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Thermomechanical behavior and constitutive modeling of tungsten-based composite over wide temperature and strain rate ranges

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ABSTRACT

Thermomechanical behavior of tungsten-based composite 93 W–4.9Ni–2.1Fe is investigated systematically over strain rates ranging from 0.001 to 3000 s^{−1}, and temperatures ranging from 173 to 873 K. Different micromechanisms are found in the evolution of microstructures between quasi-static and dynamic tests. The deformation of the tungsten particles is sensitive not only to strain rates, but also to plastic strain levels; the interaction between the grains is found to be the determining factor that cracks the grains, regardless of strain rates. Based on experimental results, two phenomenological and five physically-based constitutive models are established through a procedure of regression analysis and constrained optimization. Descriptive and predictive capabilities of these models are examined and compared. The performance of the models in characterization of work-hardening, temperature, and strain rate effects of the material is also investigated separately.

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1. Introduction

Tungsten-based composites (WBCs) have received considerable attention for both scientific and engineering applications in recent years due to their unique combination of mechanical and thermal properties. Commonly made by liquid-phase sintering mixtures of tungsten and other elemental metal powders such as nickel/iron, WBCs have a two-phase microstructure consisting of spherically shaped bcc tungsten particles embedded in the fcc W–Ni–Fe matrix. This special composite structure results in high density, high strength, moderate ductility and outstanding thermal conductivity to the materials. One of the specialized applications of WBCs is kinetic energy penetrator, for which the high strain rate and high temperature strength are of great importance in armor penetration. During ballistic events, the combined effects of strain, strain rate, and temperature dominate the process of plastic deformation, making the prediction of plastic flow behavior very challenging. The knowledge of their thermomechanical properties, deformation mechanism, and constitutive models are highly desirable over wide ranges of temperatures and strain rates, for optimization and computer simulation of the materials.

In recent years, many studies have been conducted to examine mechanical properties of WBCs, and some attempts have been made to correlate the observed plastic flow behaviors with constitutive models. Compressive flow stress of a 90 W–7Ni–3Fe WBC was tested by Coates and Ramesh (1991) over a range of strain rates from 10^{−4} s^{−1} up to about 7000 s^{−1} without consideration of temperature effects. Little strain hardening was observed at high strain rates. Yadav and Ramesh (1995) investigated high strain rate behavior of two different WBCs in compression and shear over the strain rate range of 10³–10⁵ s^{−1}, and determined several phenomenological models for one material. Kim et al. (1998) developed two techniques to study adiabatic shearband of a WBC. The strain, strain rate, and temperature dependency of the flow stress was separated and quantified. Several WBCs have been examined by (Lee and Chiou, 1996; Lee et al., 1998, 2000) in shear or compression

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over different temperature and strain rate ranges. They found that the deformation of grains varied with strain rate levels. Zhou et al. (2004) performed tensile impact tests on a 92.5 W–5.25Ni–2.25Fe composite at strain rates ranging from 100 to 1300 s⁻¹, and found a strain softening behavior in this material. A three-parameter nonlinear constitutive model was used to describe the strain rate dependent behavior of this material. Rohr et al. (2008) investigated a WBC using a combination of tension tests, modified Taylor-impact tests and planar-plate-impact tests with the VISAR technique. Parameters of the JC model were determined from the experimental results. From previous investigations, it is known that the dependencies of flow stress on strain rate and temperature are substantial and diverse in the deformation of different WBCs. However, systematic studies of this class of materials are few regarding combined effects of both strain rate and temperature, especially over wide ranges. Many aspects are still unknown in the micromechanism of plastic deformation under complex loading conditions. Although several constitutive models have been established, it brings inconvenience to engineering application of these models due to the limitation of the test conditions and the diversity of the determined material constants. Moreover, in most cases the descriptive and predictive capabilities of the constitutive models are not investigated, and hence the applicabilities of the models are not clear.

Up to now, a number of constitutive models have been proposed to address viscoplastic behaviors of materials. The JC (Johnson and Cook, 1983) and KHL (Huang and Khan, 1992; Liang and Khan, 1999; Khan and Liang, 1999; Khan and Zhang, 2000, 2001; Khan et al., 2004, 2007a,b; Khan and Yu, 2012; Khan et al., 2012) models are two typical phenomenological models that have been conceivably applied to characterization of different metals and alloys. Based on the KHL model, Farrokh and Khan (2009) developed a new KLF model for modeling of the post-yield material responses at wide range of grain sizes. Their results confirmed reasonable capability of this model to correlate a wide spectrum of the viscoplastic responses of fcc metals. Another one-dimensional phenomenological constitutive model (Khan et al., 2006) was developed and gave much better correlations in dynamic regime and finite deformation than a modified form of MSP model in characterization of the complex and highly nonlinear thermomechanical behaviors of polymers. Sung et al. (2010) proposed the H/V model for describing of several dual-phase steels, and found more accurate predictions of flow stress and failure strain than the other models selected in the literature. Brünig et al. (2003) proposed a phenomenological anisotropic damage model for ductile metals undergoing progressive plastic deformation induced material deterioration. Through numerical simulation of deformation behavior of ductile metals, this model was ascertained to properly describe the results of relevant tests. This model was further developed to take into account rate and temperature effects (Brünig, 2006), and to analyze adiabatic high strain rate deformation processes for a wide range of stress triaxialities (Brünig and Gerke, 2011). Another group of the constitutive description is the physically (or microstructurally) based equations, which are generally founded on the micromechanics of plastic deformation. Examples are the models proposed by Bodner and Partom (1972, 1975); Hoge and Mukhejee (1977); Mecking and Kocks (1981); Zerilli and Armstrong (1987, 1990); Follansbee and Kocks (1988), (Rusinek and Klepaczko, 2001; Rusinek et al., 2007; Rusinek et al., 2009; Klepaczko et al., 2009; Rodríguez-Martínez et al., 2009), Nemat-Nasser and Li (1998), (Voyiadjis and Abed, 2005; Abed and Voyiadjis, 2005; Voyiadjis and Almasri, 2008), and Meyers et al. (2002). The Bodner–Partom model is a state variable based constitutive model, which is especially suitable for describing rate- and history-dependent behaviors of materials. A unique algorithm (Rajendran et al., 1986) was developed to evaluate the parameters of this model, and results from numerical simulations of various impact problems have been compared between this model and the JC model (Cook et al., 1992). Recently, Saimoto and Houtte (2011) developed a microstructure based strain hardening constitutive relation by equating the energy of dislocations required to generate the imposed incremental strain with that which was stored as determined from the flow stress. This model was employed by Brahme et al. (2011) in FEA to simulate formability and texture evolution in the commercial aluminum alloy 5754. It was found that this model provided the best predictions of the large strain behavior of the material subjected to various strain paths, compared to several other models. Fan et al. (2010) proposed a theoretical model to describe the nucleation and grain growth in DDRX, based on the evolution of internal state variables. A simplified form of this model (Fan and Yang, 2011) was then used in prediction of flow stress and microstructure evolution during hot working of wrought two-phase titanium alloys. Colvin et al. (2009) developed a kinetics equation for the time-dependent plastic response of the material to dynamic loading exceeding 10⁷ s⁻¹, and found this model fit the experimental data reasonably well. Another dislocation based model for fcc metals subjected to very high rates of deformation (10⁴–10⁸ s⁻¹) was proposed by Austin and McDowell (2011). Good agreement was found with experiments in computed profiles of material velocity, plastic rate of deformation, and macroscopic shear stress in the shock front using this model. For accurate numerical simulation of plastically deforming structures, selection among the existing constitutive models is necessitated. In our previous study, several well-known constitutive relations including two purely phenomenological (JC and KHL) and five physically-based (PB, NNL, ZA, VA, and RK) ones were reviewed systematically in characterizing plastic behavior of bcc (Xu and Huang, 2012a, b) and fcc (Xu and Huang, submitted for publication) metals, respectively. However, systematic studies are few concerning comparison of descriptive and predictive capabilities between different models with application to WBCs.

This paper serves two purposes. One is to investigate systematically thermomechanical properties and deformation mechanism of a 93 W–4.9Ni–2.1Fe WBC under wide temperature and strain rate ranges; the other is to study descriptive and predictive capabilities of different constitutive equations regarding the WBCs. In this research, quasi-static and dynamic compression tests are performed on the material over wide ranges of strain rates (0.001–3000 s⁻¹) and temperatures (173–873 K). Emphasis is placed on the effect of work hardening on the flow stress. Microstructural evolutions are observed for understanding of its plastic behavior and failure mechanisms. Based on the experimental results, several well-known phenomenological and physically-based constitutive models are established to simulate the flow stress. Validation of the models

is performed by strain-rate-jump tests under randomly selected loading conditions. Finally, capabilities of the models in characterizing the effects of temperature, strain rate, and work hardening on flow stress of the WBC are discussed in detail separately.

2. Experimental procedure and results

2.1. Material and experiments

The material is a commercial 93 W–4.9Ni–2.1Fe (wt.%) WBC fabricated by liquid-phase sintering of mixed elemental powders. It has a density of 17.8 g/cm³. For quasi-static compressive tests, the samples have a right-cylindrical geometry 6 mm in diameter and 9 mm in height; for dynamic tests, the samples are 5 mm in diameter and 4 mm long. All samples were removed by wire electro-discharge machining (WEDM) from the WBC plates that were polished on both sides. To reduce end friction on the samples, the sample ends were greased with different lubricants for high- and low-temperature tests.

The quasi-static compression tests, including strain rates 0.001 and 1 s^{−1}, were performed using a MTS servo hydraulic testing machine. Dynamic compression tests, with strain rates of 200, 1000, and 3000 s^{−1}, were conducted using the revised split Hopkinson pressure bar technique. In quasi-static tests, five temperatures (173, 288, 473, 673, and 873 K) were selected for each strain rate; for dynamic tests, four initial temperatures were adopted (288, 473, 673, and 873 K). The elevated temperatures were attained with electro-thermal cells. The temperature was measured using a thermocouple arrangement in the furnace for quasi-static tests and a thermocouple directly attached to the specimen for dynamic tests. The initial temperatures were attained and then maintained constant to within ±2 °C in 10 min. To avoid the heating of the Hopkinson bar ends in the high-temperature dynamic tests, the specimen was first heated inside a furnace with the bar ends kept out. Then a synchronically assembled system (Xu and Li, 2009) was used to bring the bars into gentle contact with the heated specimen just before the stress pulse reached the specimen/bar interface.

To examine microstructural evolution of the material at different deforming process, it was loaded separately to various levels of plastic strain under different temperature and strain rate conditions, and then the specimens were cut by WEDM along the axis. After polishing, the axial surfaces were corroded with aqueous solution of ferric chloride and hydrochloric acid. Then the deformed microstructures of the specimens were observed under metalloscope for analysis. Fig. 1 shows the typical microstructure of this material before testing.

2.2. Experimental results

All experimental data were first converted to true stress σ vs. true strain ε curves. Plastic strain ε_p was then determined by $\varepsilon_p = \varepsilon - \sigma/E - 0.002$, with the yield strength defined by the 0.2% offset strain. Here E is the Young's modulus, changing with temperature. Fig. 2 shows the obtained plastic flow stress σ curves as a function of ε_p at different initial temperatures and strain rates. As can be seen, the flow stress curves vary largely for different conditions. At a constant strain, the flow stress decreases rapidly with the increase in temperature, while it increases with increasing strain rate. The work-hardening effect is also evident with increase of plastic strain for each loading condition. In addition, this work-hardening effect seems independent of experimental temperature, but it varies with strain rate and plastic strain. For quasi-static tests, the work-hardening effect is weakened with increasing plastic strain levels. For dynamic tests, however, the flow stress tends to have a constant increasing tendency for the whole deformation process. Details of the work-hardening effect and microstructural evolution will be addressed in the following section.

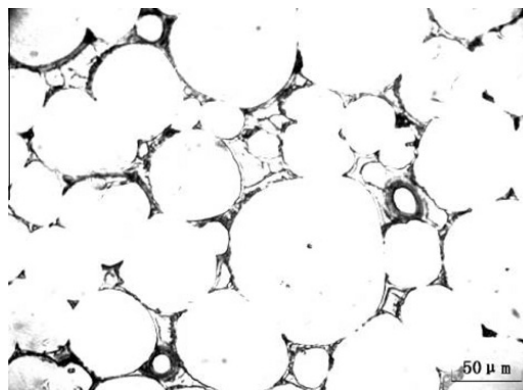


Fig. 1. Microstructure of the 93 W–4.9Ni–2.1Fe (wt.%) WBC.

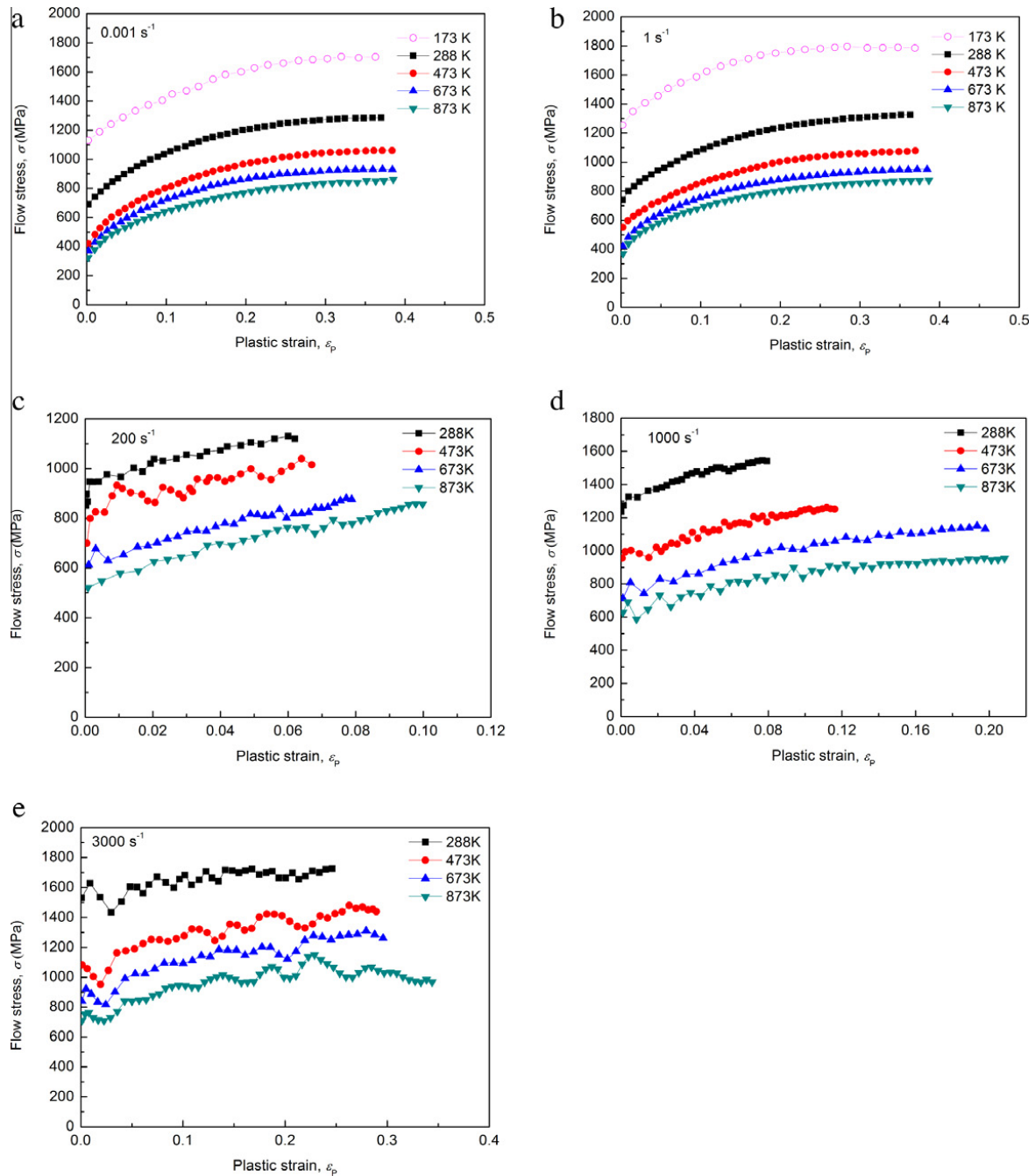


Fig. 2. Plastic flow stress curves as a function of effective plastic strain at different initial temperatures for the strain rates of (a) 0.001 s^{-1} , (b) 1 s^{-1} , (c) 200 s^{-1} , (d) 1000 s^{-1} , and (e) 3000 s^{-1} .

3. Work-hardening effect and microstructural evolution

Work hardening can be observed obviously in the flow stress of the studied material in Fig. 2. This effect shows little sensitivity to the variation of temperature, but it is especially evident at lower strain rates. For example, when the plastic strain increases from 0.02 to 0.20 at temperature 288 K, the flow stress is elevated by 51.2 and 44.7% for strain rates 0.001 and 1 s^{-1} , respectively. This significant contribution to the strengthening of flow stress with plastic strain is believed to arise from the presence of a number of dislocations during deformation, the accumulation and multiplication of which further block their motion and intensify the internal stress. The slope of the stress–strain curves in Fig. 2 can be used to define the work-hardening rate at a given loading condition. Fig. 3 plots the work-hardening rate against the plastic strain at different temperatures for strain rates 0.001 and 1000 s^{-1} , respectively. Under quasi-static loading, the work-hardening rate decreases gradually with the increase of plastic strain up to nearly 0.40. In this process, the work-hardening rate shows very

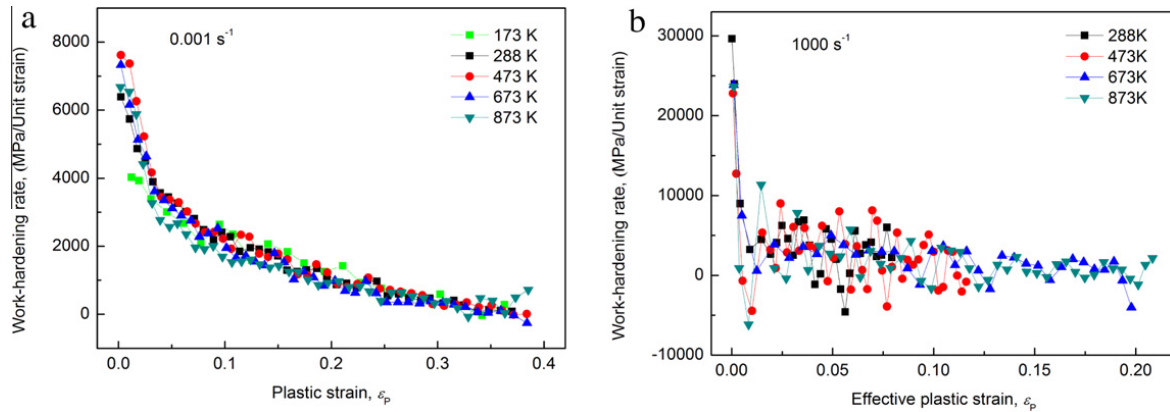


Fig. 3. Variation of work-hardening rate with plastic strain at different temperatures for the applied strain rates of (a) 0.001 s^{-1} and (b) 1000 s^{-1} .

similar tendencies for different temperatures. In case of dynamic loading, however, the work-hardening rate drops abruptly to a steady level within a plastic strain less than 0.02 for each temperature, if we neglect the oscillations in the data. The significant distinction in the tendencies of work-hardening rate between quasi-static and dynamic tests could be an indication of different mechanisms in microstructure evolution under various strain rates.

Under quasi-static compression, plastic deformation mainly starts in the comparatively soft matrix, which induces an increased stress level around the tungsten grain and at the same time elevates the flow stress. The hard tungsten particles rotate and move with the plastic flow of the matrix and concentrate, forming more tungsten/tungsten interfaces. In this period, the tungsten grains may be regarded as a minor deformation, and the mechanical behaviors of this material mainly result from the interactions between the particles and matrix. When the total deformation continues to increase, the interactions between the particles gradually dominate the process; the deformation of the particles increases, which is a crucial factor in the strengthening of the flow stress. On the whole, the plastic flow of the material is an outcome of continuous conjunct deformations in matrix and tungsten particles at different deformation zones. With increase of plastic strain levels, more particles are deformed under compression, leaving less to join the subsequent plastic flow. Therefore, the material exhibits an increasing flow stress with the increase of plastic strain, but the work-hardening rate decreases gradually (Fig. 3(a)). Under dynamic loading, however, the particles have less time to move with plastic flow of the matrix, and tend to deform more severely under the abrupt stress intensification. Therefore, compared to a quasi-static test, the work-hardening rate reaches a steady level more quickly, with a lot more tungsten grains starting to deform from the beginning of the dynamic loading (Fig. 3(b)). Fig. 4 shows the evolution of the microstructure in the material at different strain rates for small strain levels. In Fig. 4(a) the deformation of the particles is not obvious under the plastic strain of 0.05, except that the boundaries of some particles are curved under compression. In Fig. 4(b), however, as expected the deformation of the particles is more evident under a very close plastic strain level, which is loaded at 1000 s^{-1} . The result shows that the deformation of tungsten particles is sensitive to strain rate under a certain plastic strain level. When the strain level is high, however, the difference between particle deformation under different strain rates is not obvious. In Fig. 5, the material was loaded to plastic strains over 0.30 under 288 K, 0.001 s^{-1} , and 873 K, 3000 s^{-1} , respectively. For both cases, the tungsten particles deform very severely. The global tungsten grains became ellipsoidal and the matrix was extruded into strip. Strain rate effect on the deformation of the particles is no longer obvious. The various patterns of tungsten particles under different strain levels and strain rates demonstrate the existence of different mechanisms in microstructure evolution. It can also be concluded that the deformation of the tungsten particles is sensitive not only to strain rates, but also to plastic strain levels.

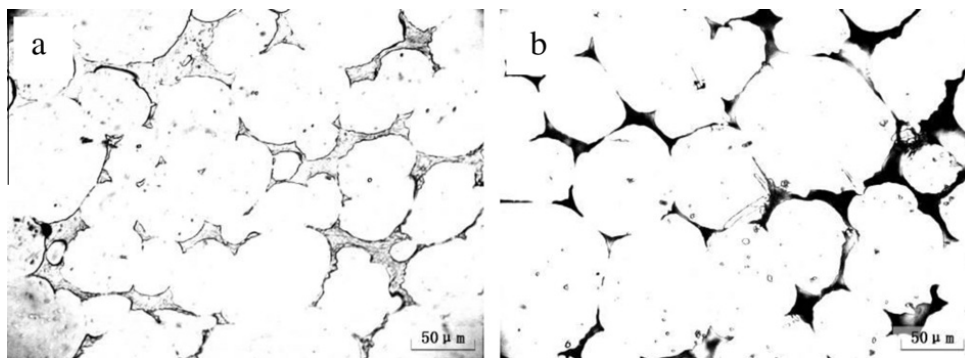


Fig. 4. Microstructures of the WBC after tests. (a) strained by 0.064 at 288 K and 0.001 s^{-1} , and (b) strained by 0.072 at 288 K and 1000 s^{-1} .

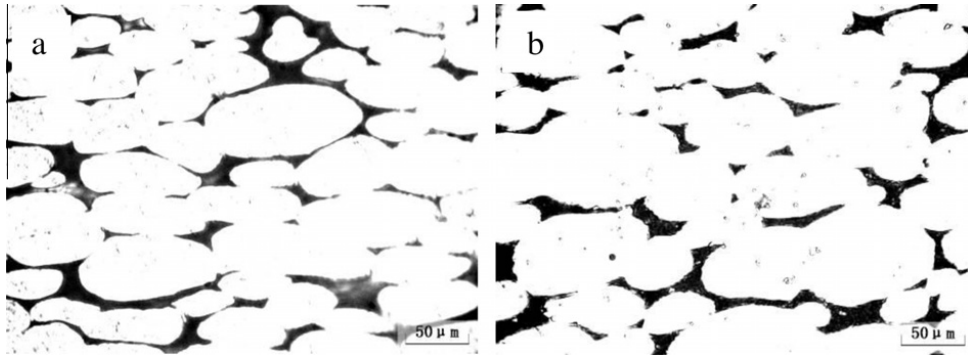


Fig. 5. Microstructures of the WBC after tests. (a) strained by 0.358 at 288 K and 0.001 s^{-1} , and (b) strained by 0.312 at 873 K and 3000 s^{-1} .

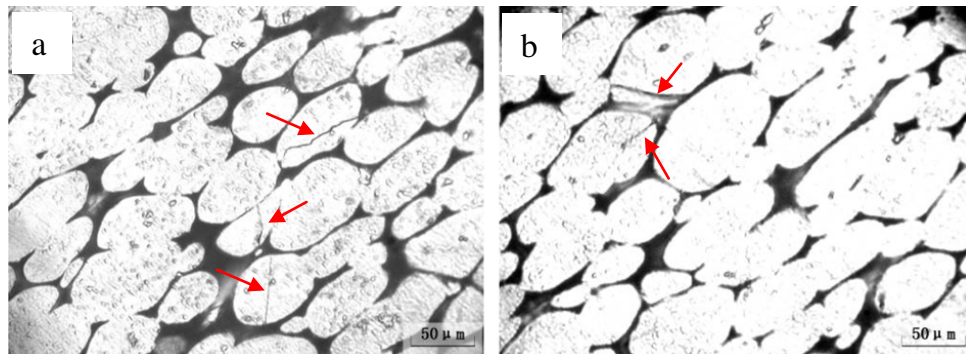


Fig. 6. Microstructures of the WBC after tests. (a) strained by 0.358 at 288 K and 0.001 s^{-1} , and (b) strained by 0.312 at 873 K and 3000 s^{-1} . Microcracks can be seen at the arrows.

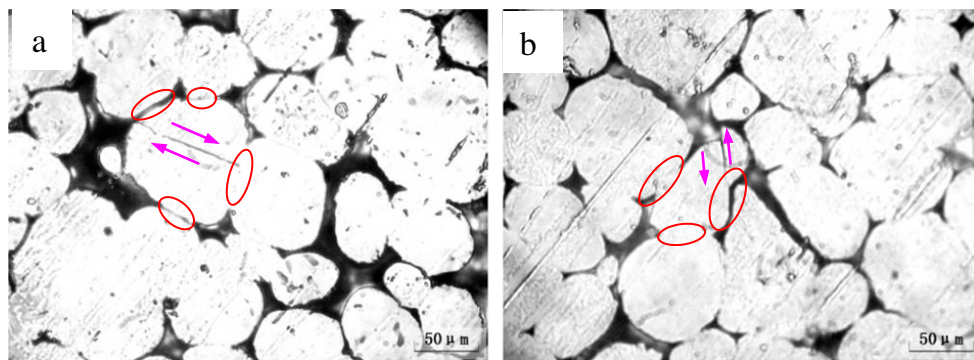


Fig. 7. Microstructures of the WBC after tests. (a) strained by 0.158 at 673 K and 0.001 s^{-1} , and (b) strained by 0.087 at 288 K and 1000 s^{-1} . The tungsten/tungsten interfaces around the cracked grains are marked by circles. The relative sliding of the fragments are marked by arrows.

In Lee and Chiou Study (1996), no microcrack was found at quasi-static deformation of a WBC subjected to shear loading. In this study, however, we observed cracked tungsten grains in the tested material for both quasi-static and dynamic tests. Fig. 6 shows microstructures of the material that was strained by 0.358 at 288 K and 0.001 s^{-1} , and strained by 0.312 at 873 K and 3000 s^{-1} , respectively. Cracked grains can be seen clearly in both cases (at the arrows). Another important phenomenon is that around each crashed grain, many tungsten/tungsten interfaces can be observed. It indicates that the cracks in the tungsten grains generate mainly from intense interactions between the grains. Fig. 7 presents some other typical microstructures in the material after quasi-static and dynamic loadings. It is clear that the grains deformed and gathered with plastic flow under compression. The interactions between the gathered grains highly elevated the flow stress. When the stress state in the tungsten grain was raised to a critical value, the grain cracked and the fragments continued to deform and move with plastic flow. In Fig. 7, relative sliding between the cracked parts of the tungsten grains can be clearly seen (at the arrows). For each cracked grain, the tungsten/tungsten interfaces around it are marked by circles. The result shows that the interaction between the grains is the determining factor that cracks the grains, regardless of strain rates of the deformation process. Therefore, cracked tungsten grains exist in the tested material for both quasi-static and dynamic cases.

4. Constitutive models

The plastic behavior of the WBC is strongly dependent on applied strain rate, temperature, and plastic strain. Therefore, accurate description and prediction of flow stress by constitutive models in consideration of its dependences on strain rate, temperature, and work hardening is of great importance. In our previous study, several well-known constitutive relations including two purely phenomenological ones (JC and KHL) and five physically-based ones (PB, NNL, ZA, VA, and RK) were compared systematically in characterizing plastic behavior of bcc (Xu and Huang, 2012a, b) and fcc (Xu and Huang, submitted for publication) metals. In the present work, the same models are also adopted to describe the flow stress of the WBC. The initial values of the model constants are determined through physical definition or experimental data fitting, and then these values are optimized using a combination of regression analysis and a constrained optimization procedure to minimize the errors between predictive and experimental results. For dynamic tests, the adiabatic temperature rise ΔT in the material during plastic deformation is calculated by Eq. (1),

$$\Delta T = \frac{\eta}{\rho C_V} \int \sigma d\varepsilon \quad (1)$$

where ρ is the mass density (17.8 g/cm³), η is the fraction of plastic work converted into heat, and C_V is the heat capacity (J/gK). There seems no agreement in publications on the value of η that works for different materials. For most metals, however, η is usually taken as 0.9 (Khan and Liang, 1999; Khan et al., 2004; Lennon and Ramesh, 2004; Wei et al., 2006). In this study, η is also assumed to be 0.9. Considering that C_V changes with temperature T , here we adopt a function of T to describe C_V , $C_V = 162.92 - 0.00995T + 1.74 \times 10^{-5}T^2$ ($\times 10^{-3}$ J/gK, for $0^\circ\text{C} \leq T \leq 1000^\circ\text{C}$) (Jiang and Zhang, 2000). Then the induced softening in flow stress is compensated in data processing. Accordingly, the calculated flow stress using the obtained models under dynamic loading should be converted back to the adiabatic condition. As the detailed procedures of parameter calibration have been introduced previously (Xu and Huang, 2012b), here the constitutive models will be addressed briefly and the determined material constants will be given directly for each model.

4.1. JC model

In the JC model (Johnson and Cook, 1983), the von Mises flow stress σ is expressed as

$$\sigma = (A + B\varepsilon^n)(1 + C \ln \dot{\varepsilon}^*)(1 - T^{*m}) \quad (2)$$

where ε is the equivalent plastic strain, $\dot{\varepsilon}^* = \dot{\varepsilon}/\dot{\varepsilon}_0$ is the dimensionless plastic strain rate for $\dot{\varepsilon}_0 = 1 \text{ s}^{-1}$, and $T^* = (T - T_r)/(T_m - T_r)$ is the homologous temperature, in which T is the absolute temperature, T_r (= 288 K) is the reference temperature, and T_m (= 1730 K) is the melting temperature of the material. The five material constants are A , B , n , C , and m . Table 1 gives the final values of these parameters after optimization. Comparison of experimental data and JC model description is shown in Fig. 8, for quasi-static and dynamic conditions respectively. In our previous study (Xu and Huang, 2012a, b, submitted for publication), the descriptive capability of the studied models for the basic data in consideration of rate dependence, temperature dependence, as well as the work-hardening of plastic behavior was assessed quantitatively using a fit error measure

$$\text{Error}(\%) = \frac{1}{n} \sum_i^n \left| \frac{\sigma_{\text{exp}}^i - \sigma_{\text{model}}^i}{\sigma_{\text{exp}}^i} \right| \times 100 \quad (3)$$

where σ_{exp} and σ_{model} are flow stresses obtained by experiment and model calculation, respectively. In this work, the same method is used to determine the descriptive and predictive errors of the models for comparison. According to Eq. (3), the descriptive errors of the JC model for quasi-static and dynamic loading conditions are 11.50 and 7.81%, respectively. Descriptive capability of the other models for the experiments conducted under quasi-static and dynamic loading conditions will also be assessed separately.

4.2. KHL model

In the KHL model (Huang and Khan, 1992; Liang and Khan, 1999; Khan and Liang, 1999; Khan et al., 2004; Khan et al., 2007a, b), the von Mises equivalent stress is expressed as

$$\sigma = \left[A + B \left(1 - \frac{\ln \dot{\varepsilon}}{\ln D_0} \right)^{n_1} \varepsilon^{n_0} \right] \left(\frac{\dot{\varepsilon}}{\dot{\varepsilon}^*} \right)^C \left(\frac{T_m - T}{T_m - T_r} \right)^m \quad (4)$$

Table 1
Determined values of the JC model parameters.

A (MPa)	B (MPa)	n	C	m
1011.1	1601.5	0.4982	0.0305	0.7263

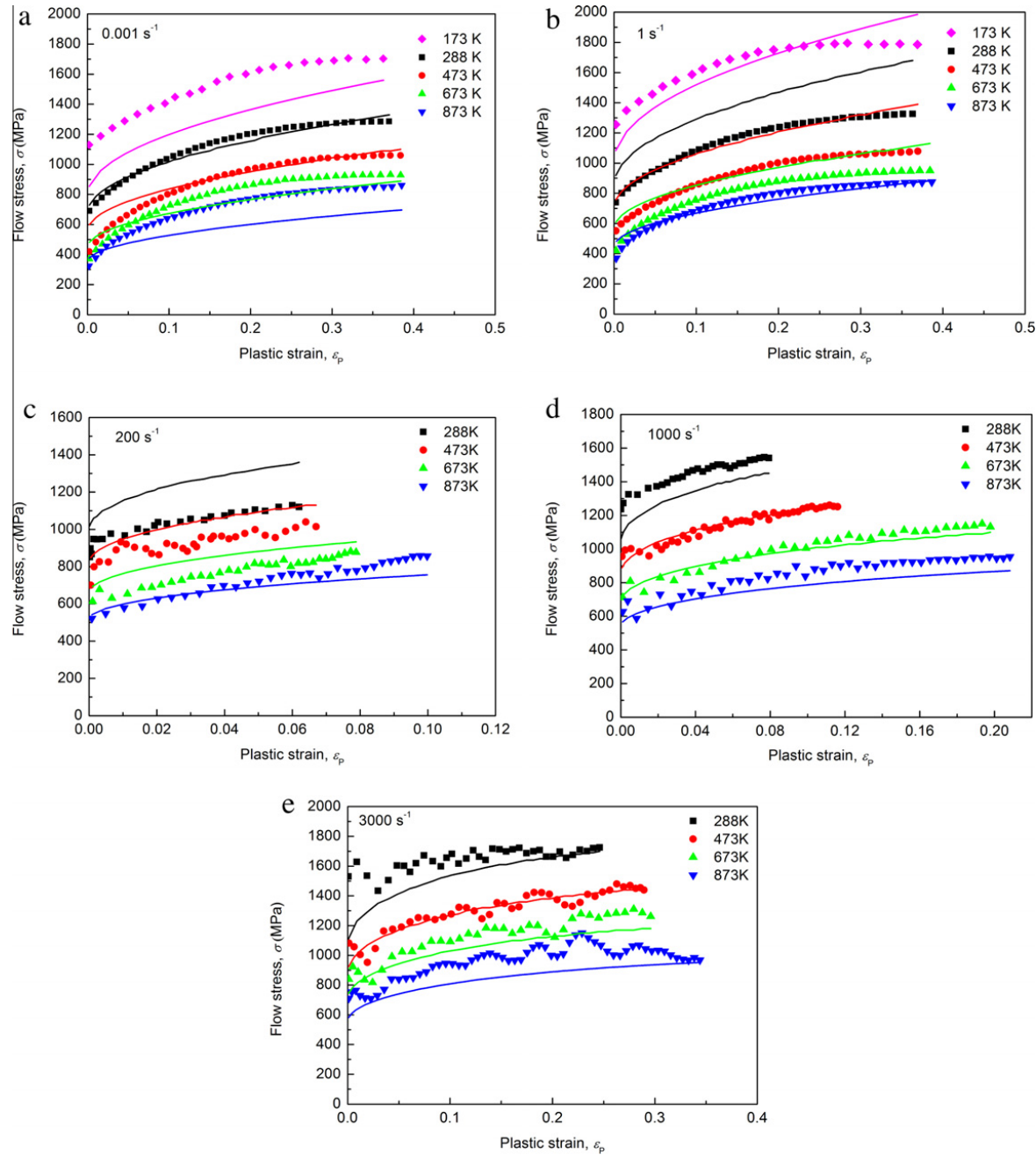


Fig. 8. Comparison of experimental data and JC model description for (a) 0.001 s^{-1} , (b) 1 s^{-1} , (c) 200 s^{-1} , (d) 1000 s^{-1} , and (e) 3000 s^{-1} (Symbols represent experimental data; solid lines are model description, with the same color for each temperature.).

Table 2
Determined values of the KHL model parameters.

A (MPa)	B (MPa)	n_1	n_0	C	m
264.6	1533.7	3.9588	0.2172	0.2196	1.1444

where $\dot{\epsilon}^*(=1\text{ s}^{-1})$ is the reference strain rate, and $D_0(=10^6\text{ s}^{-1})$ is the arbitrarily chosen upper bound strain rate. A, B, n_1 , n_0 , C, and m are material constants, and the other parameters have the same definition as the JC model. The determined values of the six material constants are shown in Table 2. Fig. 9 shows comparison of experimental data and KHL model description for different loading conditions. The descriptive errors of the KHL model for quasi-static and dynamic loading conditions are 9.49 and 9.45%, respectively.

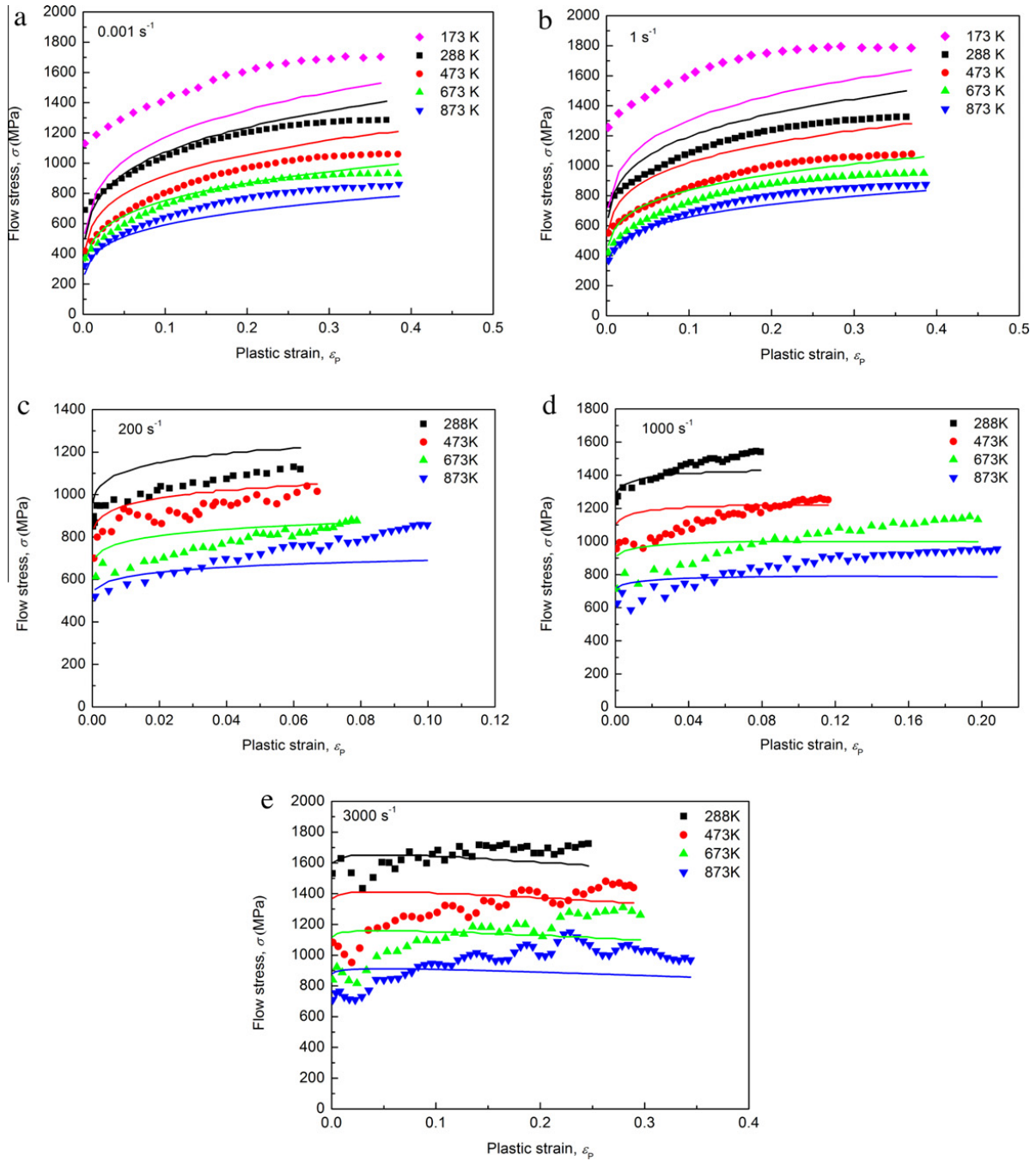


Fig. 9. Comparison of experimental data and KHL model description for (a) 0.001 s^{-1} , (b) 1 s^{-1} , (c) 200 s^{-1} , (d) 1000 s^{-1} , and (e) 3000 s^{-1} .

4.3. PB model

In this model (Nemat-Nasser and Isaacs, 1997; Klepaczko et al., 2009; Nemat-Nasser and Guo, 2000; Nemat-Nasser and Guo, 2003), the flow stress is defined by

$$\sigma = \sigma_a + \sigma^* \quad (5)$$

where σ_a and σ^* represent the athermal and thermally activated parts of the flow stress, respectively. The stress σ^* is expressed as

$$\sigma^* = \hat{\sigma} \left[1 - \left(-\frac{kT}{G_0} \ln \frac{\dot{\epsilon}}{\dot{\epsilon}_0} \right)^{1/q} \right]^{1/p} \quad \text{for } T \leq T_c \quad (6)$$

where $\hat{\sigma}$ is the threshold stress above which the barrier is overcome without any aid from thermal activation, and $\dot{\varepsilon}_0$ can be seen as a reference strain rate which characterizes the strain rate sensitivity. The parameter k/G_0 represents the temperature sensitivity of the material. The critical temperature T_c is given by

$$T_c = -\frac{G_0}{k \ln(\dot{\varepsilon}/\dot{\varepsilon}_0)} \quad (7)$$

with $\sigma^* = 0$ for $T > T_c$. The values of p and q in Eq. (9) are defined with $0 < p \leq 1$ and $1 \leq q \leq 2$. In this study, we choose $p = 0.5$ and $q = 1.5$ for the PB model. The same values are also used in the NNL and VA models.

The athermal stress σ_a is usually expressed by a power law approximation

$$\sigma_a = a\varepsilon^n \quad (8)$$

where ε is plastic strain, a is the material constant fitting the stress level, and n is the strain hardening exponent, which is also assumed to be constant in this formulation.

Thus, in PB model, the flow stress is given by

$$\sigma = \sigma_a + \sigma^* = a\varepsilon^n + \hat{\sigma} \left[1 - \left(-\frac{kT}{G_0} \ln \frac{\dot{\varepsilon}}{\dot{\varepsilon}_0} \right)^{1/q} \right]^{1/p} \quad (9)$$

For the studied WBC, the optimized values of the constitutive parameters are shown in Table 3. Fig. 10 shows comparison of experimental data and PB model description for different loading conditions. The descriptive errors of the PB model for quasi-static and dynamic loading conditions are 7.14 and 7.62%, respectively.

4.4. NNL model

In this model (Nemat-Nasser and Li, 1998), the athermal stress $\tau_\mu = a_1\gamma^n$, while the thermal part of flow stress is defined as

$$\tau^* = \tau_0 \left\{ 1 - \left(-\frac{kT}{G_0} \left(\ln \frac{\dot{\gamma}}{\dot{\gamma}_0} + \ln(1 + a(T)\gamma^{1/2}) \right) \right)^{1/q} \right\}^{1/p} (1 + a(T)\gamma^{1/2}) \text{ for } T \leq T_c \quad (10)$$

with the critical temperature given by

$$T_c = -\frac{G_0}{k} \left[\ln \frac{\dot{\gamma}}{\dot{\gamma}_0} + \ln f(\gamma, T_c) \right]^{-1} \quad (11)$$

Here $a(T)$ is an empirical function, which contributes to the definition of the strain hardening of the material,

$$a(T) = a_0 \left[1 - \left(\frac{T}{T_m} \right)^2 \right] \quad (12)$$

f is a function of the plastic strain γ and temperature T ,

$$f(\gamma, T) = 1 + a(T)\gamma^{1/2} \quad (13)$$

We choose $q = 1.5$, $p = 0.5$, then the final expression of the flow stress becomes

$$\tau = a_1\gamma^n + \tau_0 \left\{ 1 - \left(-\frac{kT}{G_0} \left(\ln \frac{\dot{\gamma}}{\dot{\gamma}_0} + \ln(1 + a(T)\gamma^{1/2}) \right) \right)^{2/3} \right\}^2 (1 + a(T)\gamma^{1/2}) \quad (14)$$

The final values of the constitutive parameters are shown in Table 4. Fig. 11 shows comparison of experimental data and NNL model description for different loading conditions. The descriptive errors of the NNL model for quasi-static and dynamic loading conditions are 6.41 and 7.40%, respectively.

4.5. ZA model

For bcc metals, the ZA model (Zerilli and Armstrong, 1987) is expressed as

$$\sigma = C_0 + C_1 \exp(-C_3T + C_4T \ln \dot{\varepsilon}) + C_5\varepsilon^n \quad (15)$$

Table 3

Determined values of the PB model parameters.

a (MPa)	n	$\hat{\sigma}$ (MPa)	k/G_0 (K ⁻¹)	$\ln \dot{\varepsilon}_0$ (s ⁻¹)	p	q
1154.4	0.2635	2061.4	3.89×10^{-5}	2×10^{10}	0.5	1.5

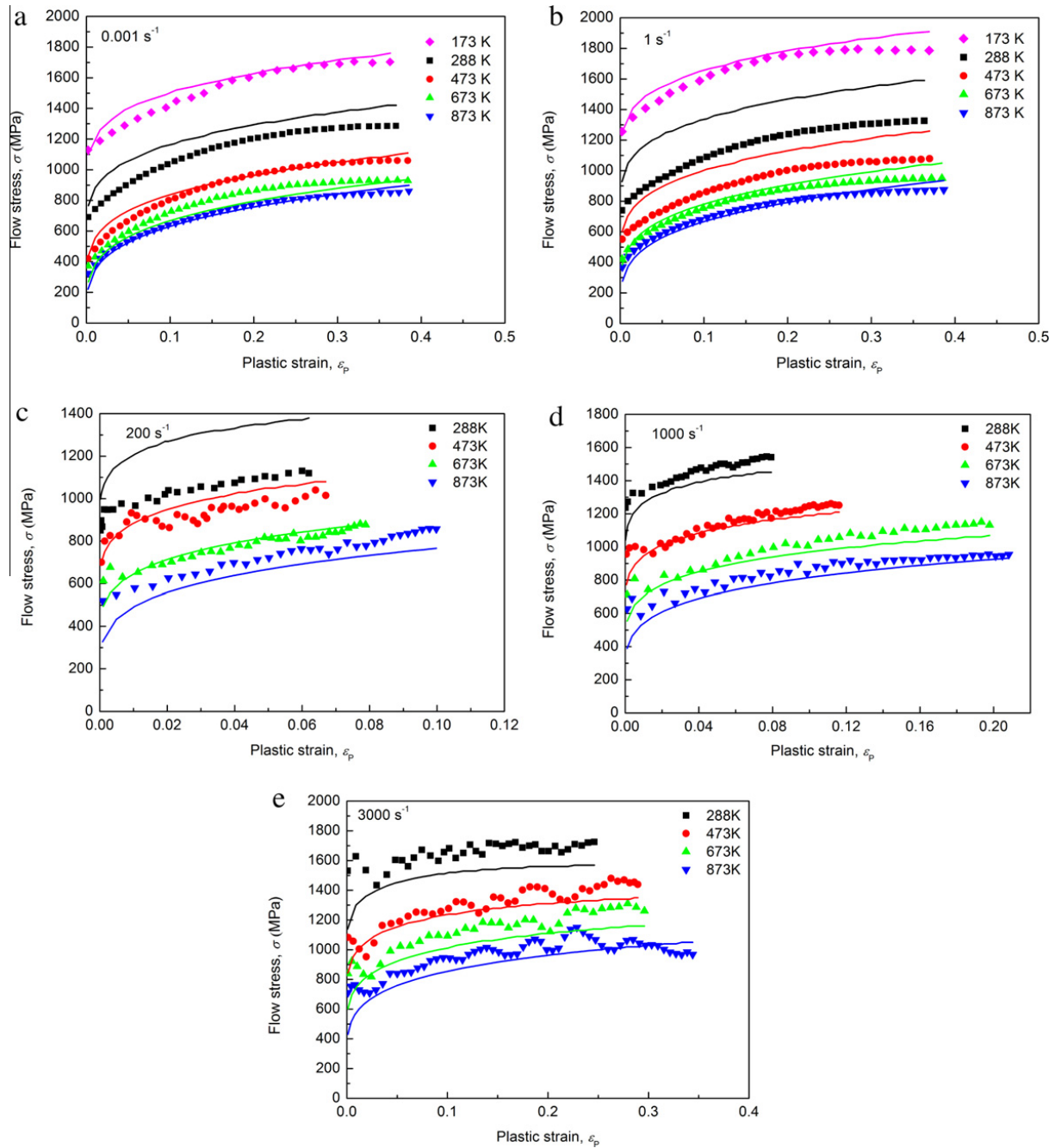


Fig. 10. Comparison of experimental data and PB model description for (a) 0.001 s⁻¹, (b) 1 s⁻¹, (c) 200 s⁻¹, (d) 1000 s⁻¹, and (e) 3000 s⁻¹.

Table 4

Determined values of the NNL model parameters.

a_1 (MPa)	n	τ_0 (MPa)	k/G_0 (K ⁻¹)	$\dot{\gamma}_0$ (s ⁻¹)	a_0	p	q
1115.5	0.2259	1865.9	3.89×10^{-5}	2×10^{10}	0.0480	0.5	1.5

In this formulation, the second term represents the thermally activated part of the flow stress, while $C_0 + C_5 \epsilon^n$ is the athermal component, with $C_0 = \sigma_0 + kl^{-1/2}$. σ_0 is an additional component of stress relating to solute and the original dislocation density, k is a microstructural stress intensity, and l is the average grain diameter.

The final values of the constitutive parameters are shown in Table 5. Fig. 12 compares experimental data and ZA model description for different loading conditions. The descriptive errors of the ZA model for quasi-static and dynamic loading conditions are 4.53 and 4.90%, respectively.

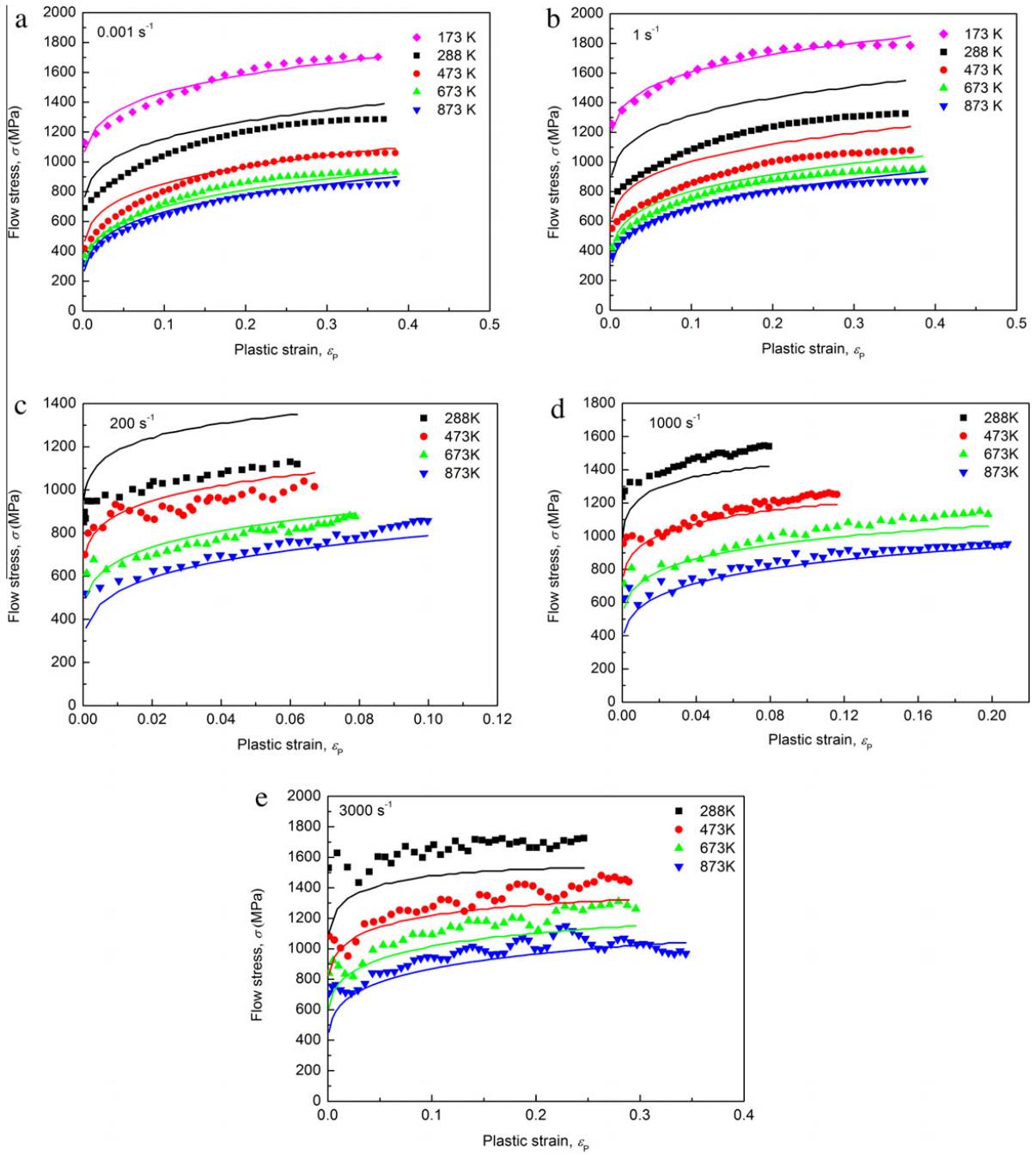


Fig. 11. Comparison of experimental data and NNL model description for (a) 0.001 s^{-1} , (b) 1 s^{-1} , (c) 200 s^{-1} , (d) 1000 s^{-1} , and (e) 3000 s^{-1} .

Table 5

Determined values of the ZA model parameters.

C_0 (MPa)	C_1 (MPa)	C_3 (K^{-1})	C_4 (K^{-1})	C_5 (MPa)	n
230.5	1864.6	0.00401	2.49×10^{-4}	1083.1	0.4355

4.6. VA model

In this model (Voyiadjis and Abed, 2005; Voyiadjis and Almasri, 2008), for bcc metals,

$$\sigma = \hat{Y}[1 - (\beta_1 T - \beta_2 T \ln \dot{\epsilon})^{1/q}]^{1/p} + B\dot{\epsilon}^n + Y_a \quad (16)$$

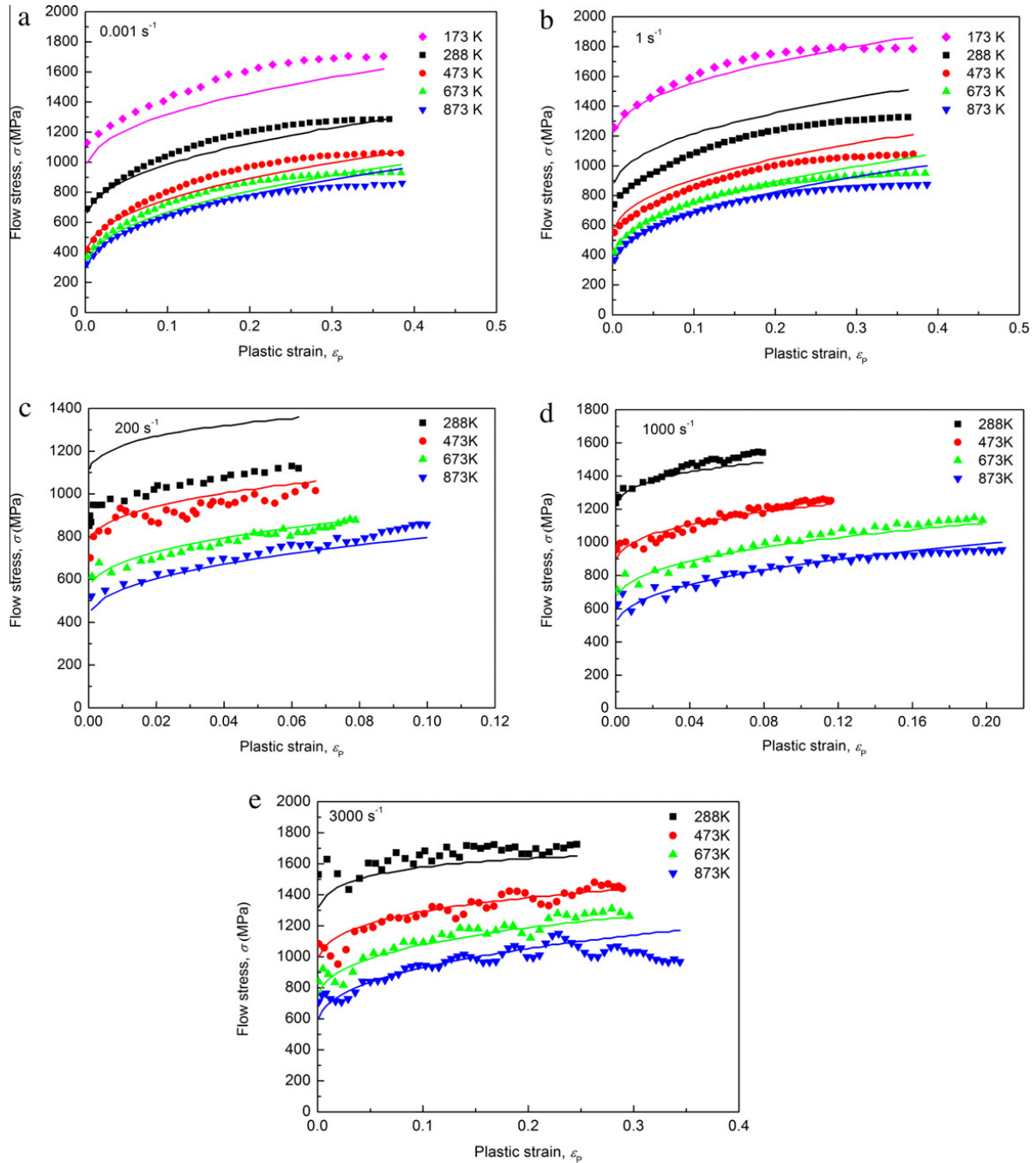


Fig. 12. Comparison of experimental data and ZA model description for (a) 0.001 s^{-1} , (b) 1 s^{-1} , (c) 200 s^{-1} , (d) 1000 s^{-1} , and (e) 3000 s^{-1} .

where the first term is the thermal component of the flow stress. $\hat{Y}$ is the threshold yield stress of the Peierls barrier to the dislocation. β_1 and β_2 are material constants. β_1 characterizes the effect of both dislocation density and the length scale through the evolution of hardening. The last two terms are the athermal component.

The final values of the constitutive parameters are shown in Table 6. Comparison of experimental data and VA model description is shown in Fig. 13. The descriptive errors of the VA model for quasi-static and dynamic loading conditions are 5.55 and 5.75%, respectively.

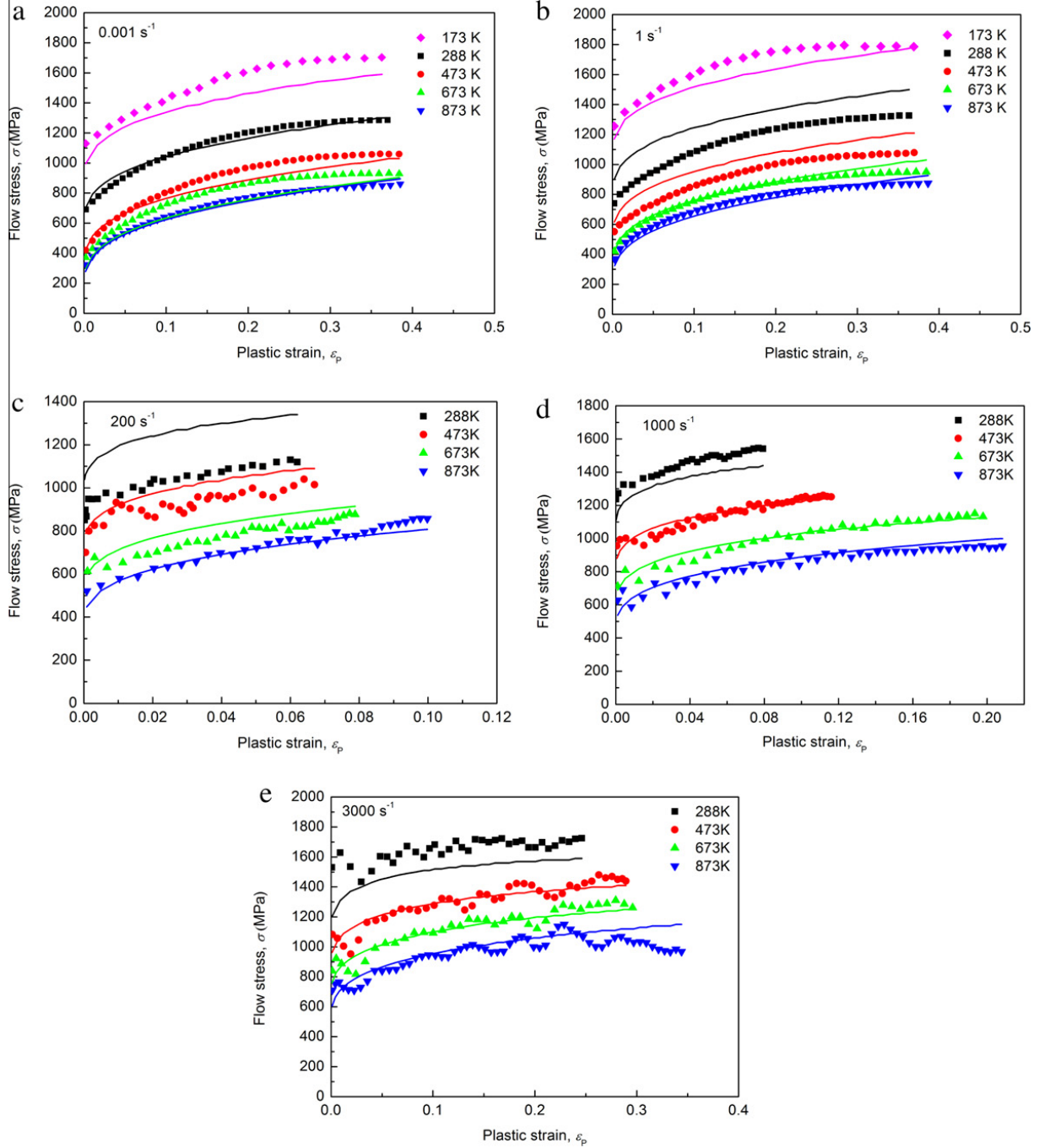
4.7. RK model

In this model (Rusinek and Klepaczko, 2001; Klepaczko et al., 2009), the flow stress is also decomposed into the athermal stress σ_a and the thermal stress σ^* . Considering the thermal softening of crystalline lattice, the both stress components are normalized by temperature-dependent Young's modulus $E(T)$.

Table 6

Determined values of the VA model parameters.

$\dot{\gamma}$ (MPa)	β_1 (K ⁻¹)	β_2 (K ⁻¹)	p	q	B (MPa)	n	Y_a (MPa)
1782.3	8.97×10^{-4}	5.33×10^{-5}	0.5	1.5	1017.4	0.3407	151.0

**Fig. 13.** Comparison of experimental data and VA model description for (a) 0.001 s⁻¹, (b) 1 s⁻¹, (c) 200 s⁻¹, (d) 1000 s⁻¹, and (e) 3000 s⁻¹.

$$\sigma = \frac{E(T)}{E_0} [\sigma_\mu(\epsilon, \dot{\epsilon}, T) + \sigma^*(\dot{\epsilon}, T)] \quad (17)$$

The expression for $E(T)$ is given by

$$E(T) = E_0 \left\{ 1 - \frac{T}{T_m} \exp \left[\theta^* \left(1 - \frac{T_m}{T} \right) \right] \right\} \quad (18)$$

where E_0 is the Young's modulus at $T = 0$ K, T_m is the melting temperature, and θ^* is the characteristic homologous temperature. The internal stress is given by

$$\sigma_\mu(\dot{\epsilon}, \dot{\epsilon}, T) = B(\dot{\epsilon}, T)(\epsilon_0 + \epsilon)^{n(\dot{\epsilon}, T)} \quad (19)$$

where ϵ_0 is the strain level which defines the yield stress in quasi-static loading. $B(\dot{\epsilon}, T)$ and $n(\dot{\epsilon}, T)$ are the modulus of plasticity and the strain hardening exponent, respectively, which are defined as

$$B(\dot{\epsilon}, T) = B_0 \left(\frac{T}{T_m} \log \frac{\dot{\epsilon}_{\max}}{\dot{\epsilon}} \right)^{-\nu}, \quad (20)$$

$$n(\dot{\epsilon}, T) = n_0 \left\langle 1 - D_2 \frac{T}{T_m} \log \frac{\dot{\epsilon}}{\dot{\epsilon}_{\min}} \right\rangle \quad (21)$$

where B_0 and D_2 are material constants. $\dot{\epsilon}_{\max} = 10^7 \text{ s}^{-1}$ and $\dot{\epsilon}_{\min} = 10^{-5} \text{ s}^{-1}$ are the upper and lower limit of the strain rate level for a particular material; these values are also used in this study. ν is proportional to temperature sensitivity, and n_0 is the strain hardening exponent at $T = 0$ K. In Eq. (21), the operator is used with the following meaning

$$\langle x \rangle = \begin{cases} x, & \text{if } x \geq 0 \\ 0, & \text{if } x < 0 \end{cases} \quad (22)$$

The thermal stress in RK model is given by

$$\sigma^*(\dot{\epsilon}, T) = \sigma_0^* \left\langle 1 - D_1 \frac{T}{T_m} \log \frac{\dot{\epsilon}_{\max}}{\dot{\epsilon}} \right\rangle^{m^*} \quad (23)$$

where σ_0^* is the effective stress at $T = 0$ K, D_1 and m^* are material constants describing temperature and strain rate sensitivities.

The final values of the constitutive parameters are shown in Table 7. The physical constants for the WBC are shown in Table 8. Fig. 14 compares experimental data and RK model description for different loading conditions. The descriptive errors of the RK model for quasi-static and dynamic loading conditions are 6.06 and 4.92%, respectively.

According to Eq. (3), the quasi-static, dynamic, and average descriptive errors of all the constitutive models are compared in Fig. 15. It is clear that the ZA model shows the lowest description errors for both quasi-static and dynamic experimental data, which indicates that the ZA model has the least minimum required fitting data (Xu and Huang, 2012b) in calibration of model parameters. The VA and RK models have similar performance in describing flow stress of the WBC, while the PB and NNL models exhibit close accuracy in each case. In characterization of the quasi-static data, the KHL model has a better performance than the JC model, but this model gives larger errors for the dynamic data than the JC model. This may result from description of decreasing flow stress with plastic strain at high strain rates (Fig. 9(e)) for this model, which is good at capturing thermal softening at high strain rates and reduction in work hardening rate of materials (Khan et al., 2004; Khan and Liang, 1999). In general, the average descriptive errors of all the models are below 10%, with the largest value of 9.66% and the least value 4.72% for the JC and the ZA models, respectively. It should be noted that a root-mean-square (RMS) error analysis was also performed in this study. The result shows that the RMS error values are approximately the same as the error values obtained by averaging deviations with Eq. (3). Hence, the averaging method of Eq. (3) is adopted here for easier comparison to previously published analyses.

Table 7
Determined values of the RK model parameters.

B_0 (MPa)	ν	n_0	D_1	D_2	ϵ_0	σ_0^* (MPa)	m^*
1775.9	0.0592	0.3674	0.2898	0.1340	0.02	1462.1	3.67

Table 8
Physical constants of the WBC.

E_0 (GPa)	θ^*	η	ρ (g/cm ³)
370	0.5	0.9	17.8

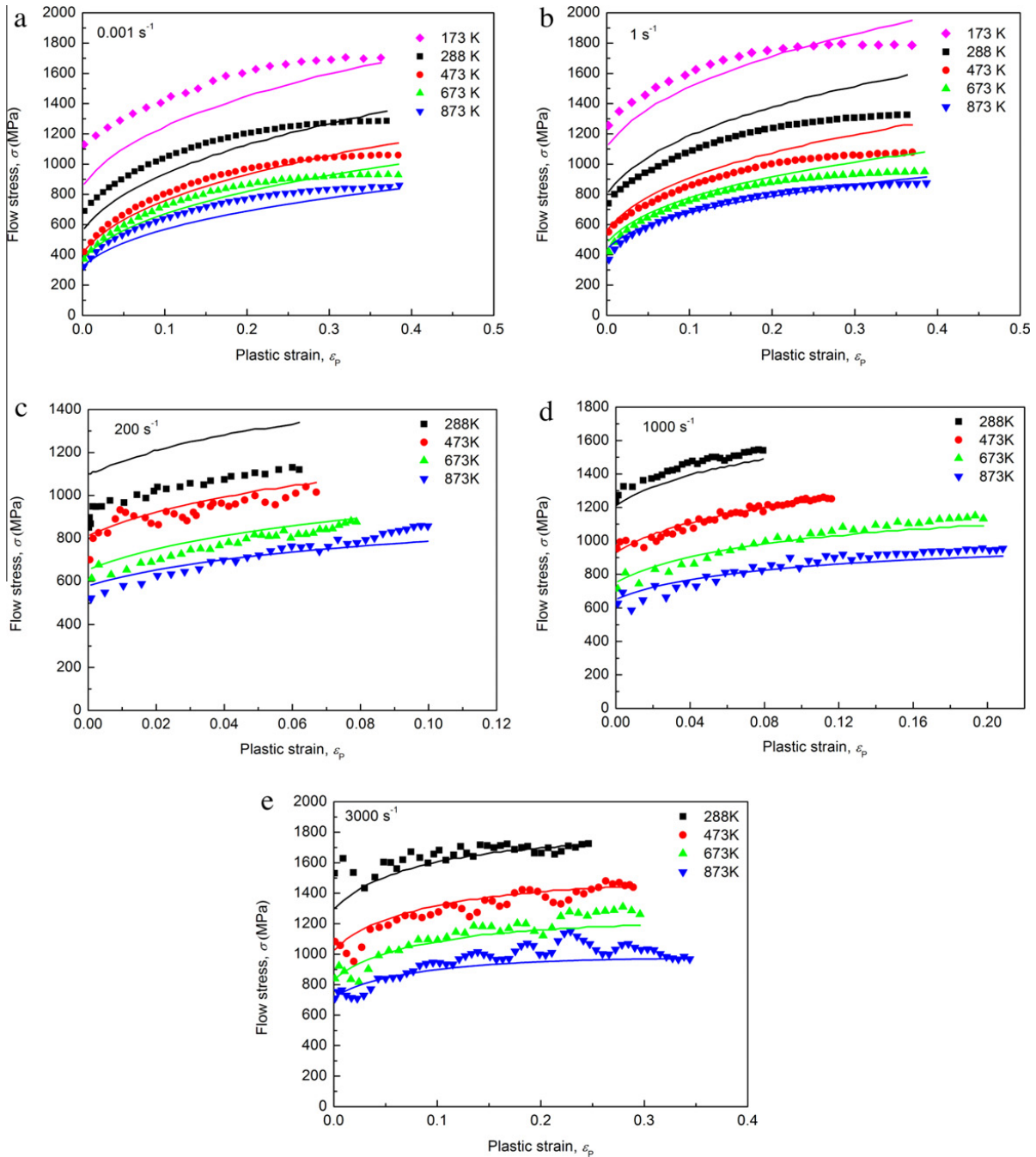


Fig. 14. Comparison of experimental data and RK model description for (a) 0.001 s^{-1} , (b) 1 s^{-1} , (c) 200 s^{-1} , (d) 1000 s^{-1} , and (e) 3000 s^{-1} .

5. Verification and comparison of constitutive models

5.1. Model verification

Besides the capability of characterizing the fundamental experimental data, the utility of a constitutive model is also dependent on its prediction for conditions that are not determined experimentally. To assess the applicability of the previously determined constitutive models, strain rate jump tests have been conducted with different strain rate and temperature sequences. These loading conditions were not included in the fundamental tests, and the loading history effect is not considered in this study. Through comparison between model prediction and experimental measurement, the precision of these models can be checked for combined strain, strain rate, and temperature conditions. In these tests, the specimen was prepared and loaded in the following process. First, the specimen ($\Phi 6 \times 9 \text{ mm}$) underwent a plastic strain by 0.078 at 793 K and 0.002 s^{-1} . Then the loading conditions were changed to 453 K and 0.5 s^{-1} , and the same specimen was sequentially strained

by 0.114. Afterward, a new specimen ($\Phi 4 \times 4$ mm) was machined from the center of the deformed specimen by WEDM, and then strained by 0.389 in the same direction at 293 K and 3800 s^{-1} with the revised Hopkinson bar technique. Finally, the experimental data were compared with the predictive results of the constitutive models under the same loading conditions. It should be noted that in the first two steps of loading, the test temperature of the specimen was attained and kept constant to within $\pm 2^\circ \text{C}$ in 10 min with the electro-thermal cells, to avoid heat treatment effect on the specimen. The metallagraphical and temperature effects induced from the WEDM of the new specimen are not considered here.

For comparison, the experimental results and the predictions of the models are plotted together in Fig. 16. For the first step of loading, the KHL and NNL models give very close prediction to the measured flow stress. The calculated flow stress curves by the other models are lower than the experimental results, but most models can still give fairly good prediction. For the second step of loading, the ZA and RK models predict the flow stress precisely, while the other models obviously overestimate the test results. It is noteworthy that only the ZA model gives very good prediction to the tendencies of the flow stress for the both quasi-static tests; for the dynamic test, most models (PB, NNL, ZA, VA, and RK) give considerably flat curves of flow stress, while the JC model presents evidently increasing flow stress with plastic strain. Only the KHL model gives agreeable prediction to the descending trend of the stress–strain curve. This descending trend flow stress results from the microstructural evolution and damage in the previous steps of quasi-static compression, as well as the vast adiabatic temperature rise in the last step of dynamic loading. The better correlation of the KHL model with the dynamic behavior of this WBC results from its good performance in characterizing thermal softening of materials at high strain rates (Khan et al., 2004; Khan and Liang, 1999).

Errors between the model prediction and the experimental results can also be measured by Eq. (3). The predictive capabilities of the obtained constitutive models for the strain rate jump tests are compared in Fig. 17. The PB model has the largest errors for both quasi-static and average results. The NNL model gives the largest error in prediction of dynamic data, which is 11.10%. Contrarily, the KHL model has the least error 5.77% in prediction of dynamic data, while the ZA model shows the best accuracy in prediction of both quasi-static and average results, the errors of which are respectively 7.69 and 6.89%. It

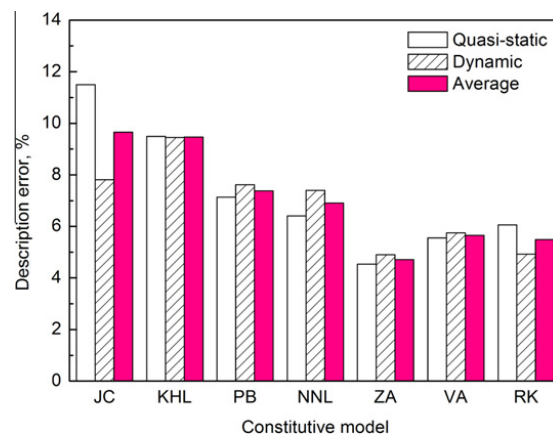


Fig. 15. Comparison of different constitutive models in description of fundamental experimental data.

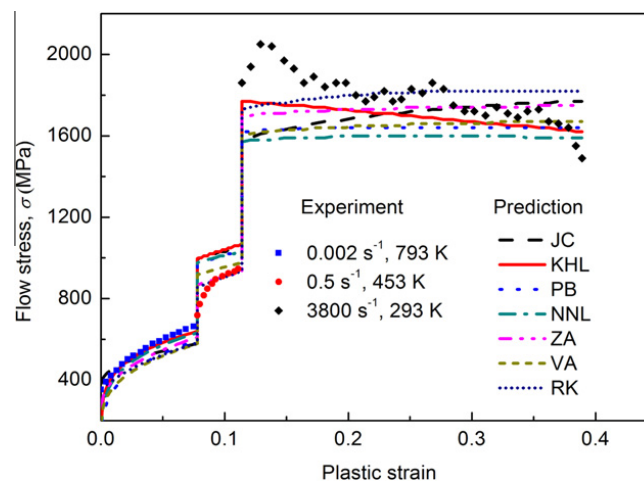


Fig. 16. Comparison between experimental data and predictive results of different models for the strain rate jump tests.

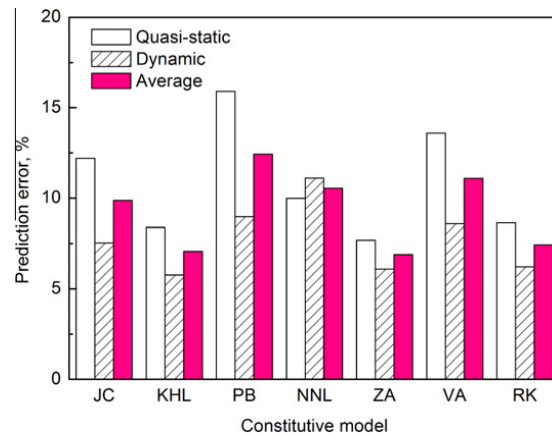


Fig. 17. Comparison of different constitutive models in prediction of the strain rate jump tests.

indicates that in characterizing plastic behavior of the studied WBC, the ZA model has the least minimum required fitting data in calibration of model parameters. Comparing the errors between basic data description and strain rate jump tests prediction, it is clear that for most of the models the errors in prediction are larger than those of description (PB, NNL, ZA, VA, and RK). It is not likely that the precision of prediction is much better than that of description. Therefore, the capabilities of the models in data prediction rely largely on their performance in basic data description. The superior performance of the ZA model to the others in both description and prediction of the WBC flow stress also indicates that the applicability of a constitutive model is dependent on both the number of its constitutive parameters and its formulation. There exists a balance between the flexibility and minimum required fitting data of a model regarding a specific material. These results agree very well with our previous study in characterizing flow stress of other metals with the same models (Xu and Huang, 2012b, submitted for publication).

5.2. Model comparison in thermomechanical description

In this section, the capabilities of the selected constitutive models in characterization of work-hardening, temperature, and strain rate effects of the WBC will be discussed separately, based on comparison of their formulation, as well as correlation with the experimental data.

The description of work-hardening behaviors of materials at different loading conditions is of great significance in the development of constitutive models. Depending on the microstructure, different materials may have distinct dependence of work hardening on strain rate and temperature. Therefore, performance of the constitutive models in description of work hardening plays an important role for the flexibilities of the models. The both phenomenological models, JC and KHL, define a coupled effect of strain rate and temperature on work hardening by the simple multiplication form. In the JC model, the work-hardening rate increases with strain rate and at the same time decreases with temperature at certain strain. Therefore, this model is widely used to describe the work-hardening behaviors of many metals. However, this model is not appropriate for modeling materials where the work-hardening rate remains constant or decreases when strain rate increases, such as tantalum (Liang and Khan, 1999). Another shortcoming of the JC model is that the intensive strengthening of flow stress at high loading rates cannot be properly described by this model, as will be shown later. Different from the JC model, the KHL model considers a softening effect of strain rate on flow stress in the first set of brackets, and improves the temperature dependence term to accommodate the case when the current temperature is lower than the reference temperature. As shown previously, the superior capabilities of the KHL model are its correlation of thermal softening at high strain rates, and the decreasing work hardening rate as the strain and strain rate increases; hence this model is especially suitable for describing viscoplastic behaviors of materials such as titanium alloys (Khan et al., 2004).

In the physically based constitutive models studied here, only the NNL model considers work hardening behavior of materials in both its thermal and athermal components. The thermal stress of different deformation levels is associated to strain rate and temperature by function f , while the athermal stress employs the power law form. For the other models, they consider work hardening only in the athermal part of flow stress, because in most BCC metals, overcoming Peierls stress resistance is the main phenomenon involved in thermally activated processes, and hence the yield stress shows strong dependence on the strain rate and temperature, while the strain hardening is less sensitive to them. Concerning the WBC studied here, the work hardening behavior is comparatively insensitive to temperature, but some dependence can still be observed on strain rate. Moreover, the PB, ZA, and VA models define a constant work-hardening behavior by their addition forms. The work-hardening rate is independent of temperature or strain rate at each specific plastic strain. This can be observed clearly from the constant intervals between the model curves in Figs. 10, 12 and 13. Therefore, these models are not appropriate for describing materials of which work-hardening behaviors are sensitive to change of temperature and strain rate (e.g. fcc metals), but they simulates the flow stress curves in the WBC fairly well. Contrarily, the RK model considers the

effects of temperature and strain rate on work hardening via the modulus of plasticity B and the strain hardening exponent n , which are both defined with dependence on strain rate and temperature (see Eqs. (20) and (21), respectively). From this point of view, the NNL and RK models are more applicable in characterization of complex work hardening behaviors. In this study, the performance of the NNL and RK models for influence of temperature and strain rate on work-hardening can be obviously seen from the increasing intervals between the model curves with plastic strain in Figs. 11 and 14, respectively.

To sum up, the JC model presents a work-hardening rate that increases with strain rate and decreases with temperature. The KHL model is good at characterization of the thermal softening at high strain rates, and the decreasing work hardening rate as the strain rate increases. The NNL and RK models have a superior capability in describing complex strain-hardening behaviors, due to their consideration of both temperature and strain rate effects. The PB, ZA, and VA models have a constant work-hardening rate, and therefore are not appropriate for simulation of work-hardening behaviors that are sensitive to temperature and strain rate.

Fig. 18 compares the description of temperature effect by the models at plastic strain 0.1, for the strain rates of 0.001 and 3000 s⁻¹. At 0.001 s⁻¹ (Fig. 18(a)), the JC and KHL models obviously underestimate the flow stress at 173 K; the JC model also gives lower prediction of flow stress for 873 K, but the KHL model predicts better at this temperature. The physically based constitutive models can simulate the decreasing trend of flow stress with temperature very well. The PB and NNL models exhibit higher values than the other models for each temperature, but the flow stress–temperature curves are very close for these models. It should be noted that for the PB, NNL, and VA models, the predicted flow stress starts to increase slightly, when the temperature approaches 900 K. This is because the highest temperature in the fundamental experimental data for model construction is 873 K. Therefore, a constitutive model should be used very carefully when the loading conditions in prediction are beyond the ranges of fundamental tests. For 3000 s⁻¹, the adiabatic temperature rise is considered in Fig. 18(b). It is seen that the physically based models also show good correlations with the experimental results. Especially, the RK and ZA models can simulate the experimental data accurately at all temperatures. Different from the situation at 0.001 s⁻¹, the PB and NNL models appreciably underestimate the flow stress for each temperature here. The JC model give good prediction at 510 K, but shows lower values for the other test results; On the contrary, the KHL model overestimates the flow stress at 510 K, but correlates well with the measurements at the other temperatures. Generally, the JC and KHL models predict more accurately at this strain rate.

On the whole, the physically based models describe the temperature effect of flow stress better than the two phenomenological models, especially for the low strain rate. The JC and KHL models do not describe the tendency of flow stress well at 0.001 s⁻¹, but their performance is better at 3000 s⁻¹. It is interesting to note that the KHL model exhibits almost linear relation between flow stress and temperature for the both strain rates. This may be a drawback of this model for describing complex temperature effects on flow stress.

The descriptive capabilities of the models on the strain rate effect at 288 and 873 K, for plastic strain 0.05, are compared in Fig. 19. At 288 K, the fast increase of flow stress for strain rates larger than 1000 s⁻¹ is only correctly described by the KHL model. The ZA and RK models also exhibit close prediction to the test results at 1000 and 3000 s⁻¹, but they fail to predict the results precisely at lower strain rates. The NNL and PB models only show good correlations at 1000 s⁻¹, but they evidently overestimate the flow stress for lower strain rates, and underestimate the result at 3000 s⁻¹. The JC and VA models only give good prediction at strain rates of 0.001 and 1000 s⁻¹. For 873 K, the PB and NNL models have very good prediction to the test data for each strain rate. The KHL, ZA, VA, and RK models overestimate the result at 1000 and 3000 s⁻¹, but they show close prediction to the lower rates. It is interesting to observe that the predicted curves of the PB, NNL, and VA models decrease slightly at very low strain rates (<0.005 s⁻¹), resulting from limited fundamental data in this range. This phenomenon may be avoided when more experimental data are used at lower strain rates in the optimization process. It is also noteworthy that the JC model describes nearly a linear relationship between flow stress and strain rate for the both temperatures; it results from a shortcoming of this model by characterizing a linear increase of flow stress with log strain rate, which is not the case in practice especially at high strain rates.

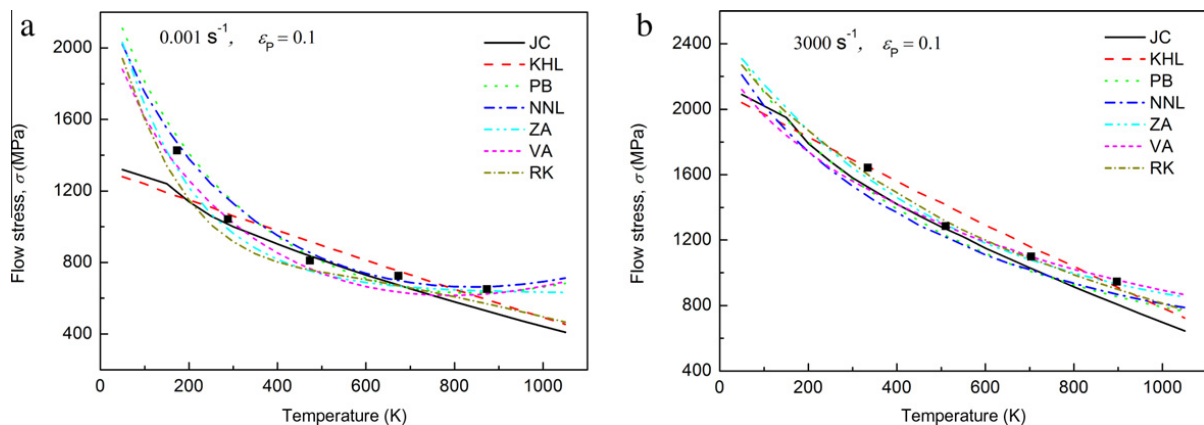


Fig. 18. Description of temperature effect by different models at plastic strain 0.1 for strain rates of (a) 0.001 s⁻¹, and (b) 3000 s⁻¹.

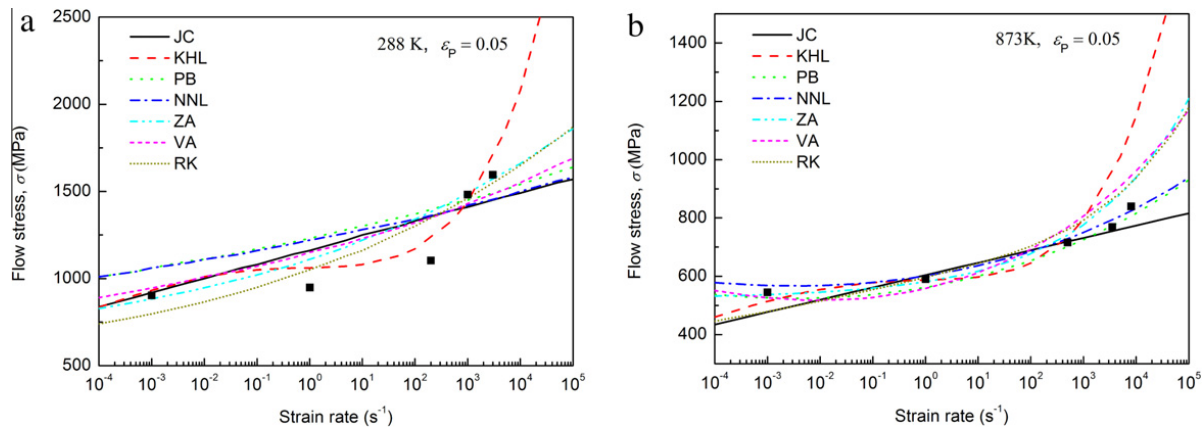


Fig. 19. Description of strain rate effect by different models at plastic strain 0.05 for temperatures of (a) 288 K, and (b) 873 K.

To sum up, the KHL model shows a better description of the strain rate effect on flow stress than the other models at 288 K, especially for the intense increase of flow stress at high strain rates. At 873 K, the PB and NNL models correlate better with the test data for all the strain rates. The other models show close prediction to test data at lower strain rates, but predict the result poorly at 1000 and 3000 s^{-1} . The JC model has the poorest description of the strain rate effect on flow stress, due to its definition of linear relationship between flow stress and log strain rate.

From the above analysis, it can be concluded that the constitutive models studied here have various specialities in the description of different aspects of material behaviors. The JC and KHL models do not consider any physical aspects of the material, and show comparatively larger errors in description of the experimental results. However, these models can still give fairly good correlations with the measurements. The notable advantages of these two models are their simple forms and easiness in calibration of model parameters. The RK and NNL models are good at describing complex strain-hardening behaviors with both temperature and strain rate effects. Concerning the studied WBC that shows little temperature effect on work hardening, however, these two models are not the best choice due to their complex formulation and material constants. The PB, ZA, and VA models have similar definitions of the stress components. They all consider work hardening only in the athermal part of flow stress, and define a constant work-hardening behavior. In the description of the studied WBC, the ZA model is more applicable than the others due to its less model parameters and better performance.

6. Conclusions

Thermomechanical behavior and constitutive modeling of a 93 W–4.9Ni–2.1Fe WBC is studied systematically over strain rates ranging from 0.001 to 3000 s^{-1} , and temperatures ranging from 173 to 873 K. In the examination of the plastic behavior, emphases are placed on the work-hardening effect and the micromechanism of deformation. The results show that the flow stress depends non-linearly on strain rate and temperature. Work hardening is comparatively insensitive to temperature, but dependence on strain rate can be observed to some extent. Through metallographic study, different mechanisms are found in the deformation of the material between quasi-static and dynamic loading conditions, resulting in distinct tendencies of work-hardening rate with an increasing plastic strain. It indicates that the deformation of the particles is sensitive not only to strain rates, but also to plastic strain levels. Moreover, cracked tungsten grains are observed in the tested material for both quasi-static and dynamic cases; the interaction between the grains is found to be the determining factor that cracks the grains, regardless of strain rates of the deformation process.

Based on the experimental data, two purely phenomenological constitutive models (JC and KHL) and five physically based ones (PB, NNL, ZA, VA, and RK) are established to simulate the plastic flow stress of the WBC. In the description of the fundamental data, the ZA model shows the lowest description errors for both quasi-static and dynamic experimental data. The average errors of the studied models are all below 10%. By comparing the model predictions and the experimental results of the strain rate jump tests, predictive capabilities of the models are assessed. The ZA model also shows the best accuracy in prediction of average results. The capabilities of the models in characterization of work-hardening, temperature, and strain rate effects of the material are also examined separately, based on comparison of their formulation, and correlation with the experimental data.

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