

Integration worksheet

Question 1

a) $\int (x^2 + 2x)(x + 1) dx$

b) $\int_2^3 e^{3x} + \sqrt{x} dx$

c) $\int_0^2 (x + 2)^3 + 2x dx$

d) $\int_4^5 \frac{2}{x^5} + 3x^2 - x + 4 dx$

e) $\int_0^1 e^x (e^x + 1) dx$

Question 2

Express $\frac{5x+11}{(x+3)(x+1)}$ in partial fractions, hence integrate $\int \frac{5x+11}{(x+3)(x+1)} dx$.

Question 3

Use integration by parts to integrate the following:

a) $\int 3x \cos 2x dx$

b) $\int x \sin 4x dx$

Question 4

Integrate the following:

a) $\int \frac{2}{3x+1} dx$ b) $\int \frac{5}{6x-1} dx$ c) $\int \frac{15x^2+2}{5x^3+2x-1} dx$

Question 5

Integrate the following:

$$\int 3 \sin^5 x \cos x dx$$

Using the substitution $u = \sin x$.

Integration Worksheet Answers

Q1)

a) $\int (x^2 + 2x)(x+1) \, dx$

$$\int x^3 + x^2 + 2x^2 + 2x \, dx$$

$$\int x^3 + 3x^2 + 2x \, dx$$

$$= \frac{x^4}{4} + \frac{3x^3}{3} + \frac{2x^2}{2}$$

$$= \frac{x^4}{4} + x^3 + x^2 + C$$

b) $\int_2^3 e^{3x} + \sqrt{x} \, dx$

$$\left[\frac{1}{3} e^{3x} + \frac{x^{1.5}}{1.5} \right]_2^3$$

Putting limits gives $= \frac{2568.13}{858.111}$

c) $\int_0^2 (x+2)^3 + 2x \, dx$

$$\int_0^2 (x^2 + 4x + 4)(x+2) + 2x \, dx$$

$$\int_0^2 x^3 + 4x^2 + 4x + 2x^2 + 8x + 8 + 2x \, dx$$

$$\int_0^2 x^3 + 6x^2 + 14x + 8 \, dx$$

$$\left[\frac{x^4}{4} + \frac{6x^3}{3} + \frac{14x^2}{2} + 8x \right]_0^2$$

gives $= 64$

$$d) \int_4^5 \frac{2}{x^5} + 3x^2 - x + 4 \, dx$$

$$\int_4^5 2x^{-5} + 3x^2 - x + 4 \, dx$$

$$\left[\frac{2x^{-4}}{-4} + \frac{3x^3}{3} - \frac{x^2}{2} + 4x \right]_4^5$$

$$\left[-\frac{x^{-4}}{2} + x^3 - \frac{x^2}{2} + 4x \right]_4^5$$

$$\text{gives} = 60.50$$

$$e) \int_0^1 e^x (e^x + 1) \, dx \quad [\text{note: } e^x \times e^x = e^{2x} \text{ (add powers)}]$$

$$\int_0^1 e^{2x} + e^x$$

$$= \left[\frac{1}{2} e^{2x} + e^x \right]_0^1$$

$$\text{gives} = 4.91$$

Q2)

$$\frac{5x+11}{(x+3)(x+1)} = \frac{A}{(x+3)} + \frac{B}{(x+1)}$$

$$5x+11 = A(x+1) + B(x+3)$$

$$\underline{x=-1}$$

$$6 = 2B$$

$$\underline{3 = B}$$

$$\underline{x=-3}$$

$$-4 = -2A$$

$$\underline{2 = A}$$

$$\int \frac{2}{(x+3)} + \int \frac{3}{(x+1)} \, dx \rightarrow 2 \int \frac{1}{(x+3)} + 3 \int \frac{1}{(x+1)}$$

$$= 2 \ln(x+3) + 3 \ln(x+1) + c$$

Q3)

a)

$$\int \underset{\substack{\uparrow \\ u}}{3x} \underset{\substack{\uparrow \\ \frac{dv}{dx}}}{\cos 2x} dx$$

$$uv - \int v \frac{du}{dx} dx$$

$$\frac{du}{dx} = 3 \quad v = \frac{1}{2} \sin 2x$$

$$= 3x \left(\frac{1}{2} \sin 2x \right) - \int \frac{3}{2} \sin 2x dx$$

$$= \frac{3x}{2} \sin 2x - \frac{3}{2} \int \sin 2x dx$$

$$= \frac{3x}{2} \sin 2x - \frac{3}{2} \left(-\frac{1}{2} \cos 2x \right)$$

$$= \frac{3x}{2} \sin 2x + \frac{3}{4} \cos 2x + C$$

b)

$$\int \underset{\substack{\uparrow \\ u}}{x} \underset{\substack{\uparrow \\ \frac{dv}{dx}}}{\sin x} dx$$

$$\frac{du}{dx} = 1 \quad v = -\cos x$$

$$= -x \cos x - \int -\cos x dx$$

$$= -x \cos x + \int \cos x dx$$

$$= -x \cos x + \sin x + C$$

Q4)

a) $\int \frac{2}{3x+1} dx$

$2 \int \frac{1}{3x+1} dx$ 'fix' the integral:

$\frac{2}{3} \int \frac{3}{3x+1} dx$ (diff bottom to get top
 $\therefore$ ans is $\ln$ of bottom)

$= \frac{2}{3} \ln(3x+1) + C$

b) $\int \frac{5}{6x+1} dx$

$5 \int \frac{1}{6x+1} dx$

$= \frac{5}{6} \ln(6x+1) + C$

c) $= \ln(5x^3 + 2x - 1) dx + C$

Q5) $\int 3 \sin^5 x \cos x dx$ let $u = \sin x$

$\int 3 u^5 \cos x \frac{du}{\cos x}$ $\frac{du}{dx} = \cos x$

Sub in: $\frac{du}{\cos x} = dx$

$\int 3 u^5 du$

$= \frac{3u^6}{6} = \frac{u^6}{3} + C \therefore = \frac{\sin^6 x}{3} + C$