
MATHEMATICAL RESILIENCE

Sue Johnston-Wilder and Clare Lee explore the notion of mathematical resilience.

As we read articles in *MT*, we see that there is a general agreement about what constitutes good practice in teaching mathematics. We also know that when teachers use that best practice, as described in articles in *MT*, the pupils get better at ‘something’, not just at passing examinations. We see that pupils gain a great deal from this good practice: they gain in confidence, become more interested in mathematics and are often more willing to carry on learning mathematics beyond the age of 16. This article is about giving that ‘something’ a name so that we can begin to talk about it, measure it, and have as an aspiration that it increases. Rather than perhaps measuring a lessening of mathematics anxiety, we would like to focus on increasing something that we want – and begin to define that ‘something’ so that it becomes measurable.

We use the term ‘*mathematical resilience*’ to describe a learner’s stance towards mathematics that enables pupils to continue learning despite finding setbacks and challenges in their mathematical learning journey. There are ways of working in mathematics that increase mathematical resilience and conversely there are ways of working that decrease pupils’ mathematical resilience. In this paper, we will first discuss what mathematical resilience is, why it is important for pupils to develop it, and then consider what ways of working increase mathematical resilience.

All learning requires a certain resilience but we contend that the resilience required for learning mathematics (mathematical resilience) is a particular construct due to the specific barriers that are presented when learning mathematics, at least in part because of the type of teaching that has often been used (e.g. tedious, isolationist, using rote learning, elitist and depersonalised, Nardi & Steward 2003, Ofsted 2008) and in part because of pervasive beliefs about the fixed nature of mathematics ability.

What is mathematical resilience?

Mathematically resilient pupils learn mathematics effectively. When mathematically resilient pupils are required to use mathematics in a new situation they will expect to find it hard at first but will have strategies or approaches to overcome the initial ‘can’t do it’ response. Hence, mathematical resilience is about understanding, but it is also about building confidence in that understanding and about being in a position to learn mathematics that is, as yet, unknown. This is true of any learning, however for reasons we will explain, it seems that it is more difficult for learners to build resilience when learning mathematics.

The simplest definition of resilience is the capacity to transcend adversity. Dictionaries tend to define the word ‘resilience’ in two ways. The first definition concerns psychological resilience, that is, the ability to recover readily from illness, depression, adversity, or the like. The second definition is more concerned with physical resilience, that is, the power or ability to return to the original form, position, etc., after being bent, compressed, or stretched. We hold that both these aspects of resilience have some contribution to make to our understanding of mathematical resilience.

Psychological resilience

Psychological resilience is defined as a dynamic process in which individuals show that they can adapt their behaviour to respond positively when they encounter significant adversity, abuse or trauma. Resilience is a two-dimensional construct concerning exposure to adversity and positive adjustment to that adversity. Someone who displays psychological resilience has a combination of personal characteristics and skills that allow them to function beyond what may be expected in the light of that person’s vulnerability or exposure to

adversity. Resilience, therefore, is a positive characteristic and implies an expectation that an individual is able to change to meet the challenges that they encounter.

What goes on in many mathematics classrooms – a focus on acquisition of skills, solution of routine exercises, preparation for tests and examinations and the need for speed (Ofsted 2008) – serve to increase anxiety and do not help to establish mathematical resilience. Such practice could be regarded as a form of cognitive abuse; abuse inflicts suffering, resulting in anxiety and avoidance patterns, diminishing the desired positive outcome which we have called mathematical resilience. Mathematics anxiety manifests itself as ‘feelings of tension and anxiety that interfere with the manipulation of numbers and the solving of mathematical problems in a wide variety of ordinary life and academic situations’ (Richardson & Suinn, 1972, p. 551). When we asked a large contingent of students training to be primary school teachers in 2008 if they recognised these feelings in themselves, about 50% of the students put up their hands. Several described breaking out into a severe panic reaction if asked to go to a mathematics class.

One particularly significant feature of mathematics anxiety is avoidance behaviour, ‘...affected individuals learn to avoid mathematics-related activity, thereby limiting their opportunities to increase their mathematical competence.’ (Ashcraft, 2002). Individuals experiencing mathematics anxiety can have severe difficulty in carrying out calculations (Hembree, 1990) and perform poorly in examinations. Those suffering in this way experience confusion and an inability to focus on the task at hand because they are thinking about how poor they are at mathematics (Ashcraft and Kirk, 2001). Mathematics anxiety seems to be an acquired response to school situations and, if it is treated, performance in exams improves (Newstead, 1998, Hembree, 1990).

In many school staffrooms and homes, there will be people who have experienced mathematics in a negative, adverse or abusive way, sometimes severe. For example, in one of our interviews, a very competent English teacher described the feelings of terror she experienced when she was asked as a school pupil to come to the board to solve a quadratic equation. This was such a negative experience that she believes it created a life-long aversion to mathematics. Some learners, however, have built ways to cope for themselves; they become mathematically resilient without formal ‘treatment’. We contend that these students have strengthened their mathematical resilience, and

that explicitly helping other students to do the same is the way to develop a numerate, empowered population.

Before we move on, we feel we should mention a couple of other negatives or difficulties that Ofsted(2008) did not spell out and that we commonly see in mathematics teaching, which add to pupils’ need to build psychological resilience:

- **Mathematics as a chameleon.** The mathematics that is learned in school disappears against the background of the real world. Pupils are told mathematics is important but they cannot see why. Where is the requirement to use quadratic equations located in the world as young people see it? Even some teachers of mathematics would not know why quadratics are in the syllabus and how they are important in modern life.
- **An over-regard for speed in calculation.** We would like all pupils to be able to calculate as fast as they are personally capable but not at the expense of undermining the mathematical thinking of pupils who respond more slowly, neurologically speaking. These are not necessarily weaker pupils. When pupils are asked to do something they believe they cannot do, a panic reaction develops, which is counterproductive to being able to think and reason.
- **Making mistakes** is often implicitly taken as a sign of carelessness or stupidity. Much of the ‘tedious work’ that pupils are required to do in mathematics classrooms is about learning one algorithm in order to achieve one right answer. Under such circumstances making a mistake is a failure to follow the ‘rules’ and therefore ‘silly’. However, if a pupil makes a mistake when reasoning mathematically, that mistake may demonstrate an opportunity for learning to take place and that the level of challenge is probably right. Much of the time, children learn by making mistakes and learning from them.
- **One right way.** Teaching mathematics as though there were one single ‘right’ way to solve a problem disables pupils from trying strategies with which they may be more comfortable. It may prevent some pupils from acquiring increased efficiency from solid understanding – it encourages an approach of helplessness rather than independence.
- **Dependence on short-term (limited) memory** – remember your tables, remember to line up your tens and units, remember two negatives make a positive. The experience that mathematics is much reliant on memory was almost

certainly the cause of the terror that the English teacher experienced in front of the board. She had to remember what to do, not think about what it might be sensible to do, not try something out and see if it works and certainly not have a conversation with her peers to develop a joint answer to the question ‘what do you do now?’

Physical resilience

Materials are resilient if they can be deformed, stretched or bent but still return to their original shape. For example, a billiard ball is ideally resilient as, although it deforms slightly when hit at speed into another, it does not permanently bend, break or crumble. Instead it is efficient in transferring energy. Is this a helpful simile for mathematical resilience? We think so. We think it would be useful for students to learn mathematics in such a way that they do not break or crumble when they come up against difficult areas where their knowledge has to be applied. We would like students to feel confident enough to transfer their mathematical knowledge and energy to other people or situations. We would like pupils to have physical resilience as well as psychological resilience when it comes to mathematics.

Resilience is used in other ways that may help us to understand what it means to be mathematically resilient. The UK government has set up a website devoted to resilience in order to reduce the risk from emergencies so that ‘people can go about their business freely and with confidence’¹. When mathematical thinking is required in any situation, we would like pupils to be resilient enough to think freely and with confidence. A resilient ecosystem can withstand shocks and surprises and, if damaged, it can rebuild itself. Resilience in an ecosystem implies the ability to build and increase the capacity for learning and adaptation. We see that, when pupils have mathematical resilience, they will be much better placed to learn and to adapt their thinking to any given situation; experience of shocks and surprises will add to their ability to learn from situations.

Mathematically resilient behaviour

Mathematically resilient pupils have strategies to use that will stand them in good stead when they encounter problems. They believe that, if they work ‘smart’, they can become better at meeting

challenges and solving problems. They will develop a ‘growth mindset’ (Dweck 2000). They will discuss and question their mathematical ideas and they will have sufficient mathematical vocabulary to enable them to work collaboratively. Therefore, they will feel part of an extended mathematical community.

Mathematically resilient pupils will know the value of asking questions and experimenting: for example, by ‘playing’ with ICT tools. They will seek help in appropriate ways, whether from their peers or from an adult, by reading or by exploring the internet, or, if all else fails, by seeking help from their teacher. They will see asking as ‘clever’ and see persistent asking as being ‘even more clever’. They will be convinced that they have to take responsibility for their own understanding and therefore actively seek understanding. They will both know how to get help and know that it is good to help others. The mathematically resilient pupil will have a realistic understanding of their own strengths and limitations and know how to improve any weaknesses.

In short, mathematically resilient pupils will assert, in their practice, their right not to be mathematically isolated or feel mathematically stupid; they will resist any expectation that they should passively accept mathematical ideas but they will demand to be allowed to work at understanding them for themselves. They will reclaim their right to progress their own mathematical thinking, using existing knowledge, skills, understanding and strategies and be confident about their ability to learn new mathematics.

Developing mathematical resilience

The Ofsted document cited above describes ‘good teaching’ as including opportunities for pupils to solve problems, engage in discussion and practical work, gain experience of more complex situations and explore meaningfulness, all of which we suggest are experiences which contribute to building mathematical resilience. Mathematical resilience is fostered, and negative adaptations become unlikely, when mathematics is seen as a social construct, one which requires logical thinking and the application of learned skills in a connected and reasoned way, often in an authentic context, and one which is active, social and reflective.

The support available for each pupil will make a difference. In a school that promotes mathematical resilience, support will include the whole commu-

nity of peers and adults with whom the pupils interact. In a school in North Warwickshire, we are exploring the role of non-mathematical staff in helping to build such a mathematically supportive community. In that school, there are mathematics coaches with no strong mathematical knowledge of their own, but with willingness to sit alongside learners and face their mathematical demons together. The coaches bring their own adult resilience, suggesting strategies and showing a readiness to listen as the pupil begins to articulate what they have tried and what they do not understand.

Mathematical resilience is fostered when pupils face, and succeed in meeting, mathematical challenge. Some learners do not develop that resilience because they never meet mathematical challenge or adversity as they seem to fly through school mathematics. At university, they meet some hard mathematics, possibly for the first time and have no knowledge of how to struggle with, and overcome, their difficulties.

Working in ways that develop mathematical resilience

The construct of mathematical resilience has its roots in the work of colleagues such as John Mason, who has written about ‘being stuck’ being the time when learning starts. His book *Thinking Mathematically* explicitly teaches strategies for managing ‘being stuck’, as does Polya’s work on problem solving. Mathematical resilience is also rooted in the work of Carol Dweck (2000), who writes about ‘fixed’ and ‘growth’ mindsets. When learners demonstrate a fixed mindset, they act as though they have the ability to learn just so much and no more, worried that they will meet or have met a fixed ceiling beyond which they cannot go. When they exhibit a growth mindset, they consider that the more effort they put in, the more their learning will grow and therefore there is no limit to what they can learn (Lee 2009). Resilience is also about recognising and overcoming learned helplessness so that learners know how to take control over, and responsibility for, their learning.

In order to develop mathematical resilience, school pupils must learn sufficient mathematical vocabulary and ways of expression to engage in mathematical conversations, to question concepts, to work collaboratively, to think mathematically and to build understanding. This means that pupils themselves have to use mathematical words and ways of expression for themselves, and not only listen to their teachers using the mathematical

language. If pupils are required to enter into mathematical conversations and question ideas or concepts, then they will think mathematically and build understanding.

Mathematically resilient pupils believe that, if they persevere and make mistakes and take wrong turns, then ultimately they will be far more likely to succeed. There will always be a place for rehearsing to acquire fluency in using mathematical skills. However, when developing mathematical resilience, explicit rehearsal of skills will not be the only or even the most frequent way that pupils will work. During mathematical learning, pupils will be asked to marshal their mathematical knowledge and experiences and to seek solutions to significant problems. Working on such problems will require pupils to try things out, to make and recognise mistakes for themselves and to work for an extended time with other people to produce a well reasoned solution. Thus, pupils will extend their ability to experiment and try out ideas in a mathematical environment and, in our experience, they will enjoy it. As pupils become more mathematically resilient, they will build an understanding that mathematics is a whole subject, where ideas are interrelated and knowledge in one area is useful in another.

The development of mathematical resilience requires pupils to acquire a reflective and thoughtful stance towards mathematics. They will know that, if they think hard, talk to others, read about mathematical ideas and reflect on the information gained, they will be able to make headway with seemingly difficult ideas and problems.

The construct we have called ‘mathematical resilience’ is important if we are to educate pupils to *use mathematics* as opposed to simply pass examinations. However, we do not think this is an either/or situation; mathematically resilient pupils will have the skills they need to decide what an examination question is asking of them but they will also have the skills needed to function mathematically in the world beyond school, and the willingness to continue their mathematical development as and when needed, and that is important. If it is important, why are we not measuring it?

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1 www.cabinetoffice.gov.uk/ukresilience.aspx

Further reading

- Daniel, B. and Wassell, S. 2002. *Assessing and Promoting Resilience in Vulnerable Children*. London: Jessica Kingsley
- Dweck, C. 2000. *Self-theories: Their role in motivation, personality, and development*. Philadelphia, PA: Psychology Press
- Lee, C. 2009. Fixed or Growth – does it matter? *MT212*
- Mason, J. Burton, L. and Stacey, K. 1985. *Thinking Mathematically*, Harlow, Essex, Addison Wesley
- Nardi, E. and Steward, S. 2003 Is Mathematics T.I.R.E.D? A Profile of Quiet Disaffection in the Secondary Mathematics Classroom, *British Education Research Journal* (29)3 pps 345-367
- Ofsted 2008. *Understanding the Score*. London, Ofsted.
- Polya, G. 1990. *How to Solve it: A New Aspect of Mathematical Method*, London, Penguin
- The Developing Thinking books offer many strategies that we suggest contribute to mathematical resilience these are:
- Graham A. 2005. *Developing Thinking in Statistics*. London: Sage Publications
- Johnston-Wilder, S. and Mason, J. (Eds.) 2005. *Developing Thinking in Geometry*. London: Sage Publications
- Mason, J., Graham, A., and Johnston-Wilder, S. 2005. *Developing Thinking in Algebra*. London: Sage Publications

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