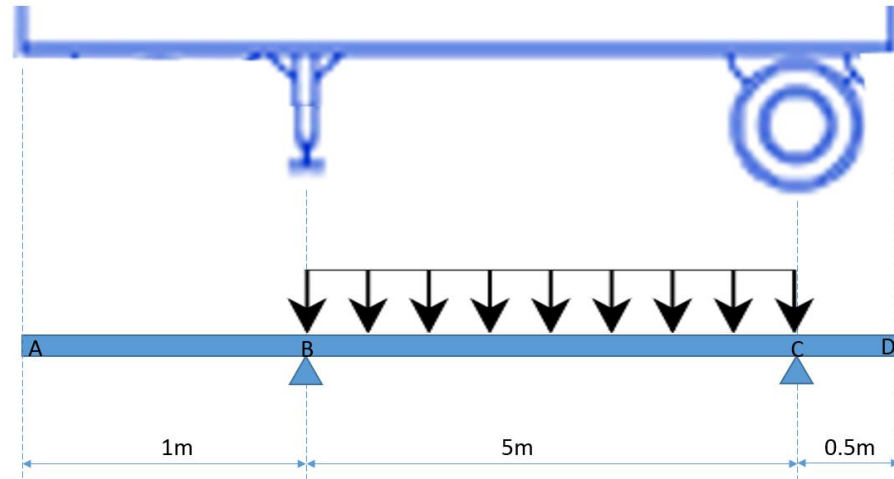
[8 Marks]

ii. Prove that the maximum slope is given by $\left(\frac{dy}{dx}\right)_{\max} = \pm \frac{WL^3}{24EI}$

[2 Marks]



b.



- i. The above figure shows a HGV trailer that may be considered to be a simply supported beam with the rear axle at location C and a supporting leg at position B. The hitch to the cab is located at position A and the rear of the trailer is at location D. In order to allow the hitch to attach to the cab it must not deform upwards by more than 10mm. As such, what flexural stiffness must the trailer supporting structure have to support a uniformly distributed weight of 5000kg between the supporting leg and the rear axle such that the trailer can still be coupled to the cab?

[5 Marks]

- ii. If the structural part of this trailer consists of an I beam that is loaded about the xx axis, using the table below, what would the mass of this I beam be if it was made from steel ($E=210\text{GPa}$) and it runs the length of the trailer?

[5 Marks]

Size D x B (mm)	Mass per metre (kg)	Thickness		I_{xx} (cm^4)	I_{yy} (cm^4)
		Web, t (mm)	Flange, T (mm)		
910.3 x 304.1	222	15.9	23.9	375924	11223
840.7 x 292.4	194	14.7	21.7	279450	9069
762 x 266.7	173	14.3	21.6	205177	6846
677.9 x 253.0	125	11.7	16.2	118003	4379
617.5 x 307.0	113	14.1	23.6	151631	11412
533.1 x 209.3	92	10.2	15.6	55353	2392
467.4 x 192.8	98	11.4	19.6	45717	2343
457.2 x 151.9	67	9.1	15	28577	878
402.3 x 142.4	46	6.9	11.2	15647	539
364 x 173.2	67	9.1	15.7	19522	1362
352.8 x 126.0	39	6.5	10.7	10087	357
310.9 x 166.8	54	7.7	13.7	11710	1061
312.7 x 102.4	33	6.6	10.8	6487	193
254 x 101.6	22	5.8	6.8	2867	120
203.2 x 133.4	25	5.8	7.8	2356	310

Formula Sheet

$$M = EI \frac{d^2 y}{dx^2}$$

$$\cos^2 \theta = \frac{1}{2}(1 + \cos 2\theta)$$

$$\sin^2 \theta = \frac{1}{2}(1 - \cos 2\theta)$$

$$\sigma_{\theta} = \frac{1}{2} (\sigma_x + \sigma_y) + \frac{1}{2} (\sigma_x - \sigma_y) \cos 2\theta + \tau_{xy} \sin 2\theta$$

$$\tau_{\theta} = \frac{1}{2} (\sigma_x - \sigma_y) \sin 2\theta - \tau_{xy} \cos 2\theta$$

$$\sigma_{1,2} = \frac{1}{2} \left[(\sigma_x + \sigma_y) \pm \sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2} \right]$$

$$\tan \theta = \frac{\sigma_p - \sigma_x}{\tau_{xy}}$$

$$\sigma_L = \frac{pd}{4t}$$

$$\Delta L = \frac{pd}{4tE} (1 - 2\nu)L$$

$$\Delta V = \frac{pd}{4tE} (5 - 4\nu)V$$

$$\sigma_H = \frac{pd}{2t\eta_L}$$

$$\sigma_L = \frac{pd}{4t\eta_C}$$