

1) Convert point $P(1, 3, 5)$ from Cartesian to cylindrical and Spherical Coordinates.

Soln: Given, $P(1, 3, 5)$ $P(x, y, z)$

$$x = 1, y = 3, z = 5$$

To cylindrical coordinates (r, ϕ, z)

$$r = \sqrt{x^2 + y^2} = \sqrt{1^2 + 3^2} = \sqrt{10} = 3.1623$$

$$\phi = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left(\frac{3}{1}\right) = 71.56$$

$$z = z = 5$$

cylindrical coordinates = $(3.16, 71.56, 5)$

To find Spherical coordinate
 (r, θ, ϕ)

$$r = \sqrt{x^2 + y^2 + z^2} \\ = \sqrt{1^2 + 3^2 + 5^2}$$

$$r = 5.92$$

$$\theta = \cos^{-1}\left(\frac{z}{r}\right) = \cos^{-1}\left(\frac{5}{5.91}\right)$$

$$= 32.31$$

$$\phi = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left(\frac{3}{1}\right)$$

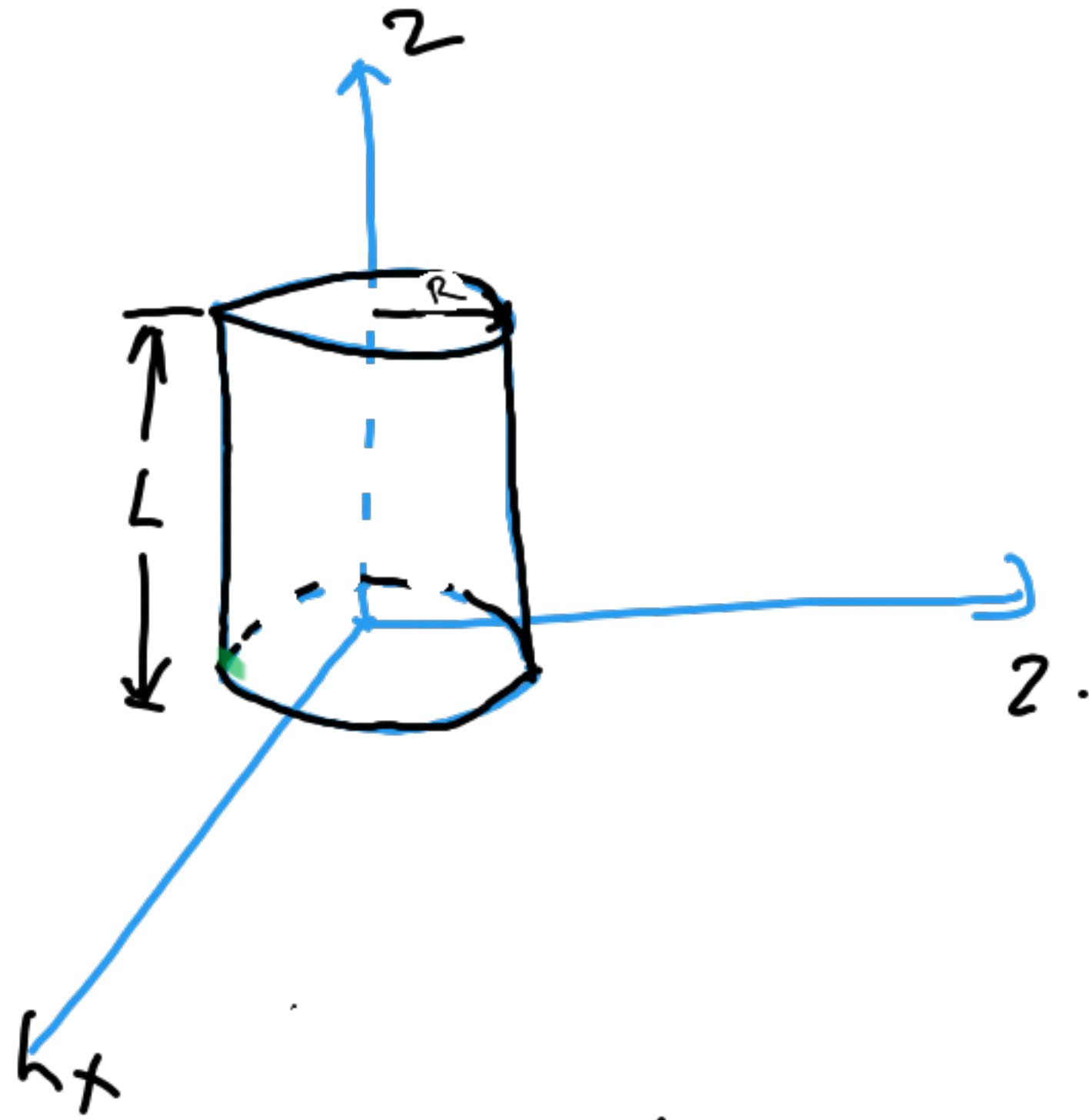
$$\phi = 71.57$$

Spherical coordinates = $(5.92, 32.31, 71.57)$

② Consider a cylinder of length L and

Radius R . Obtain its Volume by integration.

Soln



$$dv = r dr d\phi dz$$

$$r \rightarrow 0 \text{ to } R.$$

$$z \rightarrow 0 \text{ to } L$$

$$\phi \rightarrow 0 \text{ to } 2\pi$$

$$V = \int_0^R \int_0^L \int_0^{2\pi} r dr d\phi dz$$

$$= \int_0^R \int_0^L r [\phi]_0^{2\pi} dr dz$$

$$= \int_0^R \int_0^L r [2\pi - 0] dr dz$$

$$= \int_0^R \int_0^L 2\pi r dr dz$$

$$= 2\pi \int_0^L \left[\frac{r^2}{2} \right]_0^R dz$$

$$= 2\pi \int_0^L \left[\frac{R^2}{2} - 0 \right] dz$$

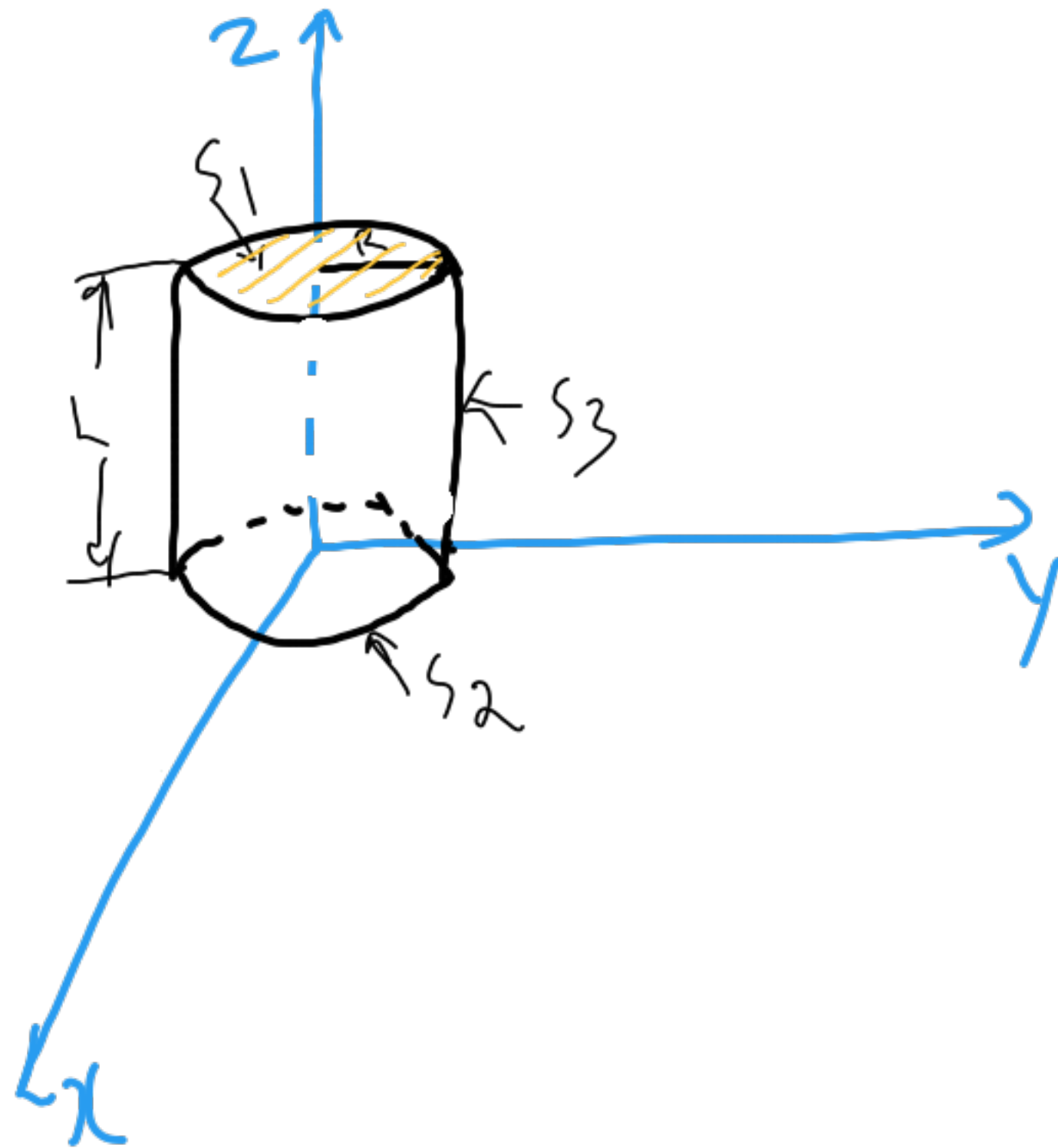
$$= 2\pi \int_0^L \frac{R^2}{2} dz$$

$$= \pi R^2 \int_0^L dz$$

$$= \pi R^2 \left[z \right]_0^L$$

$$V = \pi R^2 L$$

③ Calculate the total Surface area of the cylinder having length L and radius R by the method of integration.



Soln

Consider the upper Surface area normal (per) to z axis.

$$d\vec{S}_2 = r dr d\phi \vec{a}_2$$

$$r \rightarrow 0 \text{ to } R$$

$$\phi \rightarrow 0 \text{ to } 2\pi$$

$$S_1 = \int_0^{2\pi} \int_0^R r dr d\phi$$

$$S_1 = \int_0^{2\pi} \left[\frac{r^2}{2} \right]_0^R d\phi$$

$$S_1 = \int_0^{2\pi} \left[\frac{R^2}{2} - 0 \right] d\phi$$

$$= \int_0^{2\pi} \frac{R^2}{2} d\phi$$

$$= \frac{R^2}{2} \int_0^{2\pi} d\phi$$

$$= \frac{R^2}{2} [\phi]_0^{2\pi}$$

$$= \frac{R^2}{2} \cancel{2\pi}$$

$$S_1 = \pi R^2$$

S_2 is same as S_1 , as it is
per to z .

$$S_2 = \pi R^2$$

S_3 is per to r .

$$d\vec{s}_r = r d\phi dz \vec{a}_r$$

$\phi \rightarrow 0$ to 2π

$z \rightarrow 0$ to L

$$r = R$$

$$S_3 = \int_0^L \int_0^{2\pi} R d\phi dz$$

$$S_3 = R \int_0^L \int_0^{2\pi} d\phi dz$$

$$S_3 = R \int_0^L [\phi]_0^{2\pi} dz$$

$$= R \int_0^L [2\pi] dz$$

$$= 2\pi R \int_0^L dz$$

$$= 2\pi R [z]_0^L$$

$$S_3 = 2\pi RL$$

Total Surface area.

$$= S_1 + S_2 + S_3$$

$$= \pi R^2 + \pi R^2 + 2\pi RL$$

$$= 2\pi R^2 + 2\pi RL$$

$$= 2\pi R[R + L]$$

4) Calculate the Volume of the Sphere of radius R by integration.

Soln.

The differential Volume of the

$$\text{Sphere} \Rightarrow dv = r^2 \sin \theta dr d\theta d\phi$$

$$r \Rightarrow 0 \rightarrow R$$

$$\theta = 0 \rightarrow \pi$$

$$\phi = 0 \rightarrow 2\pi$$

$$V = \int_0^{2\pi} \int_0^{\pi} \int_0^R r^2 \sin \theta dr d\theta d\phi$$

$$V = \int_0^{2\pi} \int_0^{\pi} \left[\frac{r^3}{3} \right]_0^R \sin \theta d\theta d\phi$$

$$= \int_0^{2\pi} \int_0^{\pi} \frac{R^3}{3} \sin \theta d\theta d\phi$$

$$= \frac{R^3}{3} \int_0^{2\pi} \int_0^{\pi} \sin \theta d\theta d\phi$$

$$= \frac{R^3}{3} \int_0^{2\pi} \left[-\cos \theta \right]_0^{\pi} d\phi$$

$$= \frac{R^3}{3} \int_0^{2\pi} \left[-\cos(\pi) - (-\cos 0) \right] d\phi$$

$$= \frac{R^3}{3} \int_0^{2\pi} \left[-(-1) - (-1) \right] d\phi$$

$$\left[\int r^n dr = \left[\frac{r^{n+1}}{n+1} \right] \right]$$

$$= \frac{R^3}{3} \int_0^{2\pi} 2 d\phi$$

$$= \frac{2R^3}{3} \left[\phi \right]_0^{2\pi}$$

$$= \frac{2R^3}{3} [2\pi]$$

$$V = \frac{4\pi R^3}{3}$$

5) Given the Vectors $\vec{A} = 2\vec{a}_x - 5\vec{a}_y - 4\vec{a}_z$ and $\vec{B} = 3\vec{a}_x + 5\vec{a}_y + 2\vec{a}_z$. Find the dot product and angle between the two vectors.

Soln

The dot product

$$\begin{aligned}\vec{A} \cdot \vec{B} &= A_x B_x + A_y B_y + A_z B_z \\ &= 2 \times 3 + (-5) \times 5 + (-4) \times 2\end{aligned}$$

$$\vec{A} \cdot \vec{B} = -27$$

$$|\vec{A}| = \sqrt{2^2 + (-5)^2 + (-4)^2} = 6.71$$

$$|\vec{B}| = \sqrt{3^2 + 5^2 + 2^2} = 6.16$$

$$\begin{aligned}\theta &= \cos^{-1} \left(\frac{\vec{A} \cdot \vec{B}}{|\vec{A}| |\vec{B}|} \right) \\ &= \cos^{-1} \left(\frac{-27}{6.708 \times 6.164} \right)\end{aligned}$$

$$\theta = 130.1^\circ$$

6) Show that $\bar{A} = 4a_x - 2a_y - a_z$ and $\bar{B} = a_x + 4a_y - 4a_z$ are mutually perpendicular.

Soln

condition is $\bar{A} \cdot \bar{B} = 0$

$$\begin{aligned}\bar{A} \cdot \bar{B} &= 4 \times 1 + (-2) \times 4 + (-1) \times -4 \\ &= 4 - 8 - 4\end{aligned}$$

$$\bar{A} \cdot \bar{B} = 0$$

Hence proved.

⑦ Given $A = 2a_x + 4a_y$ and $B = 6a_y - 4a_z$.

Find the Smaller angle between them

Using (a) Cross product (b) dot product

Soln ∴ Cross product

$$\bar{A} \times \bar{B} = \begin{vmatrix} a_x & a_y & a_z \\ 2 & 4 & 0 \\ 0 & 6 & -4 \end{vmatrix}$$

$$= a_x \begin{vmatrix} 4 & 0 \\ 6 & -4 \end{vmatrix} - a_y \begin{vmatrix} 2 & 0 \\ 0 & -4 \end{vmatrix} + a_z \begin{vmatrix} 2 & 4 \\ 0 & 6 \end{vmatrix}$$

$$\bar{A} \times \bar{B} = -16a_x + 8a_y + 12a_z$$

$$|\vec{A}| = \sqrt{2^2 + 4^2 + 0^2} = 4.47 \quad -$$

$$|\vec{B}| = \sqrt{6^2 + (-4)^2} = 7.21 \quad -$$

$$|\vec{A} \times \vec{B}| = \sqrt{(-16)^2 + 8^2 + 12^2} = 21.54 \quad -$$

we know that

$$|\vec{A} \times \vec{B}| = |\vec{A}| |\vec{B}| \sin \theta$$

$$\sin \theta = \frac{|\vec{A} \times \vec{B}|}{|\vec{A}| |\vec{B}|}$$

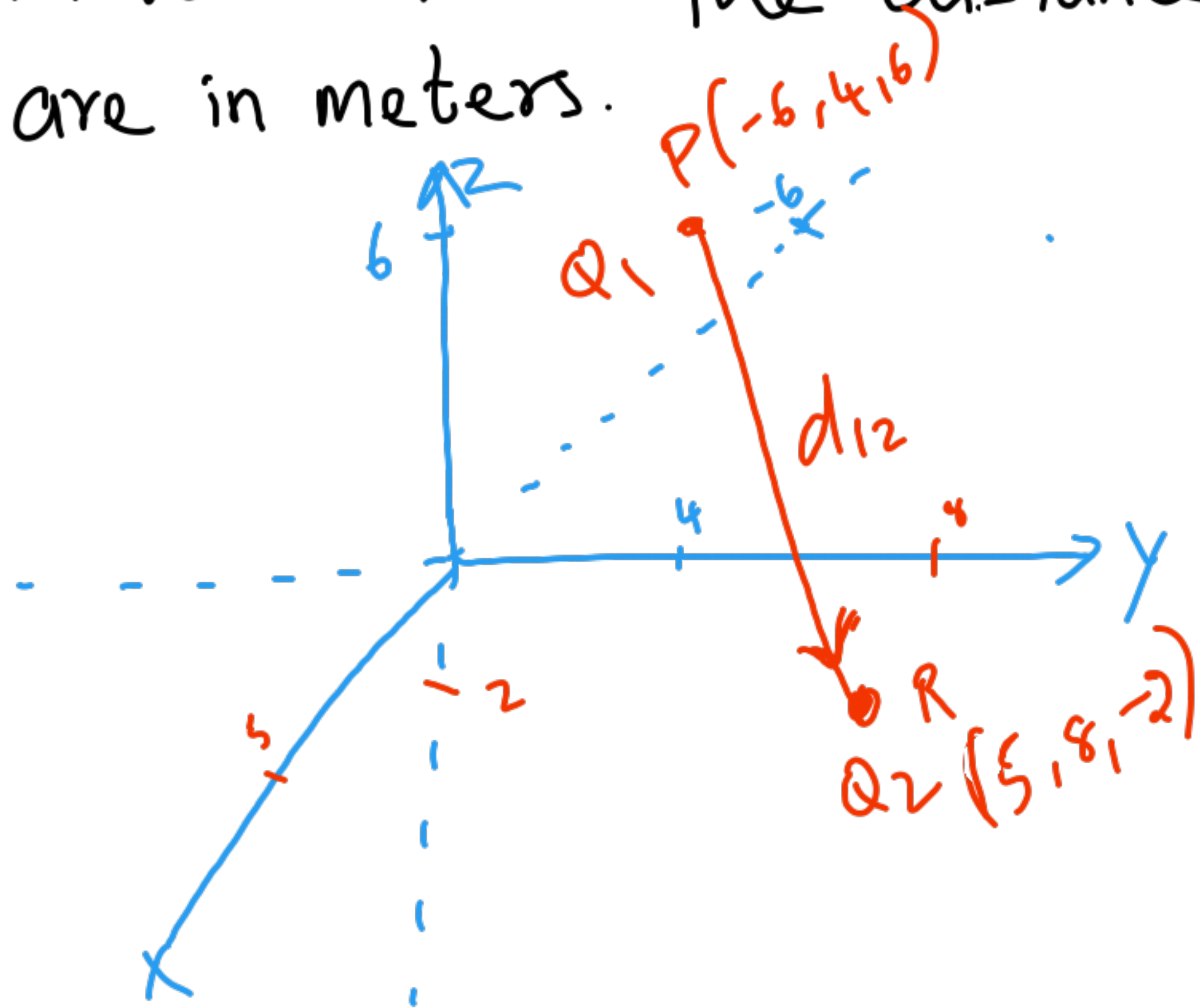
$$\theta = \sin^{-1} \left[\frac{|\vec{A} \times \vec{B}|}{|\vec{A}| |\vec{B}|} \right]$$

$$\theta = \sin^{-1} \left[\frac{21.54}{4.47 \times 7.21} \right]$$

$$= \sin^{-1}(0.668)$$

$$\theta = \underline{42^\circ}$$

8) A charge $Q_1 = -20 \mu\text{C}$ is located at $P(-6, 4, 6)$ and a charge $Q_2 = 50 \mu\text{C}$ is located at $R(5, 8, -2)$ in a free space. Find the force exerted on Q_2 by Q_1 in vector form. The distance given are in meters.



$$\vec{F} = \frac{Q_1 Q_2}{4\pi\epsilon_0 d_{12}^2} \cdot \vec{a}_{12}$$

$$\vec{P} = -6\vec{a}_x + 4\vec{a}_y + 6\vec{a}_z$$

$$\vec{R} = 5\vec{a}_x + 8\vec{a}_y - 2\vec{a}_z$$

$$\vec{d}_{PR} = \vec{R} - \vec{P} = (5 - (-6))\vec{a}_x + (8 - 4)\vec{a}_y + (-2 - 6)\vec{a}_z$$

$$\vec{d}_{PR} = 11\vec{a}_x + 4\vec{a}_y - 8\vec{a}_z$$

$$|\vec{d}_{PR}| = \sqrt{(11)^2 + (4)^2 + (-8)^2}$$

$$|\vec{d}_{PR}| = 14.17$$

$$\bar{a}_{12} = \frac{\bar{d}_{PR}}{|\bar{d}_{PR}|} = \frac{11\bar{a}_x + 4\bar{a}_y - 8\bar{a}_z}{14.177}$$

$$\bar{a}_{12} = 0.776\bar{a}_x + 0.28\bar{a}_y - 0.564\bar{a}_z$$

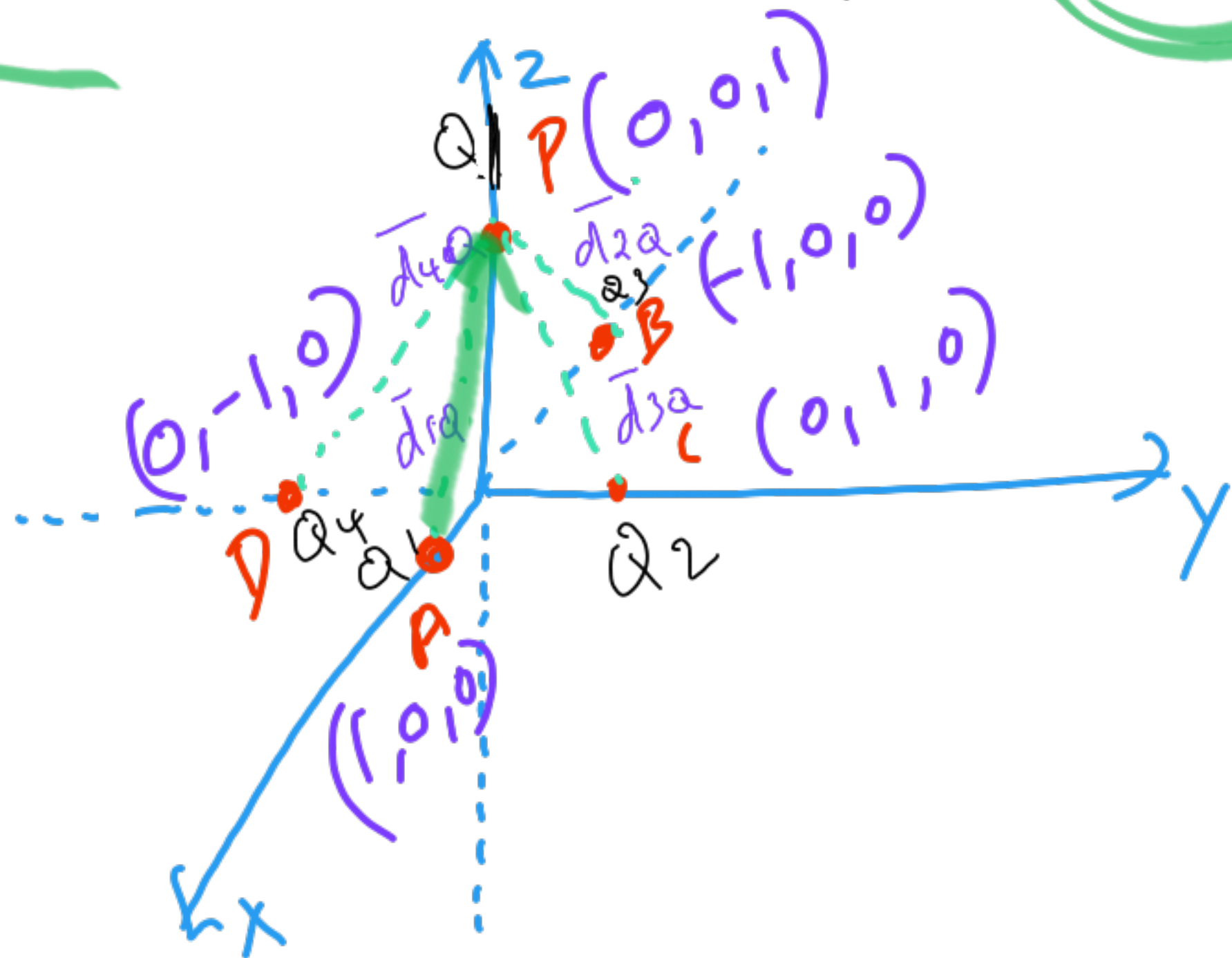
$$\bar{F} = \frac{-20 \times 10^{-6} \times 50 \times 10^{-6}}{4\pi \times 8.85 \times 10^{-12} \times (14.177)^2} \begin{bmatrix} 0.776\bar{a}_x + 0.28\bar{a}_y \\ -0.564\bar{a}_z \end{bmatrix}$$

$$\bar{F} = -0.0447 \begin{bmatrix} 0.776\bar{a}_x + 0.28\bar{a}_y - 0.564\bar{a}_z \end{bmatrix}$$

$$\bar{F} = \begin{bmatrix} -0.0346\bar{a}_x - 0.0126\bar{a}_y + 0.02522\bar{a}_z \end{bmatrix} \text{ N}$$

$$|\bar{F}| = \sqrt{(-0.0346)^2 + (-0.0126)^2 + (0.02522)^2} = 44.634 \text{ mN}$$

a) Four point charges each of $10\mu\text{C}$ are placed in free space at the points $(1,0,0)$, $(-1,0,0)$, $(0,1,0)$ and $(0,-1,0)$ respectively. Calculate the force on a point charge of $30\mu\text{C}$ located at a point $(0,0,1)$.



$$\vec{A} = \vec{a}_x ; \quad \vec{B} = -\vec{a}_x \quad \vec{C} = \vec{a}_y$$

$$\vec{D} = -\vec{a}_y \quad \vec{P} = \vec{a}_z$$

Force acting on Q due to Q_1

$$\vec{F} = \frac{Q Q_1}{4\pi\epsilon_0 (d_{1Q})^2} \vec{a}_{1Q}$$

$$\vec{a}_{1Q} = \frac{\vec{d}_{1Q}}{|\vec{d}_{1Q}|} = \frac{-\vec{a}_x + \vec{a}_z}{1.41}$$

$$\begin{aligned} \vec{d}_{1Q} &= \vec{P} - \vec{A} = (0-1)\vec{a}_x + (1-0)\vec{a}_z \\ &= -1\vec{a}_x + \vec{a}_z \\ &= -\vec{a}_x + \vec{a}_z \end{aligned}$$

$$|\vec{d}_{1Q}| = \sqrt{(-1)^2 + 1^2} = \sqrt{1+1} = \sqrt{2} = 1.41$$

$$\bar{F} = \frac{30 \times 10^{-6} \times 10 \times 10^{-6}}{4\pi \times 8.85 \times 10^{-12} (1.41)^2} \left(\frac{-\bar{a}_x + \bar{a}_z}{1.41} \right)$$

$$\bar{F} = \underline{\underline{1.349}} \left[\frac{-\bar{a}_x + \bar{a}_z}{1.41} \right]$$

$$\bar{F} = 0.955 [-\bar{a}_x + \bar{a}_z]$$

$$\frac{1.349}{1.41}$$

$$\bar{F} = -0.955\bar{a}_x + 0.955\bar{a}_z$$

$$|\bar{F}| = \sqrt{(0.955)^2 + (0.955)^2} \quad \bar{F} \approx 3 \dots$$

$$\bar{F} = 1.349 \quad N_{\bar{F}} = 1.349 \times 4$$

Total