

EXAMINATION INTERNAL VERIFICATION FORM (2017-2018)

MODULE :	PSDE5011 Control and Instrumentation	LEVEL:	5
PROGRAMME :	BSc (hons) Electrical and Electronic Engineering	TERM OF EXAM: (SPRING OR SUMMER)	Summer
LEARNING OUTCOME(S) BEING ASSESSED:	MILO1, MILO2	EXAM PAPER:	MAIN EXAMINATION AUTHOR : R. Davies

VERSION:	DATE SUBMITTED FOR VERIFICATION:	INTERNAL VERIFIER COMMENTS (Approval or detail of amendments required):	DATE:
1.	23-02-18	Minor suggestions as per email to consider, otherwise the paper conforms to the specification and is ready to be sent for external moderation.	27/02/2018
2.	03-03-18	Completed	03-03-18

INTERNAL VERIFIER PLEASE TICK TO INDICATE THAT THE EXAMINATION:	
✓	Conforms to the assessment pattern described in the module syllabus
✓	Tests the module learning outcome(s) appropriately

SIGNATURES OF FINAL APPROVAL:	SIGNATURE	DATE
INTERNAL VERIFIER: I confirm that this version of the exam paper is ready to be sent to the external examiner		03/03/2018
EXAM AUTHOR: I confirm that internal verifier and external examiner approval have been received and that this version of the exam paper is ready for printing.		

**UNIVERSITY OF WALES:
TRINITY SAINT DAVID**



**PRIFYSGOL CYMRU:
Y DRINDOD DEWI SANT**

LOCATION: COLEG SIR GAR

LLEOLIAD: COLEG SIR GAR

LEVEL 5

LEFEL 5

**EXAMINATION PERIOD
MAY 2018**

**CYFNOD ARHOLIADAU
MAI 2018**

BSc ELECTRICAL AND ELECTRONIC ENGINEERING

MODULE CODE: PSDE5011

CONTROL AND INSTRUMENTATION

TIME: 2 HOURS

ANSWER ANY THREE QUESTIONS

TOTAL MARKS = 102

NUMBER OF PAGES = 6

Equipment or materials required – Calculators are permitted

Adnoddau neu ddefnyddiau sydd eu hangen – Caniateir cyfrifiannell

The back of this assignment contains a formula sheet.

Mae cefn yr aseiniad hwn yn cynnwys daflen fformiwla.

QUESTION 1

- (a) A resistance temperature detector (RTD) has a first order response to a step change in its ambient temperature. The following information is given about the probe and the liquid into which it is placed.

$\theta_L(t)$ = temperature of the liquid at time t ($^{\circ}\text{C}$)
 $\theta_R(t)$ = temperature of the RTD at time t ($^{\circ}\text{C}$)
 M = mass of the RTD (in Kg)
 C_p = specific heat capacity of the RTD ($\text{J/Kg } ^{\circ}\text{C}$)
 U = overall heat transfer coefficient ($\text{W/m}^2/ ^{\circ}\text{C}$)
 A = surface area of the probe (m^2)

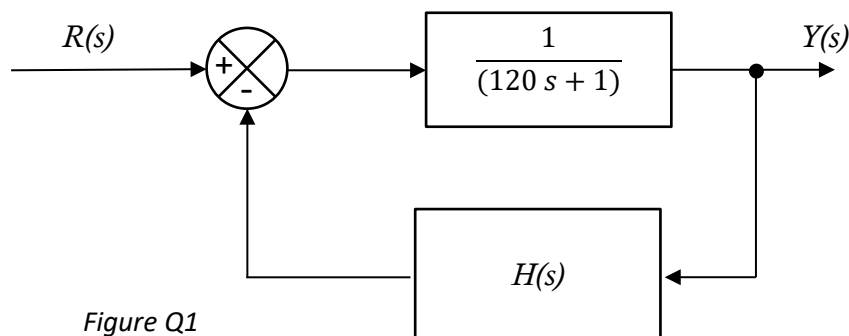
Given the following differential equation for the transfer of heat to the probe:

$$UA (\theta_L(t) - \theta_R(t)) = M C_p \frac{d\theta_R(t)}{dt}$$

- (i) Use the Laplace transform method to find the transfer function of the probe $H(s)$. [8 marks]
- (ii) Use inverse Laplace transform to find the time dependent equation for the probe temperature for a step response when, having been kept in ice, the probe is plunged into a liquid at a temperature θ_L at time $t = 0$. [7 marks]
- (iii) Sketch and describe the shape of the dynamic response curve of the probe temperature $\theta_R(t)$. [4 marks]
- (iv) A particular RTD has: $M = 0.1\text{Kg}$, $C_p = 3250 \text{ J/Kg } ^{\circ}\text{C}$, $U = 783.13 \text{ W/m}^2/ ^{\circ}\text{C}$, $A = 0.05\text{m}^2$ and $\theta_L = 75 ^{\circ}\text{C}$, calculate the temperature of the RTD at $t = 25$ seconds. [3 marks]

- (b) Figure Q1 shows a block diagram of a heating system that uses the above RTD in its feedback loop.

- (i) Find the overall transfer function of the system and show the system is overdamped. [8 marks]
- (ii) Using the final value theorem, determine the steady output temperature for a step input of 75. [4 marks]



Total for this question is 34 marks

QUESTION 2

- (a) Figure Q2.1 shows a control system involving a motor with both position and velocity feedback. Determine the damping factor with and without the velocity feedback. Hence explain the effects of velocity feedback on the output of the system in response to a unit step input.

[18 marks]

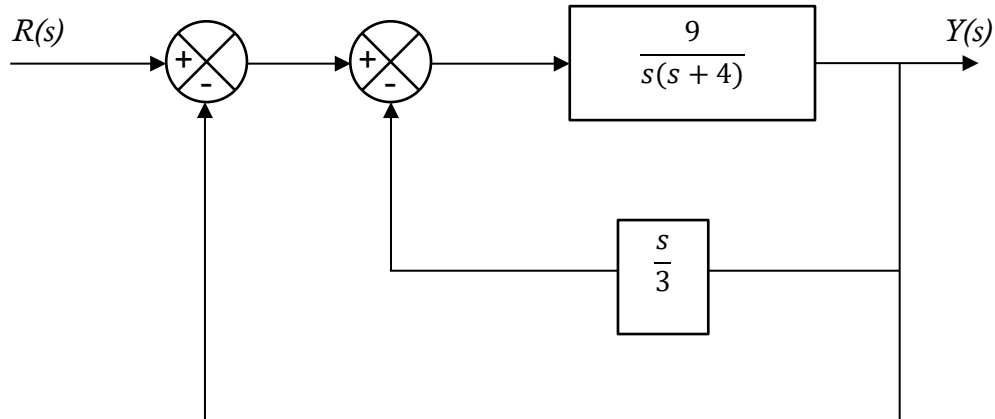


Figure Q2.1

- (b) Using a simple sketch, explain how the relative stability of a control system can be determined from magnitude and phase Bode plots using GAIN MARGIN and PHASE MARGIN.

[11 marks]

- (c) Using the method explained in part (b), determine the stability of the system from its Bode plot shown in figure Q2.2.

[5 marks]

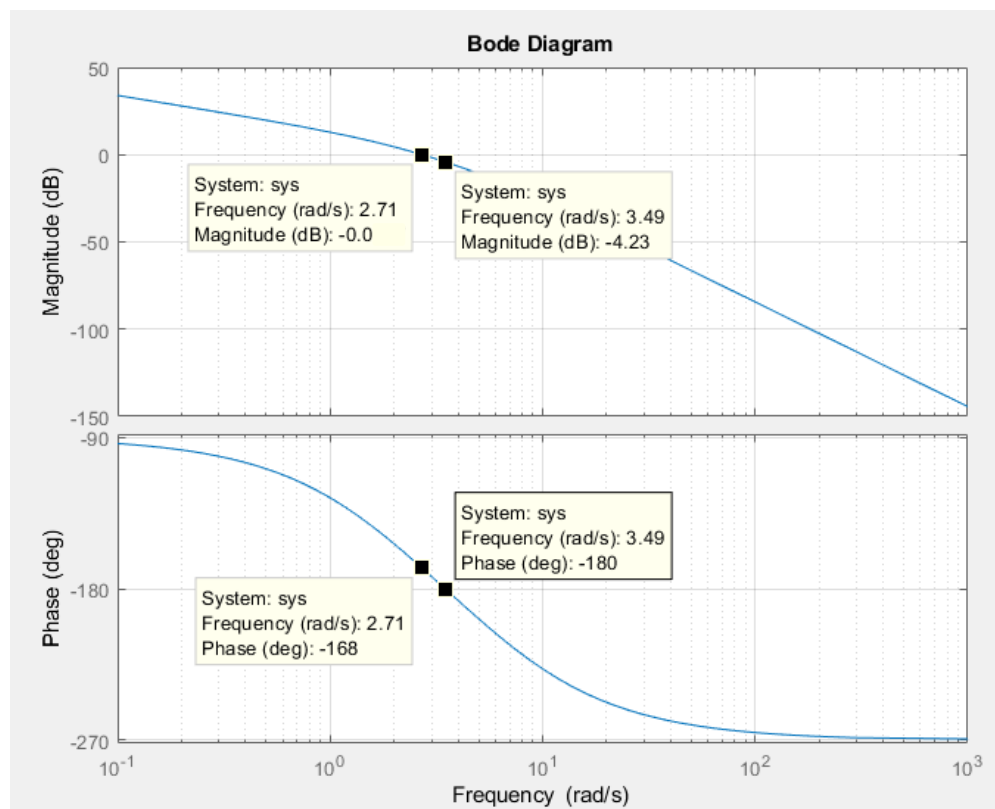


Figure Q2.2

Total for this question is 34 marks

QUESTION 3

The block diagram of a proposed control system is shown in figure Q3.1,

- (a) From the root locus diagram shown in figure Q3.2, determine (i) the open-loop transfer function $G(s)$, (ii) the natural frequency and (iii) damping factor.

[12 marks]

- (b) Determine the closed-loop transfer function in terms of K_1 and the value of K_1 for critical damping.

[8 marks]

- (c) For $K_1 = 3.92$, calculate (i) the oscillation frequency, (ii) pole locations and (iii) percentage overshoot to a step input.

[12 marks]

- (d) Explain why this system should be stable for all values of K_1 .

[2 marks]

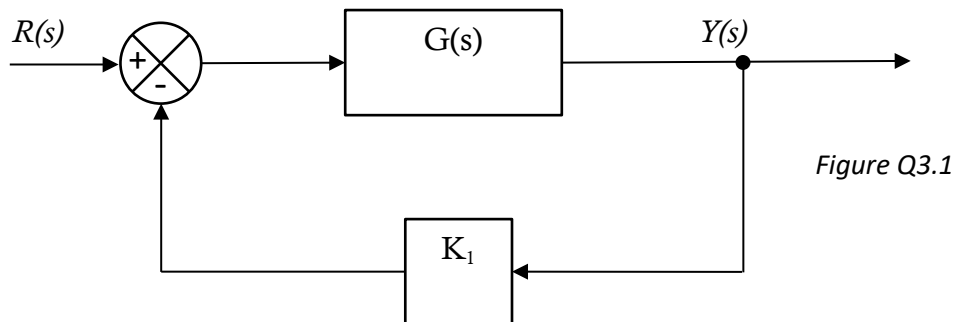


Figure Q3.1

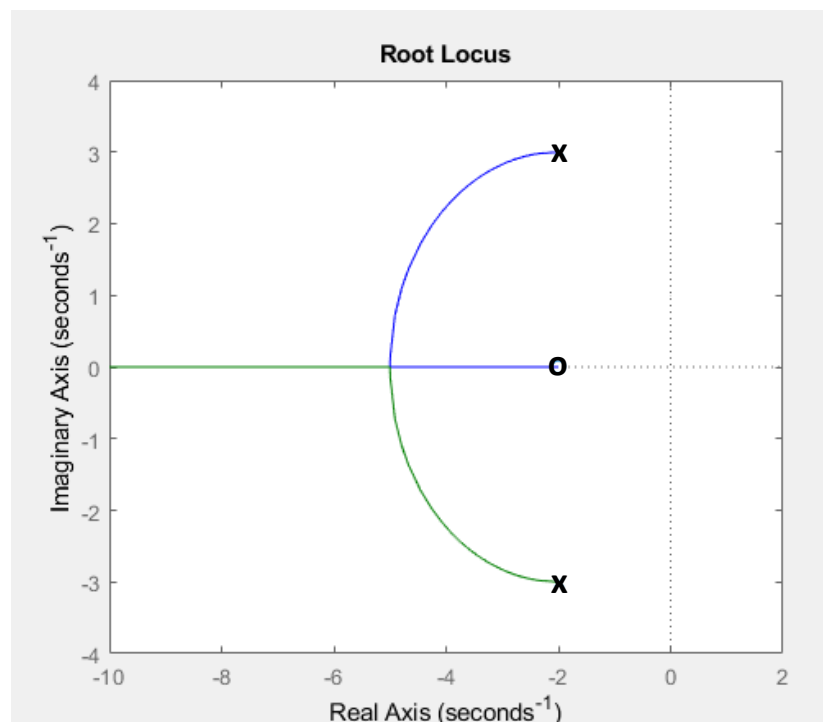


Figure Q3.2

Total for this question is 34 marks

QUESTION 4

(a) Define the following terms for a transducer using the data from table Q4 as an example:

- (i) FSD
- (ii) Static sensitivity
- (iii) Non-linearity error
- (iv) Hysteresis error

[11 marks]

(b) Table Q4 (on the next page) shows data for the calibration check of a pressure transducer with readings taken in Bar and Volts. Using the table and graph paper on the next page, fill in the empty columns and plot the calibration curve for rising and falling pressures. Determine the maximum non-linearity error and the maximum hysteresis error as percentages of the FSD, and identify them on the graph. **Don't forget to detach the table and the graph, and insert into your answer booklet.**

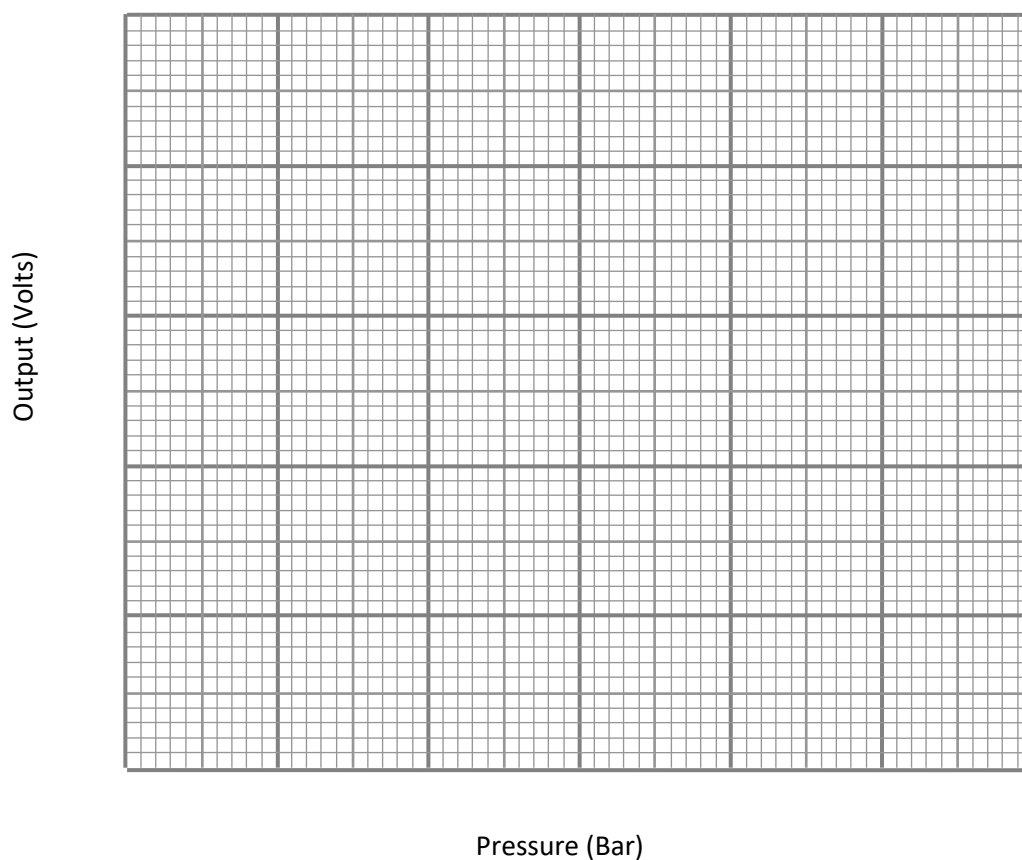
[23 marks]

Total for this question is 34 marks

Table Q4

Measured Pressure (Bar)	Output voltage (Volts)		Linear Value (Volts)	Non-linearity error (Volts)		Hysteresis Error (Volts)
	rising	falling		rising	falling	
0	0	0				
0.5	0.96	1.03				
1.0	1.89	2.12				
1.5	2.81	3.17				
2.0	3.77	4.28				
2.5	4.68	5.33				
3.0	5.61	6.25				
3.5	6.73	7.16				
4.0	7.88	8.10				
4.5	8.94	9.05				
5.0	10.0	10.0				

Graph for calibration results for QUESTION 4.



THIS PAGE MUST BE DETACHED FROM THE QUESTION PAPER AND INSERTED INTO YOUR ANSWER BOOKLET IF YOU ATTEMPT QUESTION 4.

LAPLACE TRANSFORM SHEET

Time function $y(t)$	Laplace Transform $Y(s)$
1	$\frac{1}{s}$
k	$\frac{k}{s}$
k t	$\frac{k}{s^2}$
e^{at}	$\frac{1}{s - a}$
e^{-at}	$\frac{1}{s + a}$
$1 - e^{-at}$	$\frac{a}{s(s + a)}$
$\frac{dy(t)}{dt}$	$sY(s) - y(0)$
$\frac{d^2y(t)}{dt^2}$	$s^2Y(s) - s y(0) - \frac{dy(0)}{dt}$
$\sin(\omega t)$	$\frac{\omega}{s^2 + \omega^2}$
$\cos(\omega t)$	$\frac{s}{s^2 + \omega^2}$

Final Value Theorem

$$y(\infty) = \lim_{t \rightarrow \infty} \{y(t)\} = \lim_{s \rightarrow 0} \{s Y(s)\}$$

**UNIVERSITY OF WALES:
TRINITY SAINT DAVID**



**PRIFYSGOL CYMRU:
Y DRINDOD DEWI SANT**

LOCATION: COLEG SIR GAR

LLEOLIAD: COLEG SIR GAR

LEVEL 5

LEFEL 5

**EXAMINATION PERIOD
MAY 2018**

**CYFNOD ARHOLIADAU
MAI 2018**

**MODULE CODE: PSDE5011
CONTROL AND INSTRUMENTATION**

MAIN Solutions

QUESTION 1

Q1 (a) (i) From

$$UA (\theta_L(t) - \theta_R(t)) = MC_p \frac{d\theta_R(t)}{dt}$$

$$UA \theta_L(t) - UA \theta_R(t) = MC_p \frac{d\theta_R(t)}{dt}$$

$$UA \theta_L(t) = MC_p \frac{d\theta_R(t)}{dt} + UA \theta_R(t)$$

[2 marks]

Assuming initial conditions $\theta_R(0) = 0$ at $t = 0$ of the probe RTD

Taking Laplace Transform with initial conditions of zero

$$UA \theta_L(s) = MC_p s \theta_R(s) + UA \theta_R(s)$$

[2 marks]

$$UA \theta_L(s) = \theta_R(s) [MC_p s + UA]$$

[1 mark]

$\div$ both sides by UA

$$\theta_L(s) = \theta_R(s) \left[\frac{MC_p s}{UA} + 1 \right]$$

[1 mark]

$$\text{Let } \tau = \frac{MC_p}{UA}$$

[1 mark]

$$\therefore \theta_L(s) = \theta_R(s) [\tau s + 1]$$

$$\therefore \theta_R(s) = \frac{1}{\tau s + 1} \theta_L(s)$$

[1 mark]

Q1 (a) (ii) If the probe was plunged into a medium of temperature θ_L at time $t = 0$, this would be a step input of $\frac{\theta_L}{s}$

[1 mark]

$$\therefore \theta_R(s) = \frac{1}{(\tau s + 1)} \times \frac{\theta_L}{s}$$

$$= \frac{\theta_L}{s(\tau s + 1)}$$

[1 mark]

Taking inverse Laplace transforms

$$\theta_R(t) = \mathcal{L}^{-1} \left\{ \frac{\theta_L}{s(\tau s + 1)} \right\} = \theta_L \mathcal{L}^{-1} \left\{ \frac{1}{s(\tau s + 1)} \right\}$$

[1 mark]

This is similar to $\mathcal{L}^{-1} \left\{ \frac{a}{s(s+a)} \right\} = 1 - e^{-at}$

[1 mark]

Putting our equation into a standard form

$$\frac{1}{s(\tau s + 1)} = \frac{1/\tau}{s(\tau s + 1)/\tau} \quad \text{top and bottom divided by } \tau$$

$$= \frac{1/\tau}{s(s + 1/\tau)}$$

[1 mark]

We can now see that if $1/\tau = a$, we have the same transform.

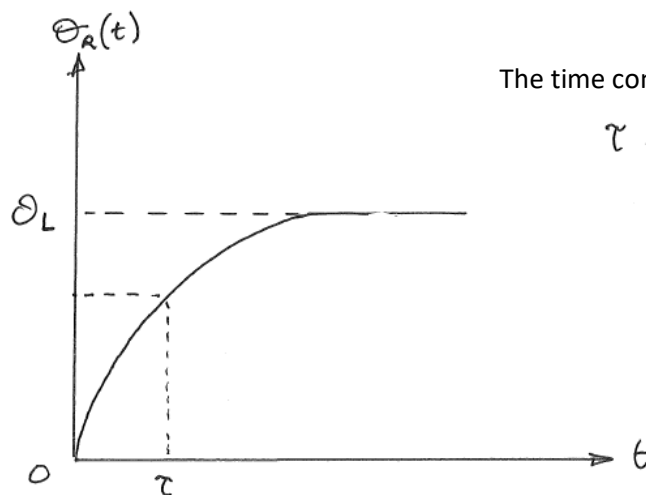
[1 mark]

$$\theta_R(t) = \theta_L \mathcal{L}^{-1} \left\{ \frac{1/\tau}{s(s + 1/\tau)} \right\} = \theta_L (1 - e^{-t/\tau})$$

[1 mark]

Q1 (a) (iii) This is an exponential rise with a time constant of τ reaching a final value of θ_L as expected.

[1 mark]



The time constant in seconds is

$$\tau = \frac{MC_P}{UA}$$

[3 marks]

Q1 (a) (iv)

$$\tau = 0.1 \times 3250 / (783.13 \times 0.05) = 8.3 \text{ seconds}$$

At $t = 25$ seconds, with $\theta_L = 75^\circ\text{C}$,

$$\theta_R = 75 (1 - e^{-25/8.3}) = 71.31^\circ\text{C}$$

[3 marks]

Q1 (b) (i)

Overall transfer function is given by

$$\begin{aligned}
 T(s) &= \frac{G(s)}{1 + H(s)G(s)} \\
 &= \frac{1}{120s + 1} \div \left[1 + \frac{1}{(8.3s + 1)(120s + 1)} \right] \\
 &= \frac{1}{(120s + 1) + \frac{1}{8.3s + 1}} \\
 &= \frac{8.3s + 1}{(120s + 1)(8.3s + 1) + 1} \\
 &= \frac{8.3s + 1}{996s^2 + 128.3s + 2}
 \end{aligned}$$

÷ top and bottom by
996

$$T(s) = \frac{0.00833s + 0.001}{s^2 + 0.1288s + 0.002}$$

[5 marks]

c.f. denominator with $s^2 + 2\zeta\omega_n s + \omega_n^2$

$$\therefore \omega_n^2 = 0.002$$

$$\omega_n = 0.0448 \text{ rad/s}$$

$$\therefore 2\zeta\omega_n = 0.1288$$

$$\zeta = \frac{0.1288}{2 \times 0.0448}$$

$$= 1.4375$$

which is > 1 $\therefore$ overdamped.

[3 marks]

1(b)(ii) For a step input of 75

$$R(s) = \frac{75}{s}$$

$$\therefore Y(s) = \frac{75}{s} \times T(s)$$

[1 mark]

Using the final value theorem

$$y(\infty) = \lim_{s \rightarrow 0} [s Y(s)]$$

$$= \lim_{s \rightarrow 0} \left[\cancel{s} \times \frac{75}{\cancel{s}} \times \frac{(0.00833s + 0.001)}{(s^2 + 0.1288s + 0.002)} \right]$$

$$= \frac{0.001}{0.002} \times 75$$

$$= 37.5^\circ \text{C}$$

[3 marks]

Total marks for question 1 = 34

QUESTION 2

without velocity feedback

(a)

$$\begin{aligned}
 T(s) &= \frac{9/s(s+4)}{1 + 1 \times 9/s(s+4)} \\
 &= \frac{9}{s(s+4) + 9} \\
 &= \frac{9}{s^2 + 4s + 9}
 \end{aligned}$$

[3 marks]

c.f. standard second order numerator

$$s^2 + 2\zeta\omega_n s + \omega_n^2$$

[1 mark]

$$\begin{aligned}
 \therefore \omega_n^2 &= 9 \\
 \omega_n &= \sqrt{9} \\
 &= 3 \text{ rad/s}
 \end{aligned}$$

$$\therefore 2\zeta\omega_n = 4$$

$$\begin{aligned}
 \zeta &= \frac{4}{2 \times 3} \\
 &= 0.667
 \end{aligned}$$

[3 marks]

which is underdamped giving oscillations

with velocity feedback

$$\begin{aligned}
 T_1(s) &= \frac{9/s(s+4)}{1 + \frac{s}{3} \times \frac{9}{s(s+4)}} \\
 &= \frac{9}{s(s+4) + 3s} \\
 &= \frac{9}{s^2 + 4s + 3s} \\
 &= \frac{9}{s^2 + 7s}
 \end{aligned}$$

[3 marks]

overall transfer function is

$$T_2(s) = \frac{9/(s^2 + 7s)}{1 + 9/(s^2 + 7s)}$$

$$= \frac{9}{s^2 + 7s + 9}$$

[3 marks]

$$\therefore \omega_n^2 = 9, \quad \omega_n = 3 \text{ rad/s}$$

$$2\zeta\omega_n = 7$$

$$\zeta = \frac{7}{2 \times 3} = 1.16$$

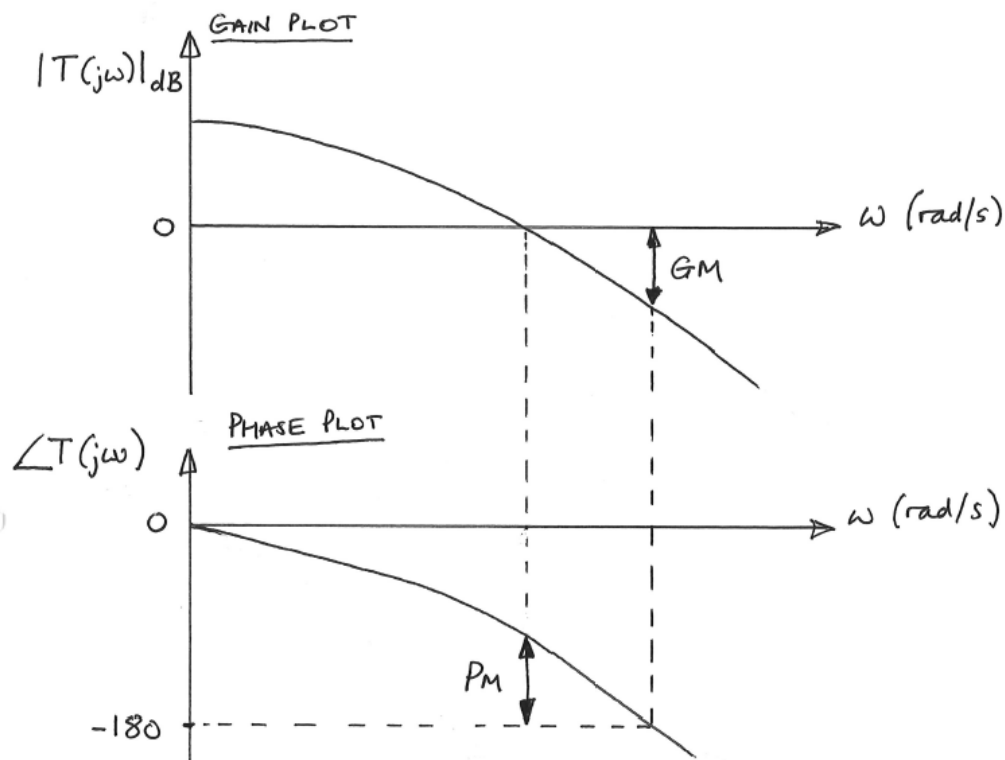
[3 marks]

which is over damped $\therefore$ no oscillations.

$\therefore$ Velocity feedback has altered the damping and removed the oscillations

[2 marks]

(b) The diagram below shows gain and phase plots.



[5 marks]

From the Bode Magnitude diagram (gain plot), the gain in dB is measured at the point where the Bode Phase diagram reaches -180 degrees. For stability, the gain at this point must be less than 0dB. The GAIN MARGIN (GM) will be the absolute value of this gain in dB and is an indicator of how close the system is to instability. Typically, a GM > 10dB gives stability.

[3 marks]

From the Bode Phase diagram, the phase in degrees is measured at the point where the Bode Magnitude reaches 0dB. For stability, the phase at this point must not be -180 degrees. The PHASE MARGIN will be the absolute value of the difference between the measured phase angle and -180 degrees and is also an indicator of how close the system is to instability. Typically, a PM of 45 degrees gives stability.

(c) From figure Q2.2,

[3 marks]

At 2.71 rad/s, Gain Margin = 4.23dB when phase = -180 degrees

[2 marks]

At 3.49 rad/s, Phase Margin = $180 - 168 = 12$ degrees.

[2 marks]

This system will be unstable since GM < 10dB and PM < 45 degrees.

[1 mark]

Total for this question is 34 marks

QUESTION 3

(a) (i)

From the root locus diagram, open loop poles occur at $-2+j3$ and $-2-j3$

[2 marks]

There is one zero at -2

[1 mark]

Therefore, the transfer function $G(s)$ is

$$\begin{aligned}
 G(s) &= \frac{s - (-2)}{(s - (-2+j3))(s - (-2-j3))} \\
 &= \frac{s+2}{(s+2-j3)(s+2+j3)} \\
 &= \frac{s+2}{(s^2 + 2s + j3s + 2s + 4 + j6 - j3s - j6 - j^2 9)} \\
 &= \frac{s+2}{(s^2 + 4s + 4 + 9)} \\
 &= \frac{s+2}{(s^2 + 4s + 13)}
 \end{aligned}$$

(a) (ii)

[5 marks]

c.f. denominator with standard equation

$$s^2 + 2\zeta\omega_n s + \omega_n^2$$

[1 mark]

$$\therefore \omega_n^2 = 13, \quad \omega_n = 3.606 \text{ rad/s}$$

[1 mark]

(a) (iii) $\therefore 2\zeta\omega_n = 4$

$$\zeta = \frac{4}{2 \times 3.606} = 0.555 \text{ underdamped.}$$

[2 marks]

QUESTION 3 continued

(b)

closed loop transfer function is given by

$$T(s) = \frac{G(s)}{1 + H(s)G(s)} \quad [1 \text{ mark}]$$

$$\text{where } G(s) = \frac{s+2}{s^2+4s+13} \quad \text{and } H(s) = K_1 \quad [1 \text{ mark}]$$

$$\begin{aligned} \therefore T(s) &= \frac{(s+2)/(s^2+4s+13)}{1 + K_1(s+2)/(s^2+4s+13)} \\ &= \frac{s+2}{(s^2+4s+13) + K_1(s+2)} \\ &= \frac{s+2}{s^2 + (4+K_1)s + (13+2K_1)} \end{aligned} \quad [3 \text{ marks}]$$

For critical damping, poles meet at -5 [1 mark]

$$\therefore T(s) = \frac{s+2}{(s+5)^2} = \frac{s+2}{s^2+10s+25} \quad [1 \text{ mark}]$$

Equating denominators

$$\begin{aligned} 4 + K_1 &= 10 & \text{or} & & 13 + 2K_1 &= 25 \\ \therefore K_1 &= 6 & & & 2K_1 &= 12 \\ & & & & K_1 &= 6 \end{aligned} \quad [1 \text{ mark}]$$

QUESTION 3 continued

(c) (i)

$$\omega = \omega_n \sqrt{1 - \zeta^2}$$

[1 mark]

$$\text{When } K_1 = 3.92$$

$$\begin{aligned}\omega_n^2 &= 13 + 2K_1 \\ &= 13 + 2 \times 3.92 \\ &= 20.84\end{aligned}$$

$$\omega_n = 4.565 \text{ rad/s}$$

[1 mark]

$$\therefore 2\zeta\omega_n = 4 + K_1$$

$$\begin{aligned}\zeta &= \frac{4 + 3.92}{2 \times 4.565} \\ &= 0.867\end{aligned}$$

[1 mark]

$$\begin{aligned}\therefore \omega &= 4.565 \times \sqrt{1 - 0.867^2} \\ &= 2.27 \text{ rad/s}\end{aligned}$$

[1 mark]

(c) (ii)

$$\begin{aligned}T(s) &= \frac{s+2}{s^2 + (4 + 3.92)s + (13 + 2 \times 3.92)} \\ &= \frac{s+2}{s^2 + 7.92s + 20.84}\end{aligned}$$

[2 marks]

Poles occur at

$$s = \frac{-7.92 \pm \sqrt{7.92^2 - 4 \times 1 \times 20.84}}{2 \times 1}$$

[1 mark]

$$\begin{aligned}
 S &= \frac{-7.92 \pm \sqrt{62.7264 - 83.36}}{2} \\
 &= \frac{-7.92 \pm \sqrt{-20.634}}{2} \\
 &= \frac{-7.92 \pm j\sqrt{20.634}}{2} \\
 S &= -3.96 \pm j 2.27
 \end{aligned}$$

[3 marks]

(c) (iii)

$$\begin{aligned}
 \%OS &= 100 e^{\frac{-\frac{3\pi}{2}}{\sqrt{1-3^2}}} \\
 &= 100 e^{-0.867\pi / \sqrt{1-0.867^2}} \\
 &= 0.43 \%
 \end{aligned}$$

[2 marks]

(d) From the root locus diagram, poles lie in the left hand plane for all values of gain.

[2 marks]

Total for this question is 34 marks

QUESTION 4

(a) (i) FSD is the full scale deflection or maximum output of a transducer. In this case FSD = 10Volts

(ii) This is defined as

[2 marks]

$$\text{Static sensitivity} = \frac{\text{output range of a transducer}}{\text{input range of transducer}}$$

$$\text{In this case} = 10 \text{ volts} / 5 \text{ Bar} = 2 \text{ Volts/Bar}$$

[3 marks]

(iii) This is defined as the difference between the linear value and the actual value.

For example, at 1.5 Bar, the linear value should be $1.5 \times 2 = 3\text{V}$ but the actual rising value = 2.81V.
The non-linearity error = $2.81 - 3 = -0.19\text{V}$

[3 marks]

(iv) This is defined as the difference between the rising and falling output voltages.

For example, at 1.5 Bar, the rising voltage = 2.81V but the falling voltage = 3.17.
The hysteresis error = $3.17 - 2.81 = 0.36\text{V}$

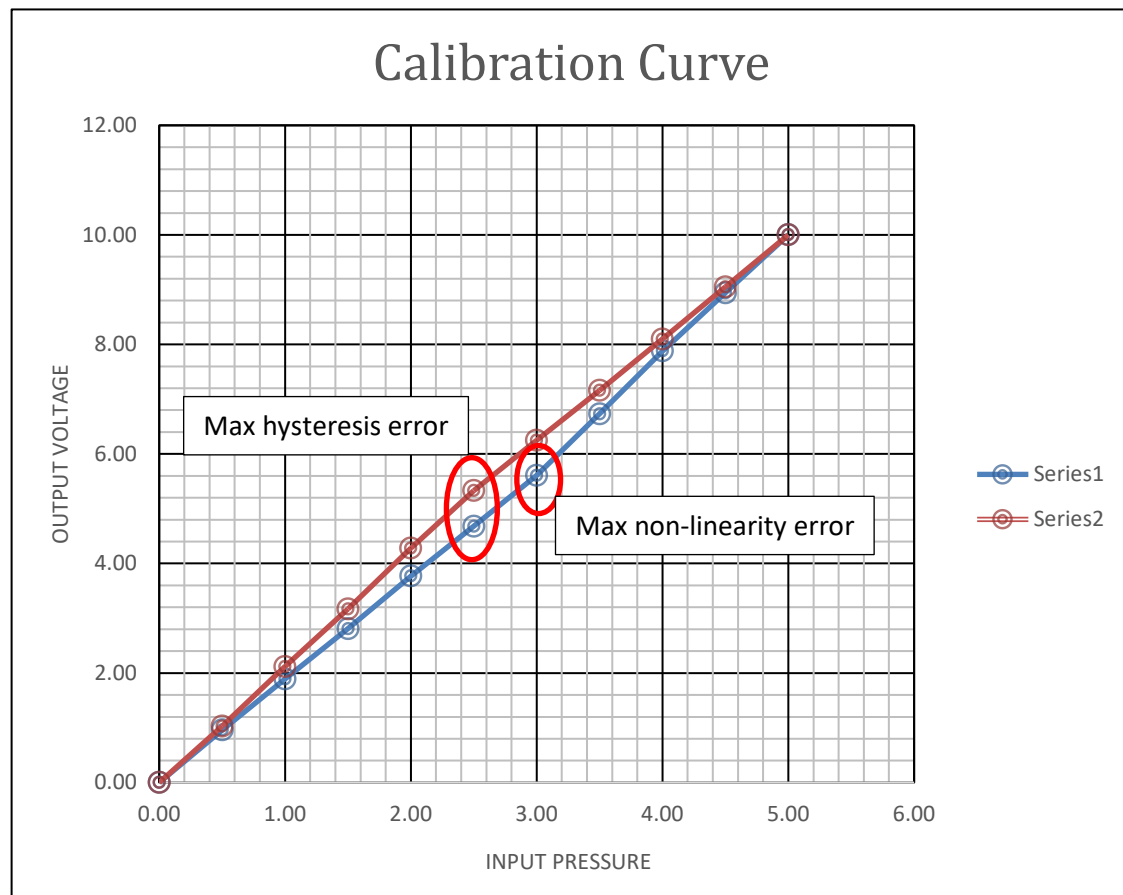
[3 marks]

(b)

Measure and Pressure (Bar)	Output voltage (Volts)		Linear Value (Volts)	Non-linearity error (Volts)		Hysteresis Error (Volts)
	rising	falling		rising	falling	
0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.50	0.96	1.03	1.00	-0.04	0.03	0.07
1.00	1.89	2.12	2.00	-0.11	0.12	0.23
1.50	2.81	3.17	3.00	-0.19	0.17	0.36
2.00	3.77	4.28	4.00	-0.23	0.28	0.51
2.50	4.68	5.33	5.00	-0.32	0.33	0.65
3.00	5.61	6.25	6.00	-0.39	0.25	0.64
3.50	6.73	7.16	7.00	-0.27	0.16	0.43
4.00	7.88	8.10	8.00	-0.12	0.10	0.22
4.50	8.94	9.05	9.00	-0.06	0.05	0.11
5.00	10.00	10.00	10.00	0.00	0.00	0.00

[10 marks]

Series 1 = rising, series 2 = falling



[10 marks]

The maximum linearity is 0.39 volt, as a percentage FSD = $0.39/10 * 100 = 0.039\%$

The maximum hysteresis error is 0.65 volts, as a percentage FSD = $0.65/10 * 100 = 0.065\%$

[3 marks]