



Stochastic modelling of abrasive waterjet footprints using finite element analysis



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ARTICLE INFO

Article history:

Received 26 January 2015

Received in revised form

5 May 2015

Accepted 10 May 2015

Available online 12 May 2015

Keywords:

Abrasive waterjet milling

Finite element modelling

Stochastic modelling

Monte Carlo methods

ABSTRACT

The proposal of erosion models to predict the jet footprint during abrasive waterjet machining is a key element for the development of this technology, but it is very challenging because of the inherent fluctuations of the process. This issue becomes critical when the size of the cutting systems is reduced, since the relative size of these deviations increases. The present paper considers for the first time a modelling framework capable of predicting the average shape of AWJM footprints and, of great novelty, the variability along the trench, combining finite element analysis and Monte Carlo methods, and verifying the model using different feed speeds and tilt angles. For that purpose, the relevance of each random parameter, such as shape (sharpness), size and relative orientation of the abrasive particles, has been investigated through parametric studies on these variables. Multiple particle simulations with randomly generated input were performed to determine the effect of operating parameters in the overall variability of the jet footprint. The process was simulated using Abaqus 6.14 as multiple garnet particles hitting a target of Ti–6Al–4V at very high velocity, eroding the target by plastic deformation and material removal. The model shows successfully the influence of single particle parameters, such as the shape, on the surface variability. The results for the footprint variability show that stochastic methods are suitable to model these fluctuations, and it is also shown that this approach yields accurate estimates of the average profile after multiple jet passes with error less than 5%.

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1. Introduction

Abrasive waterjet machining is as a non-traditional machining method that can be used to machine almost any material. With the development of innovative materials with advanced properties, high-value manufacturing companies need to consider alternative processing techniques that can cope with new requirements while keeping manufacturing costs low [1]. Abrasive waterjet has been shown to be a feasible option in many cases when other non-traditional methods, such as laser ablation or EDM, cannot be used. In short, its main advantages are (a) it can be used to machine almost any material, regardless of its properties; from ceramics [2], to polycrystalline diamond [3]; (b) the removal process involves low cutting forces on the workpiece, hence reducing the risk of mechanical damage [4]; (c) the heat affected zone produced during the process is minimal; (d) although it depends on the workpiece characteristics and the complexity of the design, AWJ is a fast manufacturing process when compared with

other non-traditional processes.

The process is powered by a high-pressure water pump, which is capable of delivering up to 600 MPa [5]. The water is forced through a small orifice, which generates a high speed water jet. At this stage, the abrasives are entrained into the system in the so-called mixing chamber and accelerated. This process involves large fluctuations at every stage, such as unpredictable variations in the pressure and uncertainties in the particle entrainment mechanism. As a result of this process, a high-energy mixture of particles hits the workpiece, generating a footprint that depends on different parameters: (i) the pressure in the pump, which affects the kinetic energy of the particles; (ii) the abrasive mass flow and the feed speed, which control the number of particles impacting the target at a given area; (iii) the impingement angle [6]. These parameters can easily be changed by the operator, and they can be chosen to optimize the process in each case.

These fluctuations in the system are one of the most significant obstacles that are impeding the development of AWJ controlled-depth milling, whereas AWJ cutting is a well-established technique. The reason is that the fluctuations are directly transferred to the milled surface, yielding large variations that are difficult to control in order to generate accurate complex features. This also

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makes it difficult to find the appropriate parameters to generate high-quality surfaces with an acceptable surface roughness for critical applications. Mathematical modelling is a key element for further development of this process: it will enable the development of better milling strategies and improve the current understanding of the process, thereby enabling manufacturers to develop AWJ machines with enhanced characteristics for controlled-depth milling.

There is a significant amount of information about the effects of the operating parameters on the AWJM process [7]. However, the information about the relation between these operating parameters and the physical properties of the jet is very limited; partly because of the difficulty of developing sensing systems that can be implemented at the machine. Balz et al. made a significant contribution by measuring the spatial distribution of the velocities of the abrasive particles [8] using laser-induced lithography; in further work, they measured the spatial mass distribution of the particles within the jet and the velocity distribution [9] using X-Ray velocimetry from a Synchrotron Light Source. This work also shows experimental evidence of the fluctuating nature of Abrasive Waterjets, reporting that values such as the velocity of the particles are not deterministic, but follow a statistical distribution. As an alternative to the lack of information of the physical conditions within the jet, several modelling frameworks have been developed to predict the jet footprint using experimental data to calibrate the model. The work presented in [10,11] is based on a preliminary calibration step to make fast predictions of the jet footprint at different operating parameters, considering multiple jet passes and tilted impact angles. These methods are focused on predicting the average jet footprint in order to develop strategies to machine complex 3D shapes. However, they do not take into account the fluctuations of the system, and this is an important limitation since it is not possible to determine the expected deviations from a target shape that are caused by the stochastic nature of AWJM.

1.1. Finite element modelling of abrasive processes

Finite element analysis is a common approach to study erosion wear by abrasive particle impacts at high velocity, since it makes it possible to study complex systems with FE packages using advanced contact algorithms and material models. AWJM can be simulated using multiple particle impacts at high velocity and neglecting the erosion effect of the water; this has been shown to be a reasonable assumption at low pressure (i.e. below 200 MPa) [12]. Early examples of this approach consisted of two-dimensional analysis of the contact between circular particles and the target [13], highlighting the relevance of the impact angle for erosive processes. More advanced models considered sharp particles, such as squares or triangles, and their relative position to the surface for

ductile and brittle materials [14]. In this study, the authors found relevant differences in the effect of the shape and the orientation between single particle and multiple particle impacts. ElTobgy et al. [15] considered a three-dimensional model with several spherical particles. The Johnson–Cook material model [16] was employed for the flow stress evolution in the target (Ti–6Al–4V), together with a shear failure criterion. This approach proved to be successful for simulating the tendency of the effect of the size, the velocity and the impact angle for high velocity single particle impacts only. Although the Lagrangian formulation is the most common approach, Takaffoli et al. presented an alternative method using Smoothed Particle Hydrodynamics to model the impact of angular particles on Al6061-T6 [17]. This led to further work towards a model for a Micro-Blaster [18,19]; they included sharp particles with random shapes travelling at 117 m/s. They obtained results that agreed qualitatively with experimental data, mainly limited by the number of particles used for the simulation and the difficulty to simulate the real conditions in the Micro-Blaster. However, this piece of research was focused on predicting the average eroded volume rather than predicting its mean shape and the statistical deviations from it. Furthermore, this model considered impact velocities below 100 m/s. In Abrasive waterjet machining, the velocity of the abrasive particles can reach up to 750 m/s [9], making the contact problem much more difficult to address, mainly due to the short contact time during the impact. In [20], an attempt was made to simulate the impact conditions during AWJM with single spherical particles at different velocities and impact angles. The comparison with experimental data showed promising results, but the use of spherical particles resulted in deviations from the experimental results, particularly at low velocities where the shape of the abrasive particles is more relevant. Anwar et al. presented the first complete model for AWJ machining [21], simulating the real conditions of the process by using more than 500 garnet particles, with different sharp shapes, hitting the target at 368 m/s. It must be noted that, at pressures below 200 MPa, the effect of the workpiece can be neglected, since it is experimentally observed that the pure waterjet does not erode the surface under such conditions. This work was successful in predicting the average jet footprint for different operating parameters, as shown in Fig. 1, and no attention was paid to the deviations observed in the footprint; it is considered by the authors that it is necessary to include such fluctuations in order to make accurate estimations of AWJ footprints.

Several researchers have attempted to develop different stochastic modelling frameworks for Abrasive Jet Processes. Nicholls et al. presented a unit-event based simulation model by considering multiple discrete damage events taking into account the size, its velocity and the position from statistical distributions [22]. In a later model, Verspui et al. [23] modified this unit-event

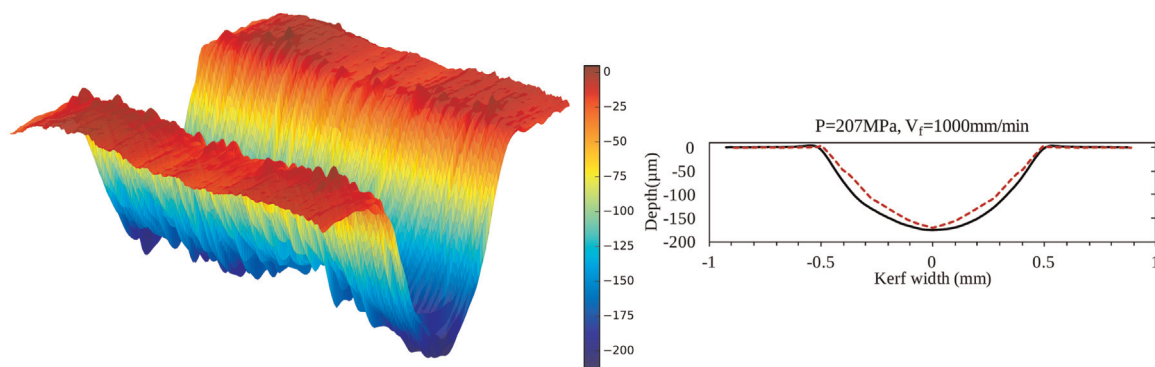


Fig. 1. Abrasive waterjet footprint data taken from previous work [21]. Only average jet footprints were investigated previously, although the variations in the surface can be clearly appreciated.

framework to a surface by adding-up single particle impacts. Two simulation frameworks for AWJ cutting were developed taking into account different fluctuation sources [24,25]. In [26], the authors include a step to recalibrate the model in order to account for variations in the mass flow rate between different jet passes. There are also earlier approaches considering a stochastic input into finite element models. In [27], the impact of several particles randomly positioned is modelled to study the damage of small particles on aerospace components. Takaffoli et al. introduced random shapes and uniformly distributed positions for multiple particles to investigate Microblasting Processing [19]; Li et al. presented a model involving multiple spherical particles simulations [28,29]. Although these approaches consider uncertainties in the jet, the results presented information on average quantities, such as the erosion rate or the eroded volume.

Considering that there is no detailed study into surface variability during AWJM, a combination of finite element analysis and Monte Carlo methods is proposed here as a tool to understand how the fluctuations in the jet may affect the milled surface quality. Previous studies have been focused on studying the average erosion rate or kerf profile; it is now attempted to model deviations from the mean and study which are the optimum conditions to reduce such effects. It is shown in this paper how this combination is a powerful tool to understand the variability of AWJ milled footprints. Using this technique, not only the average footprint profile is considered, but also single profiles along the trench that do not have a smooth shape and are a more realistic prediction of the jet footprint.

2. Finite element modelling

Although there is a significant amount of work to predict the erosion rate or the average jet footprint, the high variance observed in experimental tests has not been addressed in the literature. Given the fluctuating nature of the system, an AWJ cannot be regarded as a deterministic system. Therefore, instead of considering fixed values for the model parameters, such as the velocity of the particles, they must be regarded as probability distributions. Then, since the input is not a deterministic value, a single simulation (i.e. a single jet footprint) will not be a representative result under a given set of initial conditions. Monte Carlo methods are used to obtain representative data by running several simulations for a given set of conditions (feed speed, impact angle, particle shape). This approach not only yields a more accurate average footprint by introducing more realistic conditions to model the jet, but also reproduces the evolution of the footprint variability for different feed speeds and impact angles. Moreover, the model can be used to investigate how the noise is distributed across the jet axis and how these fluctuations propagate when multiple jet passes are overlapped.

It is a great challenge to combine Monte Carlo methods with finite element models for several reasons: (i) there is a limited amount of information on the physical conditions in the jet, and the experimental setups required for this purpose are very complex; (ii) FEA software packages are not designed to let the user specify statistical distributions as an input, and Monte Carlo simulations cannot be setup manually since it requires the capability to automate the generation of inputs under random initial conditions; (iii) the FE models developed for AWJM involve computationally expensive simulations, therefore limiting the number of cases that can actually be tested. For these reasons, the model presented in [12] has been developed further to include probability distributions instead of fixed values, therefore providing a tool to inspect the relation between the uncertainties in the system and the deviations from the average kerf profile in the

workpiece. It is therefore proposed here a suitable method to study the relevance of different parameters to the variability of the footprint.

2.1. Material models

The target material chosen for this study is Ti-6Al-4V. The Johnson–Cook visco-plastic material model [16] is a common choice for ductile materials to model high-velocity impact [15,30]. The model states that the flow stress, σ , depends on the equivalent plastic strain, ε the equivalent plastic strain rate, $\dot{\varepsilon}$, and the temperature, T , through

$$\sigma = \left[A + B\varepsilon^n \right] \left[1 + C \ln \frac{\dot{\varepsilon}}{\dot{\varepsilon}_0} \right] \left[1 - \left(\frac{T - T_r}{T_m - T_r} \right)^m \right]. \quad (1)$$

The constants A , B , n (work hardening parameter), C and m are determined experimentally and depend on the material. The reference strain rate, $\dot{\varepsilon}_0$, is used to normalize the logarithmic term in the equation, and it is the strain rate at which the material constants have been found. Apart from plastic deformation, a significant amount of material is removed during AWJ milling. Therefore, the Johnson–Cook ductile failure criterion is also incorporated [31]. This is the cumulative damage law

$$D = \sum \frac{\Delta \varepsilon}{\varepsilon^{pl}}, \quad (2)$$

where $\Delta \varepsilon$ is the change in the effective plastic strain for an increment in the load, and

$$\varepsilon^{pl} = \left[d_1 + d_2 e^{-d_3 \eta} \right] \left[1 + d_4 \ln \left(\frac{\dot{\varepsilon}_{pl}}{\dot{\varepsilon}_0} \right) \right] \left[1 - d_5 \left(\frac{T - T_r}{T_m - T_r} \right)^m \right], \quad (3)$$

where η is the pressure-effective stress ratio; $\dot{\varepsilon}_0$ is the reference strain rate at which the material constants were obtained, used to normalize $\dot{\varepsilon}_{pl}$ and d_{1-5} are constants determined experimentally. An element is removed from the system (fails) when $D=1$. The material constants are taken from [32,33].

A tensile failure criterion was considered for the abrasives particles in former investigations [12]. However, particle breakage during impact is not significant [34], and including a tensile failure criterion to model this feature makes the mesh sensitivity uncontrollable. Therefore, an isotropic linear elastic model is considered as in [14,29]. Controlling the mesh sensitivity is critical because this model is designed to predict the variability of the trench profile. In previous work, [12,21], this was not an issue because small variations may not affect significantly the average footprint profile.

2.2. Mesh

The choice of the size and the type of the elements is critical to achieve consistent results. It is very important to work with a mesh size much smaller than the problem size, although this significantly increases the computation time. This issue has been studied by performing parametric simulations varying the mesh size, concluding that the elements in the target must be C3D8R, 8-node linear brick, with a size $5 \times 5 \times 5 \mu\text{m}$ using reduced integration with hourglass control. The mesh size is made larger far away from the impact area, as shown in Fig. 9, in order to reduce computation time.

Although failure mechanisms introduce some mesh sensitivity, refining the mesh shows that a convergence can be achieved. As shown in Fig. 2, reducing the mesh leads to larger erosion, but the solution converges when the element size is reduced.

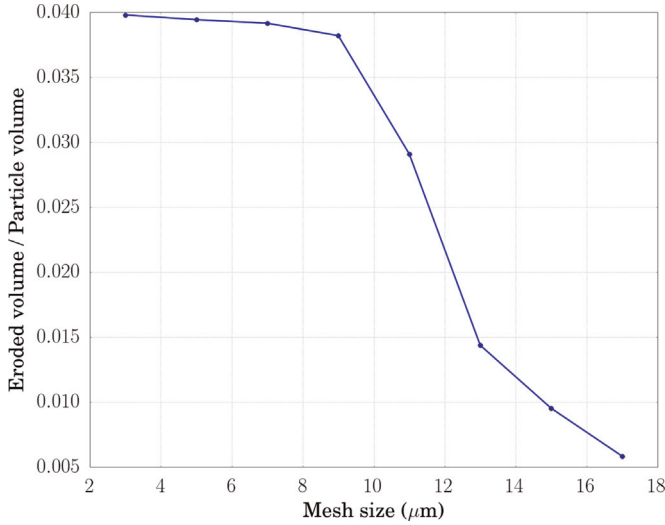


Fig. 2. Mesh sensitivity study for a single particle with a mean diameter of 80 μm and a velocity of 350 m/s, showing that there is almost no mesh dependency between 8 μm and 3 μm.

2.3. Contact model

The general contact algorithm, based on a penalty method, of Abaqus/Explicit is used to model the interaction between the particle and the target. Since this algorithm is designed to model all the interactions between free surfaces in the model, it makes it possible to cope with material removal, since it automatically defines the contact between the particles and the newly exposed elements. This is carried out by modelling contact between element-based surfaces. The contact methodology has been taken from [21], assuming a coefficient of isotropic friction of 0.1. The contact between different particles is not defined to avoid unnecessary interactions between them; this is a reasonable assumption since it has been reported that the abrasive particles are well spaced from each other during the process [8,9].

2.4. Monte Carlo simulations to enable the study of the footprint variability

As stated previously, the model parameters are not single values, but probability distributions. In this model, the following parameters are considered as uncertain: the velocity, v_i , the mass, m_i , the orientation and the relative position within the jet of each particle, $\mathbf{x}_i$. The algorithm to generate the jet parameters from probability distributions is presented in this section.

The jet path, with length L , is discretized using n circular bins. Each bin represents the jet at a given time step, and the method consists in filling each bin with particles until the accumulated mass reaches the maximum value, m_{bin} , given in Eq. (5).

If the mass of all the simulated particles is given by

$$m_L = \frac{\dot{m}_a L}{v_f}, \quad (4)$$

where $\dot{m}_a$ is the mass flow rate, then each bin will have approximately

$$m_{bin} = \frac{m_L}{n}. \quad (5)$$

For a fixed number of bins, it is easy to see how different parameters will affect m_{bin} , increasing or decreasing the number of particles.

As well as its size, each particle was assigned a relative position within the bin, $\mathbf{x}_i$. If $\mathbf{x}_0$ is the initial position of the center of the jet,

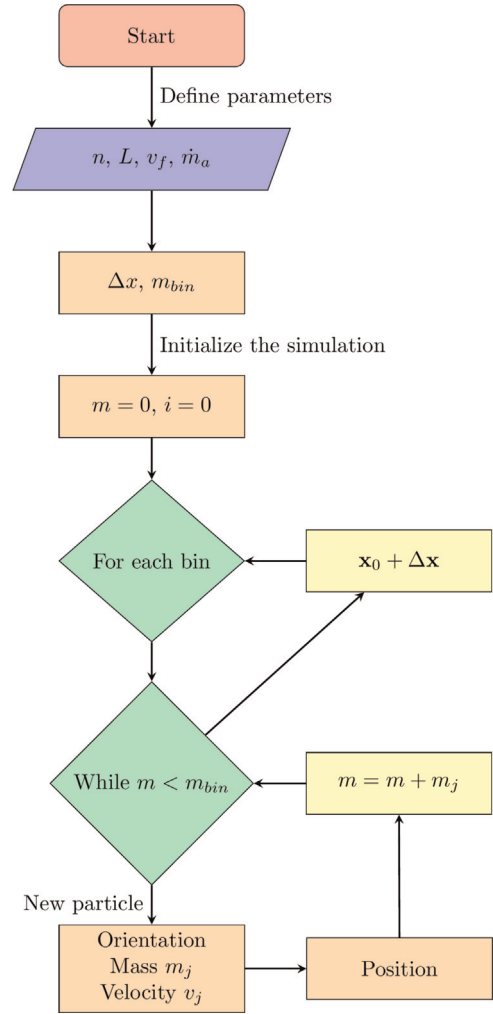


Fig. 3. Flowchart showing the main steps to generate the jet data for one simulation.

the position of a particle is given by

$$\mathbf{x}_i = \mathbf{x}_0 + j\Delta\mathbf{x}, \quad (6)$$

where

$$\Delta\mathbf{x} = \frac{L - d}{n - 1}, \quad (7)$$

j is the bin number and d is the beam diameter. This is explained in the flow diagram in Fig. 3, and an example of the generation process is shown in Fig. 4.

2.5. Statistical distributions within the jet

2.5.1. Particle distribution across the jet axis

The distributions used for this work are based on the experimental data presented in [9]. A Gaussian distribution is used to model the location of the particles within the jet according to these investigations, since a uniform distribution, as used in [19,28], would make it impossible to simulate the jet footprint. The standard deviation of this distribution is one of the model parameters that can be adjusted depending on the nozzle diameter, and it is the key element to adapt this modelling framework to other systems and to account for other parameters such as the stand-off-distance. An example of the distribution of the particles across the cross-section of jet is shown in Fig. 5.

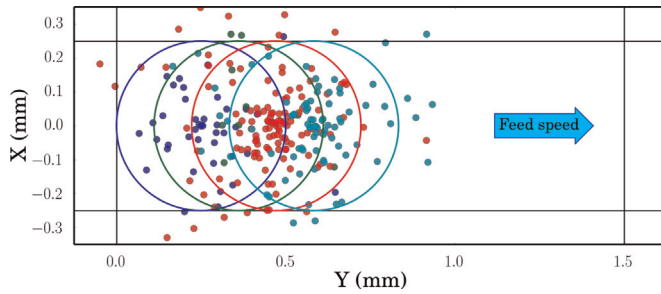


Fig. 4. Illustration of the generation of the particle positions for $v = 2000$ mm/min. Each circle represents one bin, and the particles generated for each bin have the same color as the corresponding circle. The mass flow rate used in this example is 1 g/min in order to help visualize the method. (For interpretation of the references to color in this figure caption, the reader is referred to the web version of this paper.)

2.5.2. Velocity distribution of the particles

Unlike other machining processes, it is not easy to understand the relation between the operating parameters and physical parameters such as the velocity of abrasive particles. Owing to the complex experimental setup required to measure the particle conditions in the jet [8,9], Computational Fluid Dynamics seems to be a powerful tool for predicting the velocity of the particles. However, modelling an AWJ is extremely difficult, since it is a three-phase flow and the particles cannot be considered as point-like. Accurate three-dimensional CFD models for injection abrasive waterjet systems are still in the development stage. In order to obtain reliable estimates for the probability distributions of each parameter, a one-dimensional CFD model for a given set of parameters (pressure, cutting system, mixing chamber dimensions, particle mean size, stand-off-distance), which is capable of predicting the average velocity of the particles and has been validated experimentally [35], is used. This is combined with 3D

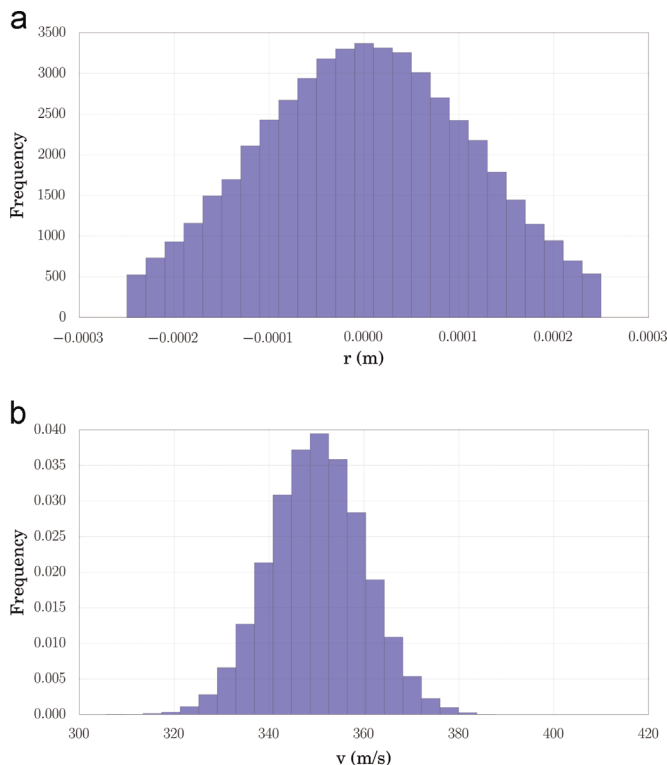
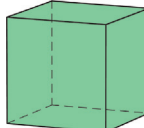
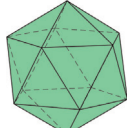
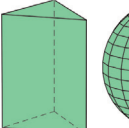
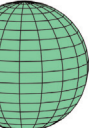


Fig. 5. Statistical distributions across the jet axis: (a) example of the Gaussian distribution for the positions of the particles within the jet; (b) distribution of the velocity of the particles around the average.

Table 1

Four different basic shapes are used: icosahedrons, cubes, triangular prisms and spheres. This is intended to relate the particle sharpness with the erosion power and the variability of the jet footprint.

Shapes used				
Average values	Sphere diameter 80 μ m		Mean velocity 353 m/s	

experimental data [9] to make assumptions about the uncertainties. It is important to remark that the mean velocity is considered constant across the jet axis, but it is a random variable with a Gaussian distribution as shown in Fig. 5.

2.5.3. Particle shape

The shape of the abrasive particles is known to be very irregular [34]. The authors in [28,29] assume that the particles are spherical; such an assumption might not be reasonable because of the differences obtained in the erosion of sharp particles even at a single impact level. Takaffoli et al. [18,19] addressed a similar problem by generating random shapes. However, a set of shapes similar to the mix of particles presented in [21] has been chosen for this work. The reason for this is that by controlling the shape of the particles we can relate the erosion efficiency and the footprint variability with the shape of the particle, and determine the relevance of this parameter for the overall machining process. Then, combinations of different shapes in different proportions to evaluate the shape effect are utilized. The chosen shapes, taken from [7], are a sphere, a cube, an icosahedron and an equilateral triangular prism, as shown in Table 1.

Each shape has a different number of corners, the size of the flat surfaces is different and therefore the interaction with the target will change. This is intended to introduce into the model particles with different sharpness, since the erosion power of a spherical particle, an icosahedron and a cube will be different if particles with equal mass impact at the same velocity. It is also shown in this paper that not only the shape of the particles, but also the relative orientation with the target, affects the erosion in both single and multiple particle impact simulations. To perform these tests, each case is tested on 1500 simulations with the same conditions (velocity, size, shape) and random orientation. The surface is extracted to compute the volume of the eroded crater each time, yielding consistent information on the behaviour of single particles. Although the particles will not be hitting a flat surface in the multiple particles case, it is shown here that taking into account different shapes and orientations is useful to model the variability of the footprint.

2.5.4. Relative orientation of the particles with the target

Apart from different particle shapes, a random relative orientation is considered. Although this will not significantly affect the overall erosion, it makes an impact on the variance of the simulated trenches; if this element is not included, the variability of the simulated footprint decreases, leading to less realistic predictions of the surface variance.

The relative orientation is generated by defining a random axis between (0,0,0) and a random point on a unit sphere [36]. This random point is picked using two random numbers, $\theta \in [0, 2\pi]$ and $u \in [-1, 1]$; then the second point of the axis will be given by

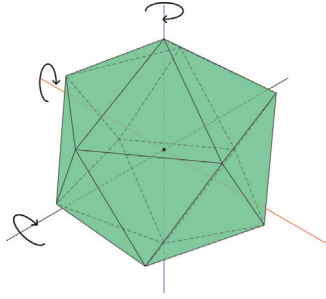


Fig. 6. The relative orientation of the particle with the target is changed by randomly rotating it. Icosahedral particle showing three different examples of random rotation axis.

$$\begin{aligned} x &= \sqrt{1 - u^2} \cos \theta \\ x &= \sqrt{1 - u^2} \sin \theta \\ z &= u. \end{aligned} \quad (8)$$

After this, a third random number is randomly picked, $\phi \in [0, 2\pi]$ to rotate the particle around the axis. An example of this is shown in Fig. 6.

Before performing simulations with multiple particles, single particle impact simulations have been carried out to test the relevance of the shape and the relative orientation.

2.6. Particle size

The abrasive particles used for this study were Garnet #220. The size distribution was measured *as received* and as they exit the cutting system. For the latter, the abrasives were collected directly while the machine was running by placing a container in the tank for several minutes. Then, the particles were sieved and the distributions shown in Fig. 7 were obtained; this distribution is used as an input for the probability distribution of the particle size in the model presented here.

As reported by other authors [34], it is confirmed that the particles undergo significant breakage during the entrainment process and as they go through the focusing tube. Therefore, the particle size distribution of the abrasives as they leave the cutting system has been used for the model.

In contrast to other distributions, the particle size distribution cannot be assumed to be Gaussian. Then, it is not possible to

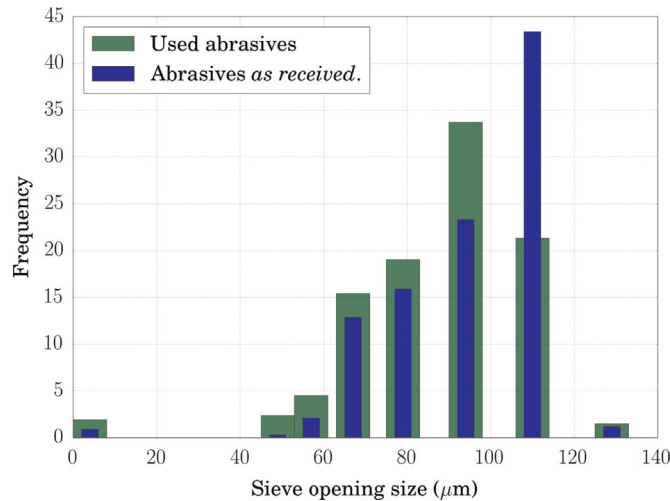


Fig. 7. Particle size distributions of the particles before and after going through the cutting system. This shows the significant breakage that the larger particles undergo during the entrainment and acceleration processes.

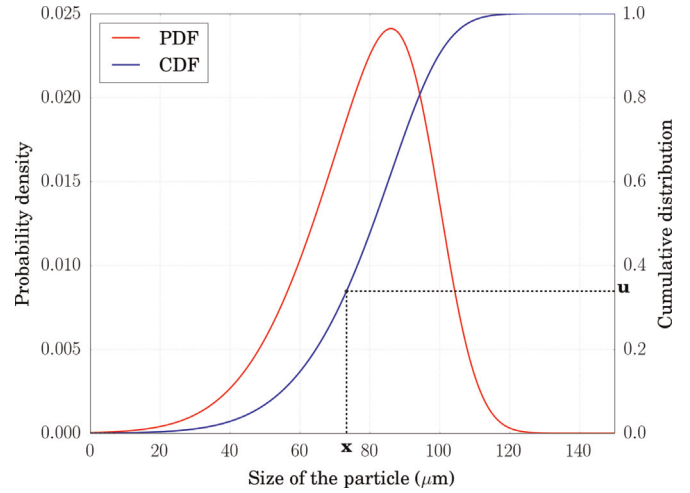


Fig. 8. Example of the generation of a random number using inverse transform sampling.

simulate it by simple generation of random numbers following a Gaussian distribution. The inverse transform sampling method is used in order to generate pseudo-random numbers following this probability distribution [37]. For this, the cumulative distribution function (CDF), F , from the distribution shown in Fig. 7 is computed. Then, random numbers from the standard uniform distribution $u \in [0, 1]$ are picked and, finally, random numbers that follow the distribution in Fig. 7 are obtained by computing the value, x , such that

$$F(x) = u. \quad (9)$$

Hence, the computed values x follow the desired distribution. An example of this is shown in Fig. 8; a value u is first generated and the associate value x is obtained using Eq. (9).

2.6.1. Impingement angle of the jet

The pattern of the erosion efficiency as the impingement angle is varied is a good method to compare the applicability of the model to different experimental conditions. The erosion efficiency for ductile materials is known to be larger at non-normal impact angle [6]. For AWJM models, this is an important aspect because the tilt angle needs to be changed along the tool path in order to generate complex 3D features. At the same time, tilting the jet implies breaking the symmetry of the system; then, it is also important to understand how the fluctuations may become larger at certain areas of the footprint since this issue needs to be taken into account when designing optimum tool path strategies. In order to tilt the jet, the coordinates of the particles are rotated about the axis defined by the jet path.

2.7. Overlapping multiple jet passes

Modelling overlapping jet passes is required in order to develop a model that can be used to understand how the process must be controlled to generate complex 3D forms. Parallel trenches have been considered for this study to investigate how the variability of the footprint evolves as several jet passes are overlapped. Since the jet footprint cannot be considered as a single average profile, but as a probability distribution, determining how these distributions will overlap as each jet pass is performed is of great importance to address the problem of how the jet must be moved to achieve a given surface.

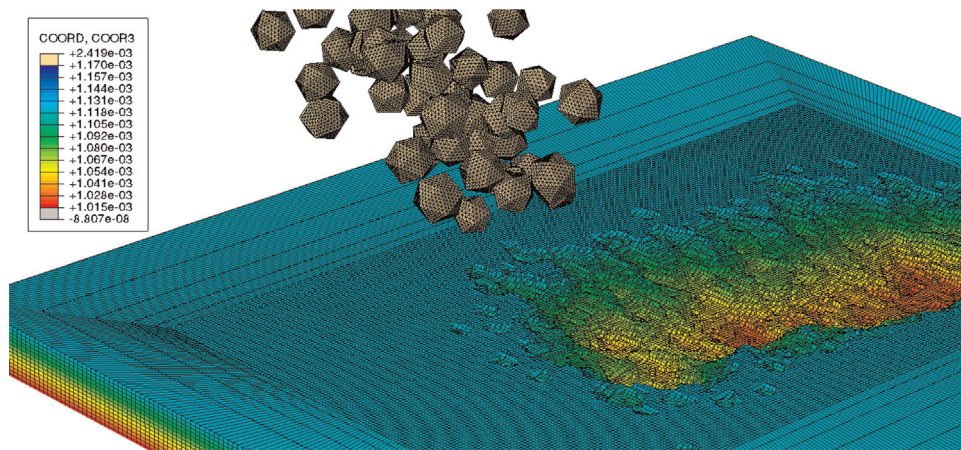


Fig. 9. Snapshot of a simulation of one single trench using icosahedral particles for $v = 2000$ mm/min with a tilted angle of 40° ; the mesh size is larger far away from the impact to reduce computation time.

2.8. Analysis procedure

One of the most challenging parts of this work has been embedding Monte Carlo routines into Abaqus 6.14. The position within the jet, velocity, particle size and orientation are used as parameters for the Monte Carlo simulation. The probability distributions of these variables that have been used as input for the Monte Carlo simulations are shown in Figs. 5a, b and 7; in the case of the orientation, the distribution is uniform as explained in Section 2.5.4. Python scripts have been written to interact with the software and develop several routines that make it possible to specify values, such as the position, velocity or size of a given particle after generating this information. An example of this is shown in Fig. 9. Another relevant aspect is computation time, which is reduced by splitting the simulation into blocks of 200 particles. In order to avoid delays, a new simulation was set to start automatically when the previous job finished. Then, the data from a simulation can be processed while running a new one. For each case, 15 simulations are carried out, optimising the target dimensions to achieve a full jet pass without using unnecessary resources, as in [21]. On average, one simulation with 2352 particles takes approximately 22 h on a 2×8 -core 2.6 GHz processor. The simulation involves approximately 2.5×10^6 elements, although this may vary depending on the feed speed and the mass flow rate. Instead of running several simulations with the same conditions, longer targets could have been used to obtain several profiles along the channels. However, running independent simulations gives a more realistic framework for modelling profiles which are far away from each other in the experimental samples.

The data extraction is performed by tracking the node coordinates of the target from the beginning, and then finding those elements that form the milled trench. This process is challenging because, owing to the material removal criterion, the initial and final surfaces do not have the same elements.

Table 2
Experimental parameters for model validation.

P (bar)	1380
$\dot{m}_a$ (g/min)	30
Nozzle diameter (mm)	0.5, 0.8
v_f (mm/min)	1000, 2000, 3000
θ	0, 10, 20, 30, 40, 50

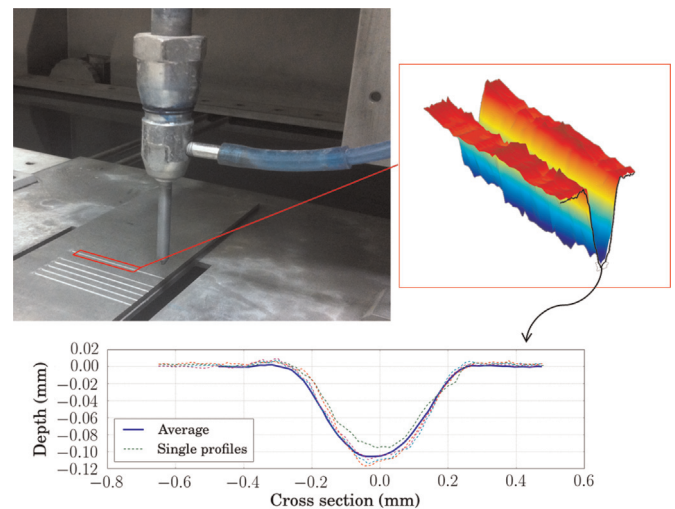


Fig. 10. Overview of the generation of AWJM trenches, showing an example of an average profile with several single profiles.

3. Generation of experimental data

In order to validate the proposed approach, experimental trials on Ti-6Al-4V at different feed speeds and impingement angles have been performed; the values for each parameter are presented in Table 2. The jet footprint can then be measured and the average and its variability compared against the simulations under different conditions; an example of this process is shown in Fig. 10.

The experimental data for model validation has been generated with a microwaterjet 3-axis machine developed by Waterjet AG; several cutting heads, from 0.2 to 0.8 mm can be mounted in the system. The machine is empowered by a KMT streamline SL-V100D ultra-high pressure pump, capable of delivering a pressure range from 700 to 4000 bar. The apparatus has been designed for high accuracy cutting applications, with a cutting accuracy of ± 0.01 mm, a positioning accuracy of ± 0.003 mm, and maximum traverse speed of 4000 mm/min. The abrasive particles used for this study are BARTON HPX #220. As the machine has three axes, a tilting angle table has been mounted as shown in Fig. 11. The trenches are measured using a Talysurf CLI 1000 metrology analyser, extracting several profiles along the milled feature. In order to reduce measurement errors, a contact probe with a radius tip of $2.2 \mu\text{m}$ was used.

In order to study the variability, 15 profiles were extracted from the trench as shown in Fig. 10. Then, the average was computed as

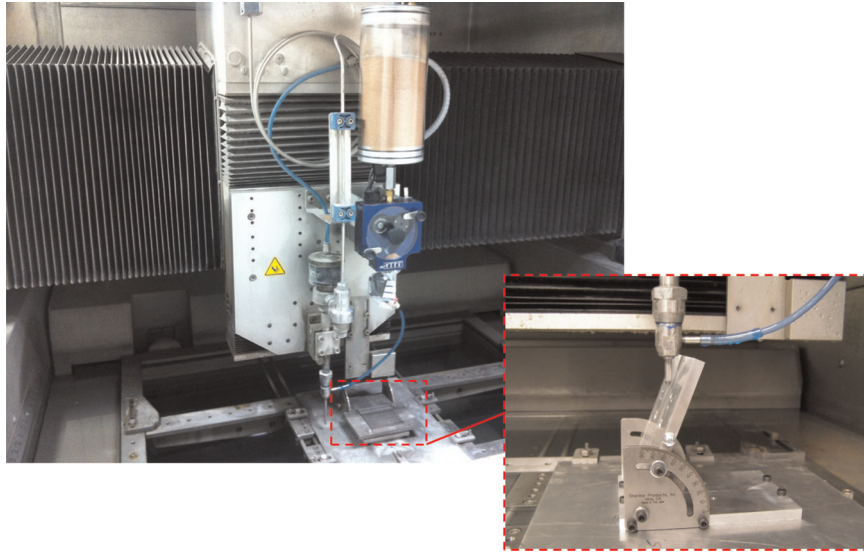


Fig. 11. Abrasive microwaterjet machine used for model validation. A tilting angle table is used to perform tests with tilted impact angle.

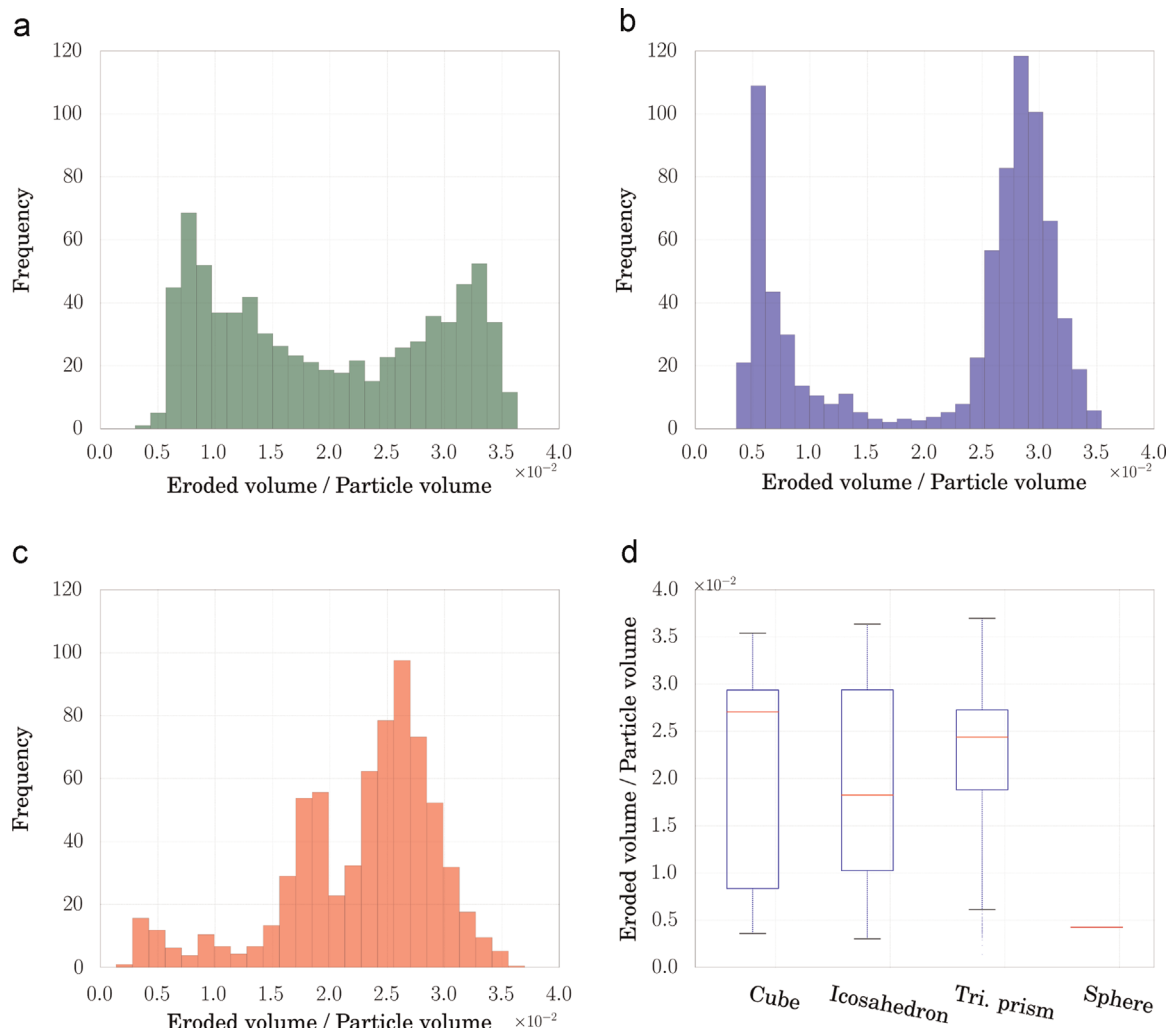


Fig. 12. Single particle simulations with a particle size of 80 μm and a velocity of 350 m/s (1500 cases): (a) Icosahedral particles; the region between the modes is densely populated as the shape is close to a sphere. (b) Cubic particles; two well-differentiated modes, showing the largest differences between high and low erosion regions as expected from sharp particles. (c) Triangular prismatic particles; a third mode appears in the middle as a result of a secondary impact of the particle. (d) Boxplot summarizing the results.

well as the standard deviation of the samples across the jet axis. This is consistent with the number of simulated profiles obtained for each case.

4. Results and discussion

The most significant results of this work are presented here, with emphasis on the patterns found in the data and comparison of the simulations with the experimental results:

- the results of Monte Carlo simulations on single particle impacts for different particle shapes are analysed;
- simulations with multiple particles, regarding as random variables those parameters that were shown to be relevant at a single particle level, and including the operating parameters, such as the feed speed and the impact angle, are performed;
- the effect of the random variables (particle velocity, orientation) is shown by fixing them to compare their effect on the footprint variability;
- the model is finally tested by simulating more complicated scenarios, such as jet passes with tilted impingement angle and overlapped trenches.

4.1. Single particle impact study

The effect of the relative orientation of the particle to target for different particle shapes is shown in this section. As mentioned previously, using four standard shapes makes it possible to relate the characteristics of the particles to the results of the simulation. In order to show this, the size of the particles is kept constant while the relative orientation is randomly varied. This makes it possible to show the effect of the particle shape independently of other parameters. In Fig. 12 the shape effect for the chosen shapes is compared after performing Monte Carlo simulations using random relative orientation to the target, as shown in Fig. 6. It is clear that both the mean and the standard deviation vary with the shape from the boxplot comparing all four shapes. The spheres yield a very low value of erosion when compared with the mean values of the others.

More detailed information is shown in each individual histogram in Fig. 12. It is remarkable that they present several modes: a high erosion mode corresponding to impact with a sharp corner, and a low-erosion mode related to impact with flat areas of the particle. This shows the difference between hitting the target with a flat face or a sharp part of the particle, such as a corner or an edge. Although this effect does not significantly influence the average erosion rate when multiple particles are used, it introduces uncertainties in the simulated trench; therefore it is a relevant property to take into account in order to model the variability.

It must be noted that a high number of cases (1500) is required to obtain consistent data, since the possible cases are not only a flat face or a corner, but any mixed situation. Furthermore, the particles rotate as they hit the target in some cases, generating a very wide variety of single-impact craters. For the icosahedral particles, Fig. 12a, the region between each mode is much more populated than in the other two cases; this was expected since this shape is much closer to a sphere than the other shapes, and the difference between the high and low erosion areas is lower. For cubic shapes, Fig. 12b, a completely opposite behaviour is observed: two very separate modes for low and high erosion that are the result of impacts through flat and sharp areas respectively. The triangular prismatic shapes, Fig. 12c, yield a third mode between the low and high erosion zones. It has been observed that there is

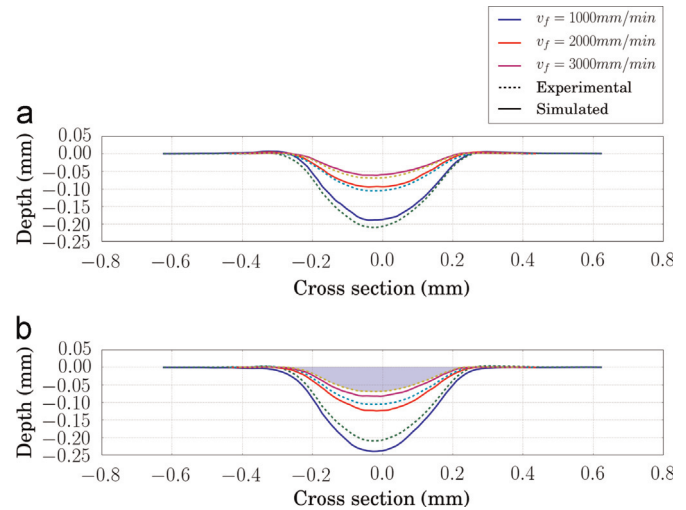


Fig. 13. Comparison of simulated and experimental profiles at different jet feed speeds using different particle shapes. (a) Results using only icosahedral particles; (b) results using cubic particles only.

a tendency for secondary hits to occur (the particle produces two small craters) for this shape, and this leads to a reduction of the low erosion mode and the generation of a mid-erosion mode. Similar behaviour has been observed for rectangular prisms. The overall differences can be observed in Fig. 12d, where the results from using spherical particles have also been included; it must be noted that the spherical particles yield the same result for any orientation. From Fig. 12d, it is observed that the highest erosion efficiency is achieved with cubic particles, although the crater with largest volume from all the simulations was obtained using a triangular prism.

Studying in detail the performance of the model at a single-particle level is a key element to ensure its consistency when multiple particles are included at the same time. It has made it possible to identify the relevance of different shapes and relative orientations in the multiple particles case in order to model the uncertainties in the jet.

4.1.1. Single jet passes at normal impact angle

The results obtained for single straight jet passes with normal impact angle show that the predictions of the eroded volume are close to experimental data, and that the footprint has a similar shape. It is particularly important to remark that both experimental and simulated data are the result of averaging multiple profiles; this is shown in Fig. 1, and how the average results from the simulation capture the piled up material on the edges and the shape of the footprint. In Fig. 13 experimental and simulated profiles at different jet feed speeds are compared; these have been simulated using the probability distributions shown in Figs. 5a, b and 7 as an input for the Monte Carlo simulation and using different shapes and combinations of shapes to study the effect of the particle shape on the performance of the model. The pattern observed in Fig. 12d can be now appreciated on real profiles by comparing Fig. 13a and b; using cubes results in more erosion, and this leads to overestimation of the size of the jet footprint, whereas the icosahedral shapes are less aggressive and lead to an underestimation of the erosion; this agrees with the results for single particle impacts. The filled area in Fig. 13b shows an example of how the erosion rate for an average profile is calculated by computing the integral across the profile. This is used in Figs. 14 and 15 to compare the results from different simulations.

The experimental results are in excellent agreement with the simulated data. In fact, the results in Fig. 14 suggest that the

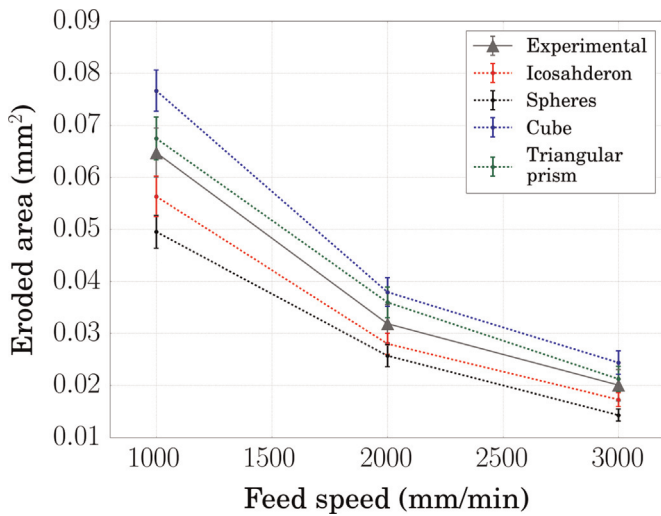


Fig. 14. Comparison of simulated and experimental erosion at different feed speeds using different particle shapes.

observed values could be achieved by using a given combination of shapes. This shows the relevance of the particle sharpness on the erosion process. It is not the scope of this work to find a combination of particles that results into the desired erosion rate; but to show the relevance of the shape in the simulation and its potential effects in the footprint variability. For this, the model is tested using several combinations of particle shapes as presented in Table 3.

The values for the normalized erosion values are shown in Fig. 15a; the combination of different shapes is consistent with the expectations from single shape cases. In Fig. 15b; the normalized values of the variability are shown for each case. It is noted that the results obtained with triangular prismatic particles are the closest to experimental values. An additional situation has been considered in order to study the effect of considering the velocity and relative orientation of the particles as a probability distribution; if the velocity of the particles is considered as a deterministic value instead of a probability distribution, the variability of the milled surface decreases. This is shown in Fig. 15b as a special case (*). These results show that considering the uncertainties at a single particle level is a key element to model the variance in the

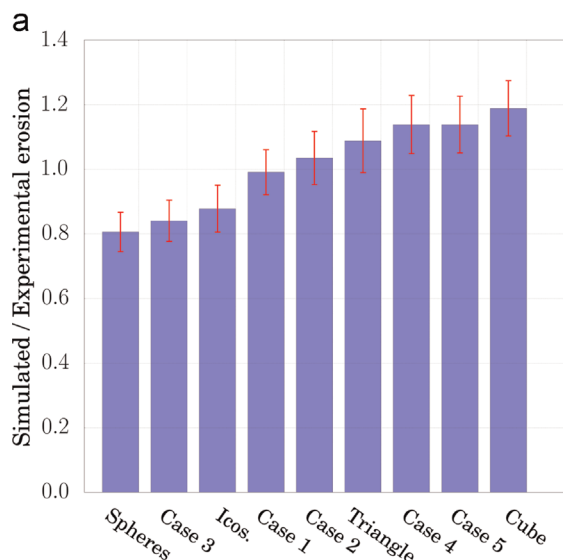


Table 3

Combinations of particle shapes used to test the effect of sharpness on the simulations.

Simulated cases	Shape (%)			
	Icosahedron	Cube	Spheres	Tri. Prism
Case 1	25	25	25	25
Case 2	50	50	0	0
Case 3	50	0	50	0
Case 4	0	50	0	50
Case 5	0	0	50	50

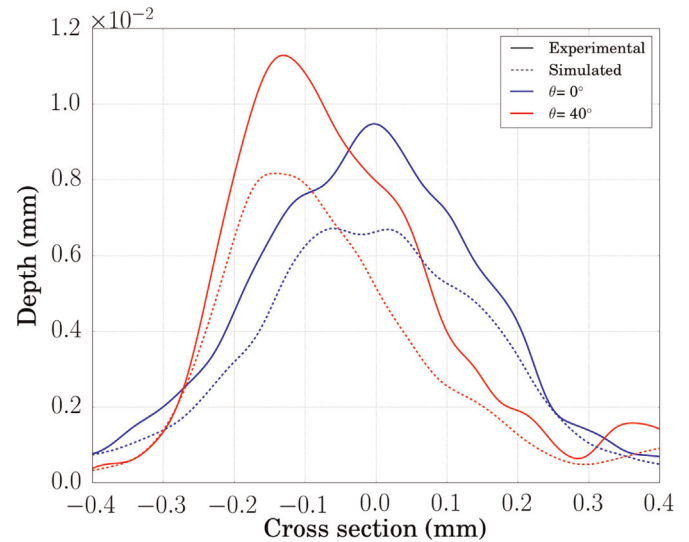


Fig. 16. Comparison of the jet footprint variability across the trench for normal and tilted impact at $\theta = 40^\circ$ at $v_f = 2000$ mm/min.

final surface.

Apart from the total deviation from the average profile, it is interesting that the noise is larger near the center of the trench. This was expected since the erosion is higher near the jet axis, and therefore the fluctuations in the jet result in larger deviations in the surface as would be expected in a stochastic process. This is shown in Fig. 16 for $v_f = 2000$ mm/min.

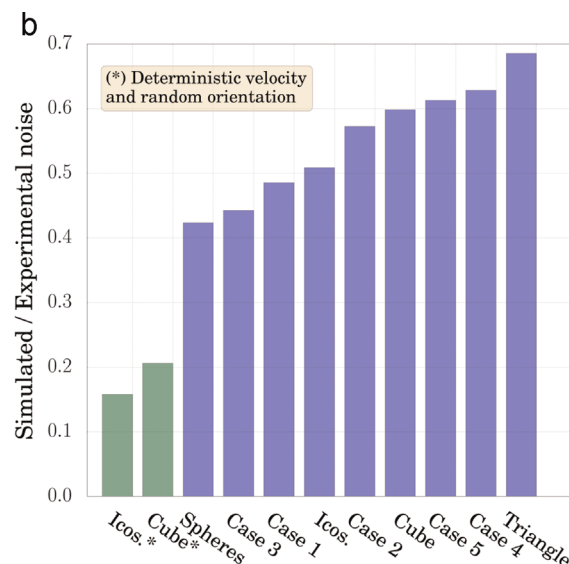


Fig. 15. Normalized erosion and variability values for the cases proposed in Table 3: (a) Average erosion values; the error bars show the Standard Deviation; (b) Standard Deviation for each case, showing two particular cases with deterministic values for the velocity of the particles (*).

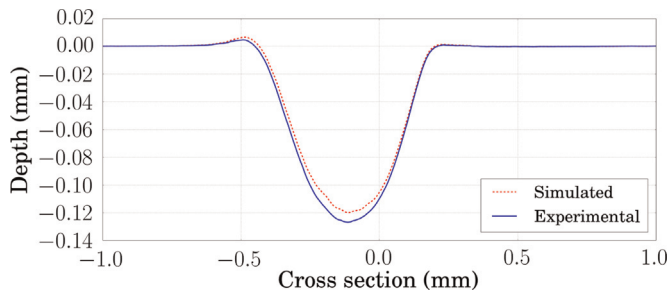


Fig. 17. Average jet footprint using icosahedral shapes at $v_f = 2000$ mm/min with an impingement angle of 40° .

Although the variability is slightly underestimated, the shape of the standard deviation across the jet is predicted by the model. The higher erosion occurs near the center of the trench, and this is also where the fluctuations are higher; this means that the evolution of the noise is related to the erosion rate at a given point. The values shown in Fig. 15b are obtained by integrating the standard deviation across the jet for each case.

4.1.2. Tilted jet passes

The effects of passes with tilted jet instead of passes at normal impact angle are shown in this section. The purpose of doing this is to validate the model in a more difficult scenario by breaking the symmetry of the problem. Three relevant features can be seen in both simulated and experimental data: first, piled up material is only observed on one side of the profile, since the material is pushed by the impinging particles coming from one side as shown in Fig. 17; second, the erosion increases up to 50° , see Fig. 18, as expected from the literature [38]; third, the surface variability is not symmetric as in the previous case, as shown in Fig. 16, but it is shifted to one side. These observations must be taken into account when generating tool path strategies for complex 3D shapes, since considering different impingement angles may introduce unexpected fluctuations in the surface.

4.1.3. Overlapping trenches

The simulation and validation of overlapping jet passes shows the relevance of considering the variability of the process within the model. Not regarding the system as a deterministic entity leads to a very accurate prediction of the average profile of overlapped trench passes, and it also yields a reasonably good estimation of the Standard Deviation of the profile.

An example with three jet passes has been simulated at

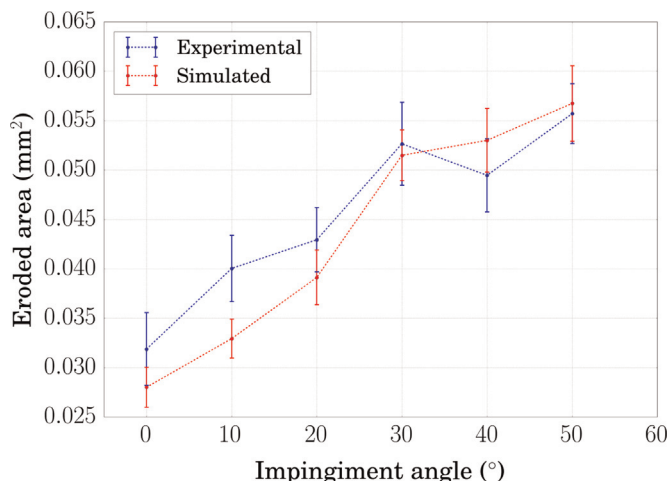


Fig. 18. Comparison of simulated and experimental erosion at different impact angles using icosahedral shapes.

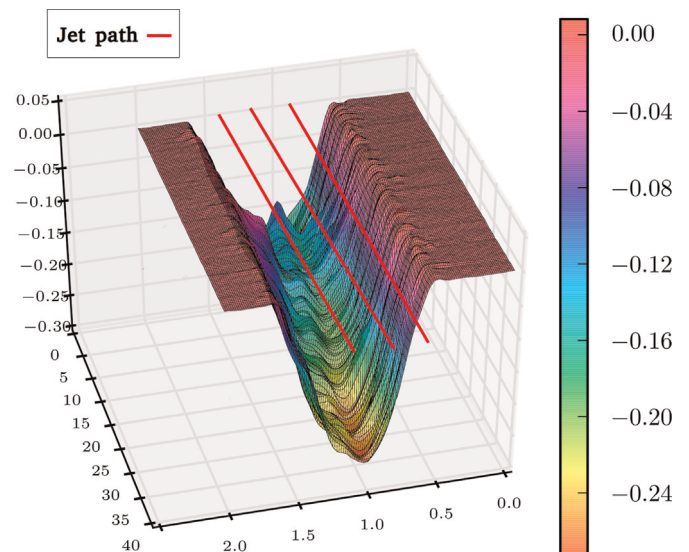


Fig. 19. Experimental data from three jet passes at $v_f = 2000$ mm/min with a step-over of 0.25 mm.

$v_f = 2000$ mm/min in order to show how the variability overlaps when multiple jet passes are performed. For this purpose, the chosen particle shapes are taken from Case 1 in Table 3, since it is the combination with best results as shown in Fig. 15a. It is shown in Fig. 19 how the chosen step-over for this example makes it possible to see the effects of each pass separately, while showing the effects of the overlapping since the depth of the trench increases with each jet pass. The fluctuations can also be observed, particularly at the bottom of the trench.

The extracted profiles from the surface shown in Fig. 19 are shown in Fig. 20. On the one hand, the estimation for the average shown in Fig. 20a predicts the shape of the overlapped jet footprints successfully, with an error less than 4%; it must be remarked that tool-path design strategies must cope with the challenge of finding an optimum step-over to avoid the bumps between jet passes if a flat surface is to be achieved. On the other hand, the prediction of the variability shown in Fig. 20a underestimates the variability, although it shows the same pattern of how the Standard Deviation accumulates near the center of each jet pass. Moreover, the prediction is closer to the observed data than the linear accumulation of the Standard Deviation. It shows that, a small step-over may lead to a larger variability. Therefore, using a small step over would make the average profile smoother but it would increase the fluctuations from the surface.

The most relevant finding from this study is that by overlapping several jet passes, the accumulated variability increases considerably. Comparing Figs. 16 and 20b, the variability increases by a factor of 4 when 3 passes are performed, and this is expected to increase as more passes are made. The largest values of the variability occur in the overlapping region between different passes, showing the importance of continuing the investigations to develop techniques to estimate these errors for larger 3D features in order to optimize the tool-path.

The work presented in this paper shows a finite element analysis model capable of predicting both the average and variability of AWJM footprints. The capability of addressing the problem of having large fluctuations can be used to design smarter tool-path strategies that minimize the deviations from the target shape with two advantages: (i) the model considered here takes into account more realistic physical conditions for the jet, and the predicted average profiles give a consistent estimation; (ii) the prediction of the variability makes it possible to determine the regions with

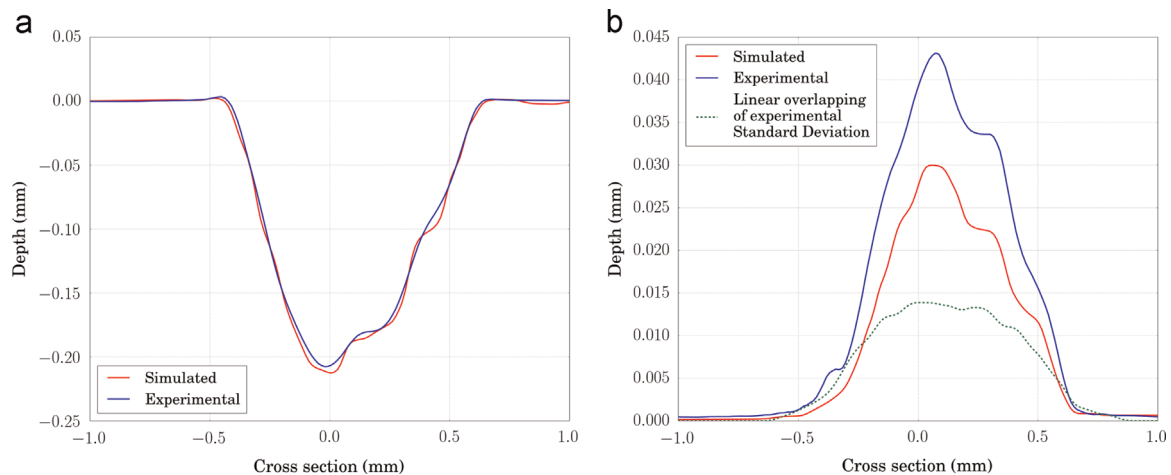


Fig. 20. Validation of the model for three jet passes at $v_f = 2000$ mm/min with a step-over of 0.25 mm. The first pass is performed on the right-hand side. (a) Average profiles from experimental and simulated data. The model prediction has less than 4% error, and it captures the features produced by the overlapping passes. (b) Comparison of the Standard Deviation between simulated and experimental jet footprints.

larger deviations, providing a powerful tool to enable further research in self-learning methods and real time monitoring during the machining process.

It has also been shown that using tilted impingement angles may lead to unexpected deviation patterns since the noise is shifted to one side. This symmetry is also broken in the average footprint profile, as seen in Fig. 17, because the material is piled up only on one edge. This feature must be taken into account, since although using a tilted jet may lead to higher efficiency for certain materials, it may introduce uncontrollable deviations in the surface. Furthermore, the results from overlapped jet passes show that a compromise must be achieved when generating tool-paths, since reducing the step-over results in a better control of the shape of the surface, but it also involves an increase of the variability of the surface.

This new modelling framework can also be used to understand the key elements to design new machines capable of overcoming certain issues. In this case, our findings suggest that lighter machines that are capable of faster accelerations need to be designed in order to develop AWJM as a competitive technology to produce high-quality 3D shapes. The presented model is limited because it is computationally very expensive and it is difficult to extend it to other materials because the available information on material models that can cope with high strain rates is limited. Furthermore, the error in the predicted Standard Deviation could be decreased by using more advanced material models taking into account the irregularities in the target.

However, there is now a better understanding of the fluctuations in the surface during AWJM, and the information shown here may be used to develop approaches that can predict both the footprint profile and the variability in very short times in order to embed these models into CAD/CAM systems that can generate tool paths in real time.

Acknowledgements

The authors acknowledge that this work is performed within the EU Initial Training Network STEEP (Grant no. 316560); C. Narayanan (ASCOMP GmbH) for providing data and software; and S. Anwar for his support.

References

- [1] D.A. Axinte, B. Karpuschewski, M.C. Kong, A.T. Beaucamp, S. Anwar, D. Miller, M. Petzel, High energy fluid jet machining HEFJet-Mach: from scientific and technological advances to niche industrial applications, *CIRP Ann.—Manuf. Technol.* 63 (2) (2014) 751–771.
- [2] D.K. Shanmugam, J. Wang, H. Liu, Minimisation of kerf tapers in abrasive waterjet machining of alumina ceramics using a compensation technique, *Int. J. Mach. Tools and Manuf.* 48 (2008) 1527–1534.
- [3] D.A. Axinte, D.S. Srinivasu, M.C. Kong, P.W. Butler-Smith, Abrasive waterjet cutting of polycrystalline diamond: a preliminary investigation, *Int. J. Mach. Tools Manuf.* 49 (August (10)) (2009) 797–803.
- [4] C.A. van Luttervelt, On the selection of manufacturing methods illustrated by an overview of separation techniques for sheet materials, *CIRP Ann.—Manuf. Technol.* 38 (2) (1989) 587–607.
- [5] M. Hashish, Observations on Cutting With 600-MPa Waterjets, *J. Press. Vessel Technol.* 124 (2) (2002) 229.
- [6] Y.I. Oka, H. Ohnogi, T. Hosokawa, M. Matsumura, The impact angle dependence of erosion damage caused by solid particle impact, *Wear* 203–204 (0) (1997) 573–579.
- [7] A.W. Mombert, R. Kovacevic, *Principles of Abrasive Water Jet Machining*, Springer, New York, 1998.
- [8] R. Balz, K.C. Heiniger, Determination of spatial velocity distributions of abrasive particles in abrasive water jets using laser-induced fluorescence under real conditions, *Proceedings of 16th WJTA-IMCA Conference and Expo*, Houston, Texas, 2011.
- [9] R. Balz, R. Mokso, C. Narayanan, D.A. Weiss, K.C. Heiniger, Ultra-fast X-ray particle velocimetry measurements within an abrasive water jet, *Exp. Fluids* 54 (3) (2013) 1–13.
- [10] M.C. Kong, S. Anwar, J. Billingham, D.A. Axinte, Mathematical modelling of abrasive waterjet footprints for arbitrarily moving jets: Part I—single straight paths, *Int. J. Mach. Tools Manuf.* 53 (February (1)) (2012) 58–68.
- [11] J. Billingham, C.B. Miron, D.A. Axinte, M.C. Kong, Mathematical modelling of abrasive waterjet footprints for arbitrarily moving jets: Part II—Overlapped single and multiple straight paths, *Int. J. Mach. Tools Manuf.* 68 (May) (2013) 30–39.
- [12] S. Anwar, D.A. Axinte, A.A. Becker, Finite element modelling of overlapping abrasive waterjet milled footprints, *Wear* 303 (12) (2013) 426–436.
- [13] K. Shimizu, T. Noguchi, H. Seito, M. Okada, Y. Matsubara, FEM analysis of erosive wear, *Wear* 250 (1) (2001) 779–784.
- [14] Q. Chen, D.Y. Li, Computer simulation of solid particle erosion, *Wear* 254 (3–4) (2003) 203–210.
- [15] M.S. ElTobgy, E. Ng, M.A. Elbestawi, Finite element modeling of erosive wear, *Int. J. Mach. Tools Manuf.* 45 (11) (2005) 1337–1346.
- [16] G.R. Johnson, W.H. Cook, A constitutive model and data for metals subjected to large strains, high strain rates and high temperatures, *Proceedings of the 7th International Symposium on Ballistics*, the Hague, the Netherlands, 1983.
- [17] M. Takaffoli, M. Papini, Material deformation and removal due to single particle impacts on ductile materials using smoothed particle hydrodynamics, *Wear* 274–275 (0) (2012) 50–59.
- [18] M. Takaffoli, M. Papini, Numerical simulation of solid particle impacts on Al6061-T6. Part I: three-dimensional representation of angular particles, *Wear* 292–293 (0) (2012) 100–110.
- [19] M. Takaffoli, M. Papini, Numerical simulation of solid particle impacts on Al6061-T6 Part II: materials removal mechanisms for impact of multiple angular particles, *Wear* 296 (1–2) (2012) 648–655.
- [20] M. Junkar, B. Jurisevic, M. Fajdiga, M. Grah, Finite element analysis of single-

- particle impact in abrasive water jet machining, *Int. J. Impact Eng.* 32 (July (7)) (2006) 1095–1112.
- [21] S. Anwar, D.A. Axinte, A.A. Becker, Finite element modelling of abrasive waterjet milled footprints, *J. Mater. Process. Technol.* 213 (2013) 180–193.
- [22] J.R. Nicholls, D.J. Stephenson, Monte Carlo modelling of erosion processes, *Wear* 186 (1995) 64–77.
- [23] M.A. Verspui, G. De With A. Corbijn, P.J. Slikkerveer, Simulation model for the erosion of brittle materials, *Wear* 233–235 (1999) 436–443.
- [24] A. Lebar, M. Junkar, Simulation of abrasive water jet cutting process: Part 1. Unit event approach, *Model. Simul. Mater. Sci. Eng.* 12 (November (6)) (2004) 1159–1170.
- [25] H. Orbanic, M. Junkar, Simulation of abrasive water jet cutting process: Part 2. Cellular automata approach, *Model. Simul. Mater. Sci. Eng.* 12 (November (6)) (2004) 1171–1184.
- [26] N. Haghbin, J.K. Spelt, M. Papini, Abrasive waterjet micro-machining of channels in metals: comparison between machining in air and submerged in water, *Int. J. Mach. Tools Manuf.* 88 (January) (2015) 108–117.
- [27] Y. Wang, Z. Yang, Finite element model of erosive wear on ductile and brittle materials, *Wear* 265 (5–6) (2008) 871–878.
- [28] W.Y. Li, J. Wang, H. Zhu, C. Huang, On ultrahigh velocity micro-particle impact on steels: a multiple impact study, *Wear* 309 (1) (2014) 52–64.
- [29] W.Y. Li, J. Wang, H. Zhu, H. Li, C. Huang, On ultrahigh velocity micro-particle impact on steels: a single impact study, *Wear* 305 (1–2) (2013) 216–227.
- [30] S. Anwar, D.A. Axinte, A.A. Becker, Finite element modelling of a single-particle impact during abrasive waterjet milling, *Proc. Inst. Mech. Eng. Part J: J. Eng. Tribol.* 225 (August (8)) (2011) 821–832.
- [31] G.R. Johnson, W.H. Cook, Fracture characteristics of three metals subjected to various strains, strain rates, temperatures and pressures, *Eng. Fract. Mech.* 21 (January (1)) (1985) 31–48.
- [32] D.R. Lesuer, Experimental Investigations of Material Models for Ti-6Al-4V Titanium and 2024-T3 Aluminum, Technical Report, Lawrence Livermore National Laboratory, September 2000.
- [33] G. Kay, Failure Modeling of Titanium 6Al-4V and Aluminum 2024-T3 With the Johnson-Cook Material Model 2003.
- [34] K.M. Babu, O.V.K. Chetty, Studies on recharging of abrasives in abrasive water jet machining, *Int. J. Adv. Manuf. Technol.* 19 (May (9)) (2002) 697–703.
- [35] C. Narayanan, R. Balz, D.A. Weiss, K.C. Heiniger, Modelling of abrasive particle energy in water jet machining, *J. Mater. Process. Technol.* 213 (December (12)) (2013) 2201–2210.
- [36] M. George, Choosing a Point from the Surface of a Sphere, *The Annals of Mathematical Statistics* 43 (1972) 645–646.
- [37] L. Devroye, Non-Uniform Random Variate Generation, Springer-Verlag, New York 29 (1987) 675–676.
- [38] Y.I. Oka, K. Okamura, T. Yoshida, Practical estimation of erosion damage caused by solid particle impact: Part 1: effects of impact parameters on a predictive equation, *Wear* 259 (1–6) (2005) 95–101.