

Complete, fold and seal
(see note 2 below)

FORENAME(S): WILLIAM
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Instructions to Candidates

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2. Enter your name in the space provided to the right, fold over and seal down (using the label provided) to hide your name.
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4. Write your answers in ink.
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8. **Every book must be handed in. In no circumstances may answer books, used or unused, be taken out of the examination venue.**

1	Student Number	2111222
2	Title of Examination Paper	control engineering
3	Module Code	SGEE 5014
4	Date and Time of Examination	9.40 9/1/2025
5	Examination Venue	
6	Seat Number	
7	Number of Answer Books handed in	1062

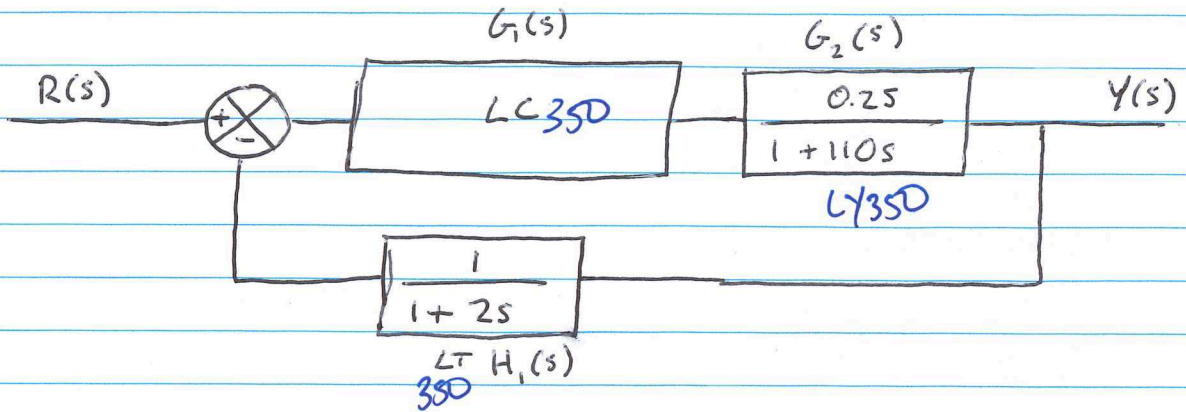
List the numbers of the questions you have attempted	Office Use
2	
3	
4	
5	
Total	62.

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62

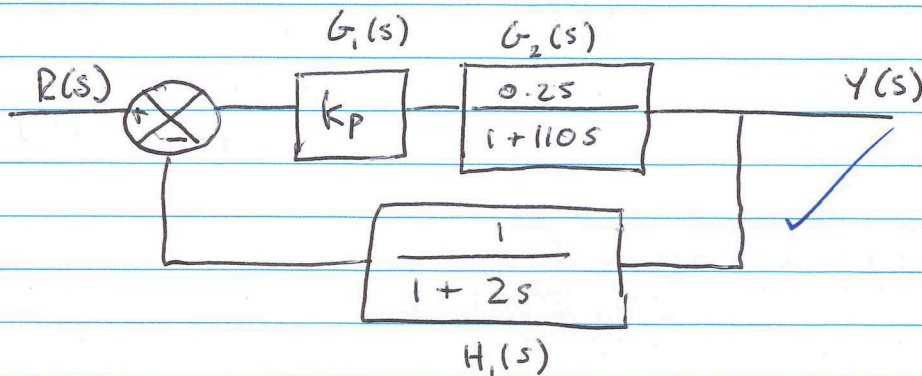
Q2a)

I)



2

II)



2

overall TFcombine G_1 & G_2

$$K_p \times \frac{0.25}{1 + 110s} = \frac{0.25K_p}{1 + 110s}$$

✓

1

simplify negative feedback

$$\frac{G(s)}{1 + G(s)H(s)}$$

↓

$$\frac{0.25K_p}{1 + 110s}$$

$$1 + \left(\frac{0.25K_p}{1 + 110s} \right) \left(\frac{1}{1 + 2s} \right)$$

=

$$\frac{0.25K_p}{1 + 110s} \div \frac{1 + \frac{0.25K_p}{(1 + 110s)(1 + 2s)}}{1 + \frac{0.25K_p}{(1 + 110s)(1 + 2s)}}$$

2

multiply by $\frac{(1+110s)(1+2s)}{(1+110s)(1+2s)}$

$$= \frac{0.25k_p(1+2s)}{(1+110s)(1+2s) + 0.25k_p}$$

expand brackets

$$= \frac{0.25k_p + 0.5k_p s}{220s^2 + 112s + 1 + 0.25k_p}$$

overall transfer function. ✓

When $k_p = 1.1$ & step input.

substituting k_p

$$\frac{0.275 + 0.55s}{220s^2 + 112s + 1 + 0.275}$$

overall gain?

$$= \frac{0.275 + 0.55s}{220s^2 + 112s + 1.275}$$

using final value theorem & step input

$$\lim_{t \rightarrow \infty} y(s) = \lim_{s \rightarrow 0} \left\{ s \times \frac{1}{s} \times TF(s) \right\}$$

$$y_{ss} = \lim_{s \rightarrow 0} \left\{ \frac{0.275 + 0.55s}{220s^2 + 112s + 1.275} \right\}$$

$$= \frac{0.275}{1.275} = 0.2 \quad \times$$

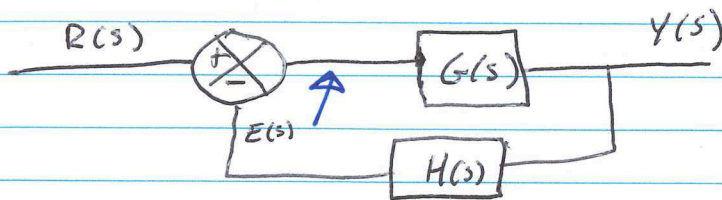
$$e_{ss} = 1 - y_{ss} \quad \text{steady state}$$

$$e_{ss} = 1 - 0.2 = \underline{0.8} \quad \text{error} \quad \times$$

4

4

Q2b)

find equation representing $E(s)$

$$E(s) = H(s)Y(s)$$

find equation representing $Y(s)$

$$Y(s) = (R(s) - E(s))G(s)$$

unconventional labelling

substitute $E(s)$

$$Y(s) = (R(s) - H(s)Y(s))G(s)$$

$$= G(s)R(s) - H(s)Y(s)G(s)$$

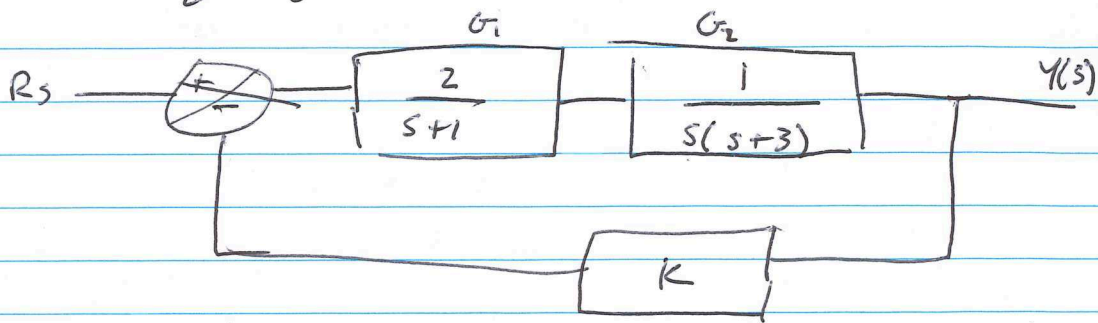
$$Y(s) + H(s)Y(s)G(s) = G(s)R(s)$$

$$Y(s)(1 + H(s)G(s)) = G(s)R(s)$$

$$\frac{Y(s)}{R(s)} = \frac{G(s)}{1 + H(s)G(s)}$$

6

Q3a) To analyse the block diagram, simplify into 1 transfer function.



Merge G_1 & G_2

$$\frac{2}{(s(s+3))(s+1)}$$

~~simplify~~ negative feedback

$$\frac{G(s)}{1 + H(s)G(s)}$$

↓

$$\frac{2}{(s(s+3))(s+1)} \times \frac{s(s+3)(s+1)}{s(s+3)(s+1)}$$

$$1 + K \left(\frac{2}{s(s+3)(s+1)} \right)$$

$$= \frac{2(s(s+3)(s+1))}{s(s+3)(s+1) + 2K}$$

expand $s(s+3)(s+1)$

$$= (s^2 + 3s)(s+1)$$

$$= s^3 + s^2 + 3s^2 + 3s$$

$$= s^3 + 4s^2 + 3s$$

2

$$\frac{2(s^3 + 4s^2 + 3s)}{s^3 + 4s^2 + 3s + 2k}$$

all values of s present and
of the same sign
 $\therefore$ the system can be
stable

1

using routh array to determine range of
 k values that produce a stable system.

$$\begin{array}{l} s^3 \quad 1 \quad 3 \\ s^2 \quad 4 \quad 2k \\ s^1 \quad 3 - \frac{1}{2}k \quad 0 \\ s^0 \quad 2k \end{array}$$



$$\frac{(4 \times 3) - (1 \times 2k)}{4}$$

$$= 3 - \frac{1}{2}k$$

$$\frac{2k(3 - \frac{1}{2}k) - (4 \times 0)}{3 - \frac{1}{2}k}$$

$$= 2k$$

$$= 2k$$

8

for a stable system each value in the first
column must be of the same sign $\therefore k \geq 0$
& not 0

$$3 - \frac{1}{2}k \geq 0$$

$$3 \geq \frac{1}{2}k$$

$$\therefore k \leq 6$$

the range of stable values of k is:

$$0 < k < 6$$

3

Q2b) transforming the diagram into unity feedback

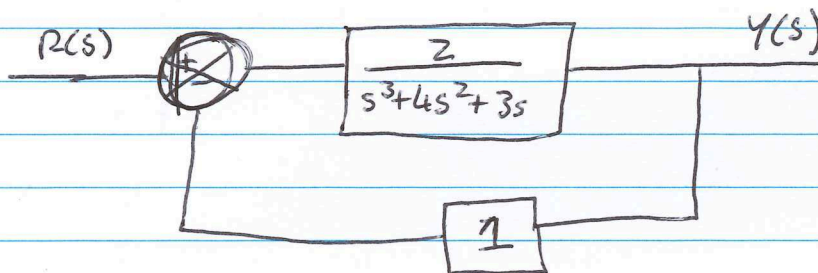
using the overall transfer function found in Q2a.

unity feedback:

$$\frac{G(s)}{1 + G(s)}$$

$$= \frac{2(s^3 + 4s^2 + 3s)}{s^3 + 4s^2 + 3s + 2k} \times \frac{s^3 + 4s^2 + 3s + 2k}{s^3 + 4s^2 + 3s + 2k}$$

For unity feedback $k=1$



$$1 + \frac{2}{s^3 + 4s^2 + 3s} \times \frac{s^3 + 4s^2 + 3s}{s^3 + 4s^2 + 3s}$$

$$= \frac{2(s^3 + 4s^2 + 3s)}{s^3 + 4s^2 + 3s + 2}$$

overall transfer function for unity feedback.

all s^n terms present in denominator without requiring factorisation $\therefore$ there are no $\frac{1}{s}$ (integrators) terms ~~added~~ within the system. $\therefore$ the system is class 0.

Q2c)
3?

I) $s^3 + 10s^2 + 2s + 9$

All s^n terms present & the same sign $\therefore$ system can be stable. ✓

II) $s^5 + 6s^4 + 9s^3 + 2s^2 - s + 4$

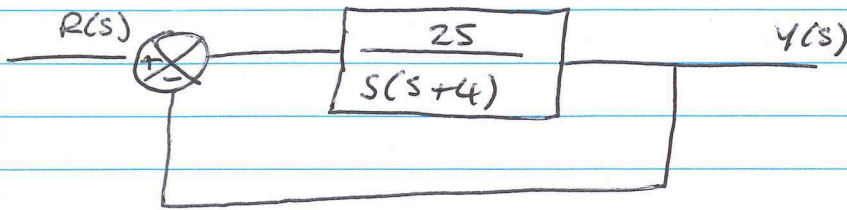
All s^n terms present, however, terms are not all of the same sign $\therefore$ system cannot be stable.

III) $s^4 + 7s^3 + s^2 + 5$ ✓

Not all s^n terms present, however, all terms are the same sign $\therefore$ system can, at best, be critically stable. ✓

3

Q4a) overall transfer function



$$\frac{G(s)}{1 + G(s)}$$

$$= \frac{\frac{25}{s(s+4)}}{1 + \frac{25}{s(s+4)}} \times \frac{s(s+4)}{s(s+4)}$$

$$= \frac{25 \cancel{s(s+4)}}{s(s+4) + 25} \quad \begin{matrix} s(s+4) \\ = s^2 + 4s \end{matrix}$$

$$= \frac{25s^2 + 100s}{s^2 + 4s + 25} \quad \times$$

Q4b) standard form for 2nd order: $\frac{K\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$

I) natural frequency: c.w standard form

$$\begin{aligned} \omega_n^2 &= 25 \\ \omega_n &= 5 \text{ rad/s} \end{aligned}$$

II) damping factor:

$$2\zeta\omega_n = 4$$

$$2\zeta\omega_n = 4$$

$$10\zeta = 4$$

$$\zeta = 0.4$$

/

2

III) damped frequency

when $\zeta \geq 1$ critically damped

$$2\omega_n = 4$$

$$\omega_n = 2$$

as ζ increases, ω_n decreases $\therefore$ damped frequency: $0 < \omega_d < 2$ X

IV) Rise time $0 \rightarrow 100\%$ final value.

$$t_r = \frac{\pi - \tan^{-1}\left(\frac{\sqrt{1-\zeta^2}}{\zeta}\right)}{\omega_n \sqrt{1-\zeta^2}}$$

$$= \frac{\pi - \tan^{-1}\left(\frac{\sqrt{1-0.4^2}}{0.4}\right)}{5 \sqrt{1-0.4^2}}$$

$$\sqrt{1-0.4^2} = 0.9165$$

$$= \frac{\pi - \tan^{-1}\left(\frac{0.9165}{0.4}\right)}{5(0.9165)}$$

$$= \frac{\pi - 1.1593}{4.5825}$$

$$= 0.4323 \text{ seconds}$$

2

v) Peak time

$$t_p = \frac{\pi}{\omega_n \sqrt{1 - \zeta^2}}$$

$$= \frac{\pi}{5(0.9165)} = \frac{\pi}{4.5825} = 0.68565$$

2

vi) % overshoot :

$$100 e^{\frac{-3\pi}{\sqrt{1 - \zeta^2}}}$$

$$= 100 e^{\frac{-0.4(\pi)}{4.5825}} = 100 e^{\frac{-1.2566}{4.5825}} = 100 e^{-0.2742}$$

1

$$= 76.0180\%$$

vii) settling time within $\pm 2\%$

$$t_s = \frac{4}{\zeta \omega_n} = \frac{4}{0.4(5)} = 2$$

2

viii) ~~no~~ oscillations during settling time

$$n = \frac{2}{\pi} \sqrt{\frac{1}{\zeta^2} - 1}$$

$$n = \frac{2}{\pi} \sqrt{\frac{1}{0.4^2} - 1} = \frac{2}{\pi} \sqrt{5.25} = 1.4587$$

2

Q2c)
4?

plot ~~pts~~ poles on argand diagram
poles when denominator = 0

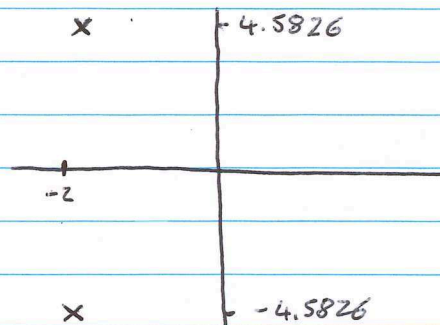
$$\therefore s^2 + 4s + 25 = 0$$

$$\S \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

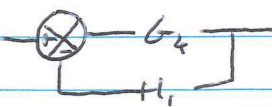
$$\frac{-4 \pm \sqrt{4^2 - 4(1)(25)}}{2(1)}$$

$$= \frac{-4 \pm \sqrt{16 - 100}}{2} = \frac{-4 \pm \sqrt{-84}}{2}$$

$$= \frac{-4 \pm \sqrt{84}}{2} = -2 \pm \sqrt{4.5826}$$



Q5a) beginning with -ve feedback

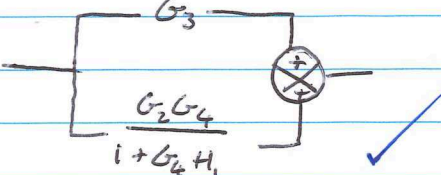
$$\frac{G_4}{1 + G_4 H_1} \quad \checkmark$$


combine with G_2

$$\frac{G_2 G_4}{1 + G_4 H_1} \quad \checkmark \quad \text{--- } G_2 \quad \text{--- } \frac{G_4}{1 + G_4 H_1} \quad \text{---}$$

2

add in parallel with G_3

$$G_3 + \frac{G_2 G_4}{1 + G_4 H_1}$$


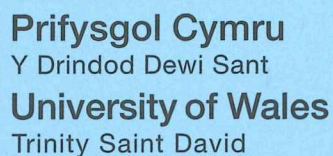
combine with G_1

$$G_1 \left(G_3 + \frac{G_2 G_4}{1 + G_4 H_1} \right) \quad \text{--- } G_1 \quad \text{--- } \frac{G_2 G_4}{1 + G_4 H_1} + G_3 \quad \text{---}$$

simplify -ve feedback

$$\frac{G_1 G_3 + \frac{G_1 G_2 G_4}{1 + G_4 H_1} G_3}{1 + H_2 G \left(G_3 + \frac{G_1 G_2 G_4 H_2}{1 + G_4 H_1} \right)}$$

1



12 Pages

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FORENAME(S): WILLIAM

CONNOR

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[illegible]

Q5b) deriving final expression given values

$$G_1 G_3 + \frac{G_1 G_2 G_4}{1 + G_4 H_1}$$

$$\frac{1 + H_2 G_1 G_3 + \frac{G_1 G_2 G_4 H_2}{1 + G_4 H_1}}{1 + G_4 H_1}$$

$$\Rightarrow G_1 G_3 = 4(2) = 8 \quad G_1 G_2 G_4 = 4\left(\frac{1}{s+1}\right)\left(\frac{1}{s}\right)$$

~~$$\frac{4}{s+1} \times \frac{4}{s}$$~~

$$= \frac{16}{s^2 + s}$$

$$1 + G_4 H_1 = 1 + \frac{1}{s} 2$$

$$= 1 + \frac{2}{s}$$

$$= s + 2$$

$\therefore$

$$= 8 + \frac{4}{s^2 + s} + \frac{16}{s}$$

$$\frac{1 + 8 + \frac{4}{s^2 + s} \times 2}{s + 2} + \frac{2}{s} + \frac{16}{s}$$

$$= 8s + 16 + \frac{4}{s^2 + s}$$

$$\frac{s + 2 + 8s + 16 + \frac{8}{s^2 + s}}{s^2 + s}$$

$$= \frac{8s + 16 + \frac{4}{s^2 + s}}{s^2 + s} \times \frac{s^2 + s}{s^2 + s}$$

$$= \frac{9s + 18 + \frac{8}{s^2 + s}}{s^2 + s}$$

$$= \frac{8s^3 + 8s^2 + 16s^2 + 16s + 4}{9s^3 + 18s^2 + 18s + 8}$$

$$= \frac{8s^3 + 24s^2 + 16s + 4}{9s^3 + 27s^2 + 18s + 8}$$

Q5c) analyse system to find poles & zeroes

$$\frac{8s^3 + 24s^2 + 16s + 4}{9s^3 + 27s^2 + 18s + 8}$$

check method.

for poles denominator = 0

factorise s

$$s(9s^2 + 27s + 18) + 8 = 0$$

$$s(9s^2 + 27s + 18) = -8$$

↓

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-27 \pm \sqrt{27^2 - 4(9)(18)}}{2(9)} = \frac{-27 \pm \sqrt{729 - 648}}{18}$$

$$= \frac{-27 \pm 9}{18} = -2 \text{ or } -1$$

X

∴

$$\frac{9s^2 + 27s + 18}{9} = (s+2)(s+1)$$

$$\therefore \frac{s}{9} (s+2)(s+1) = -\frac{8}{9}$$

$$\frac{s}{9} (s+2)(s+1) = -\frac{8}{9} \quad s = -8$$

$$s+2 = -\frac{8}{9}$$

$$s = -\frac{10}{9} = -2.89$$

$$s+1 = -\frac{8}{9}$$

$$s = -\frac{17}{9} = -1.89$$

for zeroes ~~denominator~~ $= \infty$
 numerator $= 0$

$$8s^3 + 24s^2 + 16s + 4 = 0$$

$$\div 8$$

$$s^3 + 3s^2 + 2s + \frac{1}{2} = 0$$

$$s(s^2 + 3s + 2) = -\frac{1}{2}$$

$$s((s+2)(s+1)) = -\frac{1}{2}$$

$$s = \frac{4}{3} - \frac{1}{2}$$

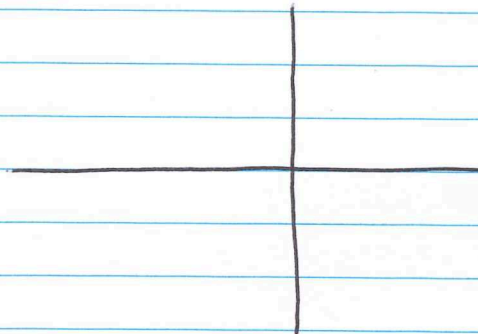
$$s+2 = -\frac{1}{2}$$

$$s = \cancel{1.5} - 2.5$$

$$s+1 = -\frac{1}{2}$$

$$s = \cancel{0.5} - 1.5$$

X



i've done all of this wrong... oops