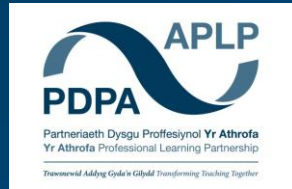




Prifysgol Cymru
Y Drindod Dewi Sant
University of Wales
Trinity Saint David



ECAD5012 – Session 1

Mathematics and Numeracy

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Aims



- ❖ Deall sut mae dealltwriaeth o rif yn datblygu yng Nghyfnod Allweddol 2.
- ❖ Datblygu dealltwriaeth o'r Cwricwlwm a deall y disgwyliadau
- ❖ Egwyddorion Asesu ar gyfer Dysgu
- ❖ Cael syniadau am sut y gallwch ddatblygu sgiliau rhif yn yr ystafell ddosbarth.

- *Understand number development in Key Stage 2
- *Develop an understanding of the Curriculum and understand the expectations
- *Principles of Assessment for Learning
- *Have some ideas on how you can develop number skills in the classroom

O ddysgu Mathemateg yn y Dysgu Sylfaen i ddysgu
Mathemateg yng Nghyfnod Allweddol 2.

From teaching Mathematics in the Foundation Phase to
teaching Mathematics in Key Stage 2.

Trafodwch:

Beth yw'r gwahaniaethau rhwng dysgu mathemateg yn y ddau
gyfnod?

Discuss:

What are the differences between teaching in both key stages?



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200
1822 - 2022

Trawsnewid Addysg, Trawsnewid Bywydau
Transforming Education, Transforming Lives

AREA OF LEARNING AND EXPERIENCE

Mathematics and Numeracy

Guidance to help schools and settings develop their own curriculum, enabling learners to develop towards the four purposes.



Contents

1. Introduction
2. **Statements of what matters**
3. **Principles of progression**
4. Descriptions of learning
5. Designing your curriculum



MAES DYSGU A PHROFIAD

Mathemateg a Rhifedd

Canllawiau i helpu ysgolion a lleoliadau i ddatblygu eu cwricwlwm eu hunain, gan alluogi dysgwyr i ddatblygu o ran y pedwar diben.



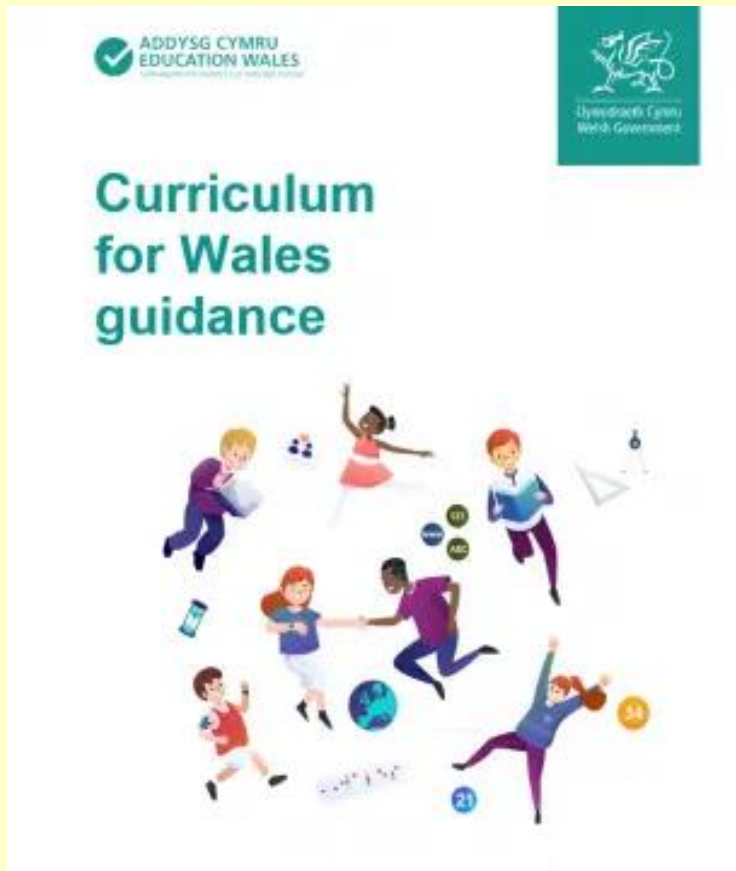
Cynnwys

1. Cyflwyniad
2. **Datganiadau o'r hyn sy'n bwysig**
3. **Egwyddorion cynnydd**
4. Disgrifiadau dysgu
5. Cynllunio eich cwricwlwm



Statements of What Matters

- The number system is used to represent and compare relationships between numbers and quantities
- Algebra uses symbol systems to express the structure of mathematical relationships
- Geometry focuses on relationships involving shape, space and position, and measurement focuses on quantifying phenomena in the physical world
- Statistics represent data, probability models chance, and both support informed inferences and decisions.



Progression in the Mathematics and Numeracy Area of Learning and Experience (Area) involves the development of **five connected and interdependent proficiencies which have no hierarchy**. These are crucial considerations for schools when designing their curriculum to ensure the progression of learners.

- **Conceptual understanding**
- **Communication using symbols**
- **Fluency**
- **Logical reasoning**
- **Strategic competence**

Conceptual understanding

Mathematical concepts and ideas should be built on, deepened and connected as learners experience increasingly complex mathematical ideas. Learners demonstrate conceptual understanding through being able to explain and express concepts, find examples (or *non-examples*) and by being able to represent a concept in different ways, flowing between different representations including verbal, *concrete*, visual, digital and *abstract*.

Communication with symbols

Learners should understand that the symbols they are using are abstract representations and should develop greater flexibility with the application and manipulation of an increasing range of symbols, understanding the conventions of the symbols they are using.

Strategic competence

Learners should become increasingly independent in recognising and applying the underlying mathematical structures and ideas within a problem, in order to develop strategies to be able to solve them.

Logical reasoning

As learners experience increasingly complex concepts, they should also develop an understanding of the relationships between and within these concepts. They should apply logical reasoning about these relationships and be able to justify and prove them. Justifications and *proof* should become increasingly abstract, moving from verbal explanations, visual or concrete representations to abstract representations involving symbols and conventions.

Fluency

As learners experience, understand and effectively apply increasingly complex concepts and relationships, fluency in remembering facts, relationships and techniques should grow, meaning that facts, relationships and techniques learned previously should become firmly established, memorable and usable.

Camau Cynnydd Progression Steps



Progression step 2

I can read, write and interpret larger numbers, up to at least 1000, using digits and words.

I can understand that the value of a number can be determined by the position of the digits.

I have engaged in practical tasks to estimate and round numbers to the nearest 10 and 100.

I am beginning to estimate and check the accuracy of my answers, using [inverse](#) operations when appropriate.

I can order and sequence numbers, including odd and even numbers, and I can count on and back in step sizes of any [whole number](#) and simple [unit fractions](#).

Progression step 3

I can use a range of representations to develop and secure my understanding that the value of a digit is related to its position. I can read, record and interpret numbers, using figures and words up to at least one million.

I can use a range of representations to extend my understanding of the number system to include negative values, decimals and fractions. I can accurately place [integers](#), decimals and fractional quantities on a number line. I can apply my understanding of number value to round and approximate appropriately.

Progression step 2

I have explored [additive relationships](#), using a range of representations. I can add and subtract [whole numbers](#), using a variety of written and mental methods.

I can use my understanding of multiplication to recall some multiplication facts and tables starting with tables 2, 3, 4, 5 and 10 and I can use the term 'multiples'.

I have explored and can use my understanding of [multiplicative relationships](#) to multiply and divide [whole numbers](#), using a range of representations, including sharing, grouping and [arrays](#).

I can understand the equivalence and value of coins and notes to make appropriate transactions in role play.

Progression step 3

I can verify calculations and statements about number by [inverse](#) reasoning and approximation methods.

I can use the four arithmetic operations confidently, efficiently and accurately with [integers](#) and decimals, and I can combine these using [distributive](#), [associative](#) and [commutative](#) laws where appropriate.

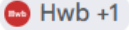
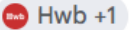
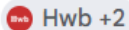
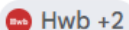
I have extended my understanding of [multiplicative](#) reasoning to include the concept and application of ratio, proportion and scale.

I can fluently recall multiplication facts up to at least 10 x 10 and use these to derive related facts.

I have experienced and explored simple multiplicative relationships that allow me to discuss the properties of number, including factors, multiples, prime and square numbers.

I can demonstrate an understanding of income and expenditure, and I can apply calculations to explore profit and loss.

Assessment for Learning

- **Learner-Centered Progression:** Assessment is designed to support the 3-16 continuum of learning, ensuring learners are consistently supported and challenged at their own pace. 
- **Active Participation:** Learners are encouraged to be partners in the learning process by engaging in self-assessment, peer-assessment, and taking responsibility for their next steps. 
- **Holistic Development:** AfL goes beyond testing. It builds a comprehensive picture of a learner's strengths, attitudes, and development across the breadth of the curriculum. 
- **Integrated Process:** Assessment is indistinguishable from day-to-day teaching and learning. It is embedded naturally through observation, discussion, and authentic contexts. 
- **Shared Understanding:** Practitioners use professional dialogue to understand progression, ensuring that both parents and learners are engaged in the process. 

[assessment for learning principles hwb - Google Search](#)

The 4 Elements of AfL

Sharing criteria with learners	Quality of questions/ quality of talk	Use and quality of feedback	Peer and self assessment
<i>Learners know what will make their work good.</i>	<i>All learners actively think up answers to good, open questions.</i>	<i>Ongoing feedback keeps learning on track.</i>	<i>Learners benefit from evaluating each other's work.</i>
<i>Learners know where they are, and the next step, according to the criteria.</i>	<i>Learners explain ideas to each other, reinforcing their learning and understanding.</i>	<i>Learners act on the feedback there and then. Learners take the next step.</i>	<i>Learners gradually become more independent by learning how to self-assess.</i>



AfL

- Success Criteria
- Questioning
- Peer Assessment



- Teacher assesses where learners are and adjusts teaching accordingly
- Share Success Criteria
- Improve quality of questions - How many ways can you think of to make 100? Give them a choice of answers. The answer is 40. What are the questions
- Explore wrong ideas and answers- this is so important
- Peer and self-assessment

Revision of the CPA approach- Jerome Bruner

The abstract nature of maths can be confusing for children, but by using **concrete materials** they are **able to 'see' and make sense of what is really going on.**

Previously, there was the misconception that concrete resources are only for learners who are struggling with maths. In fact, concrete resources can be used in a wide variety of ways at all levels. All children, regardless of ability, benefit from the use of practical resources to ensure that understanding goes beyond procedural learning.

Practical resources promote reasoning and discussion, enabling children to express and explain a concept. Teachers are also able to observe the children to gain a better understanding of where misconceptions lie and to establish the depth of their understanding.

The CPA Approach

Concrete

Using physical objects to explore mathematical problems



Pictorial

Using visual models to explore mathematical problems



Abstract

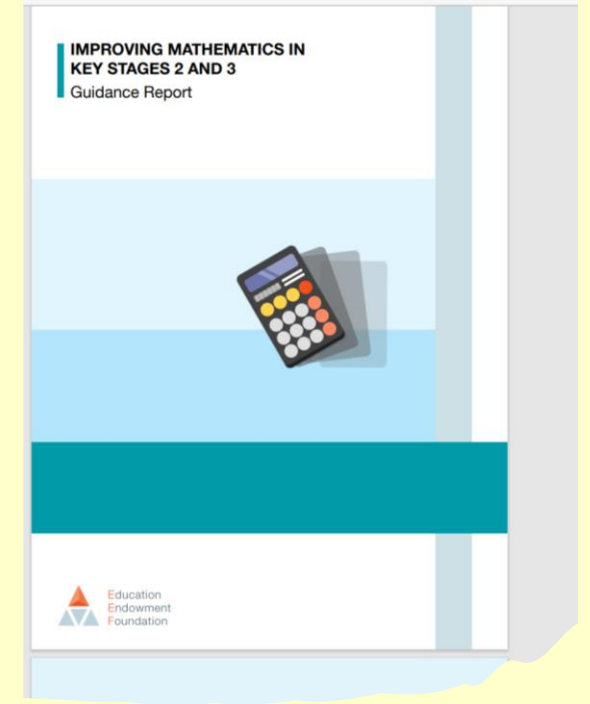
Using only mathematical symbols and numbers to explore mathematical problems

$$2 + 3 = 5$$



Tudalen 14-15

Pages 14-15



Research:

Education Endowment Foundation (EEF) (2022)
'Improving Mathematics in Key Stages 2 and 3'
Guidance Report suggests some key actions that can help pupils to improve their skills in problem-solving.

[EEF-Improving-Mathematics-in-Key-Stages-2-and-3-2022-Update.pdf](#)

Recommendation 1	Use assessment to build upon pupils' existing knowledge and understanding	8
Recommendation 2	Use manipulatives and representations	10
Recommendation 3	Teach strategies for solving problems	14
Recommendation 4	Enable pupils to develop a rich network of mathematical knowledge	16
Recommendation 5	Develop pupils' independence and motivation	18
Recommendation 6	Use tasks and resources to challenge and support pupils' mathematics	20
Recommendation 7	Use structured interventions to provide additional support	26
Recommendation 8	Support pupils to make a successful transition between primary and secondary school	28

Box D: What is a problem-solving strategy?

A problem-solving strategy is a general approach to solving a problem. The same general strategy can be applied to solving a variety of different problems. For example, a useful problem-solving strategy is to identify a simpler but related problem. Discussing the solution to the simpler problem can give insight into how the original, harder problem may be tackled and the underlying mathematical structure.

A strategy is different from an algorithm, which is a well-established sequence of predetermined steps that are executed in a particular order to carry out a commonly-required procedure.

Evidence summary

- The review identified three relevant meta-analyses concerned with the teaching of problem-solving. These provide some evidence for the use of problem-solving, although the effects were varied.
- A systematic review by the US What Works Clearinghouse provides evidence for specific teaching approaches. The evidence strength is judged to be strongest in support of the use of visual representations and worked examples, and encouraging pupils to monitor and reflect on the problem-solving process.

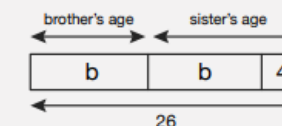
Box E: Using the bar model to compare strategies

A class was working on this problem:

*'A sister is 4 years older than her brother.
The total of their ages is 26.
How old are they?'*

One pupil used the bar model to solve the problem:

So the brother is 11 years old and the sister is 15 years old.



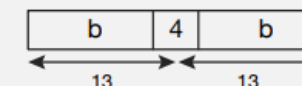
$$b + b + 4 = 26$$

$$\text{so } b + 4 = 22$$

Another pupil said, 'Couldn't we just halve 26 and then add and take away 4? So their ages are $13 + 4$ and $13 - 4$, which is 17 and 9.'

Another pupil said, 'No, you don't add and take away 4, you add and take away 2'. The teacher drew this revised bar model diagram:

'How does this help us to see whether it's 4 or 2 that you add and subtract?'



A pupil said, 'There are two lots of b in the 26, there is only one 4, so it was correct to add and subtract 2, not 4.'

Tasg

Darllenwch dudalennau 16-17.
Beth yw'r strategaethau
dysgu ar gyfer datrys
problemau?

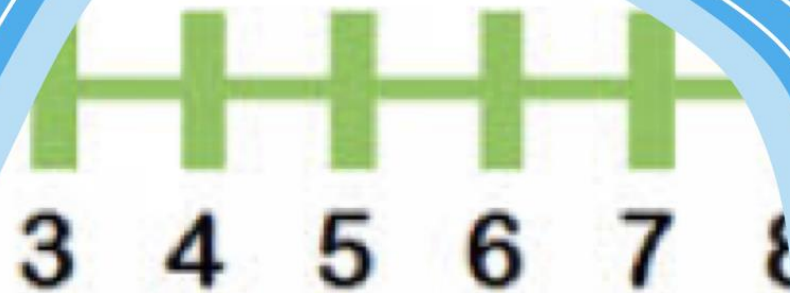
Task

Read pages 16-17.
What are the learning strategies
for problem solving?





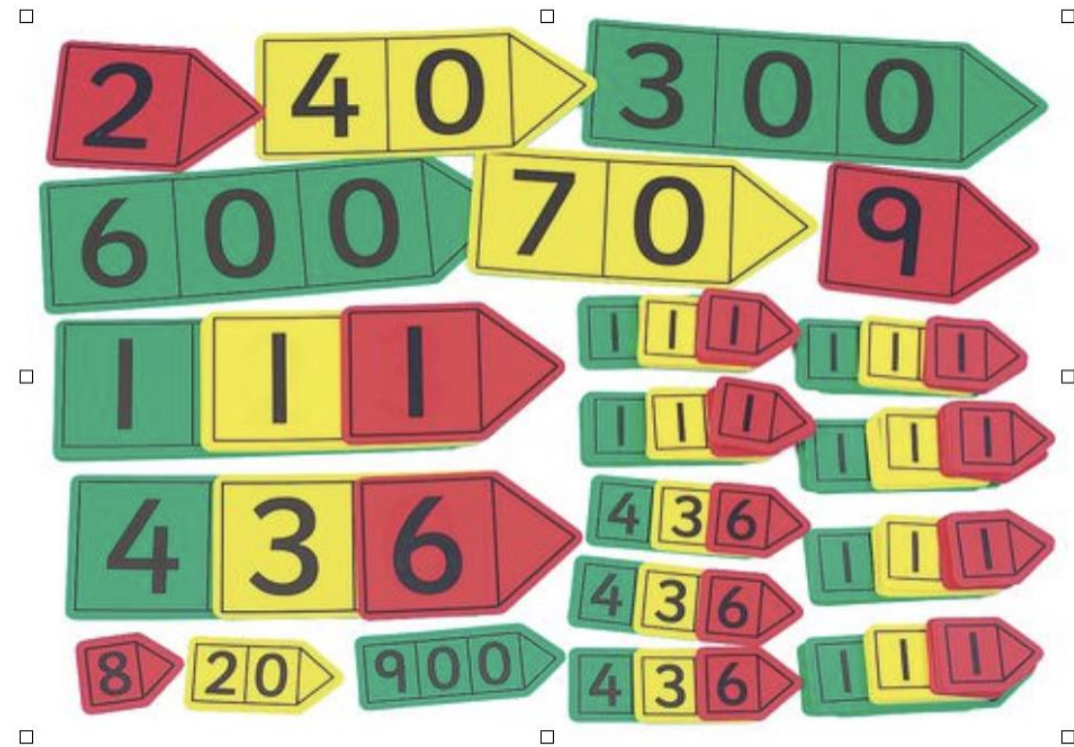
Adnoddau - Resources





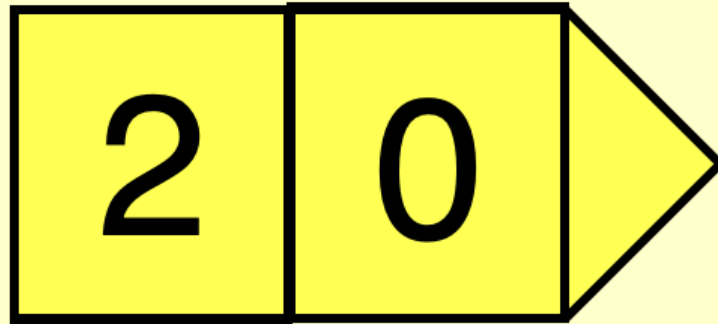
Cardiau Gwerth Lle

Place Value Cards



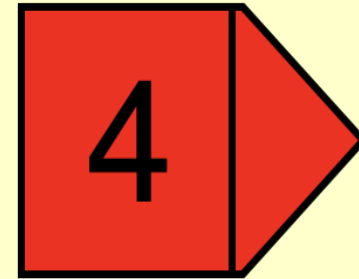
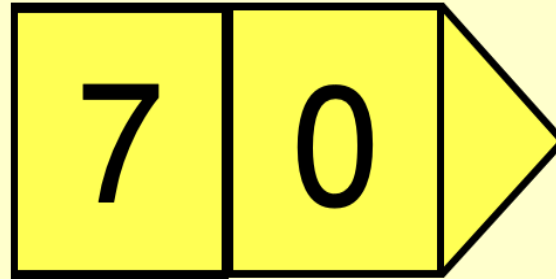
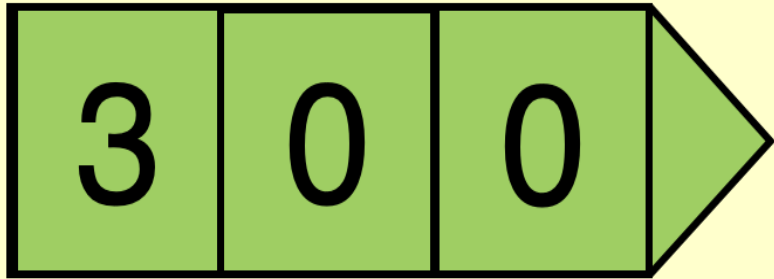
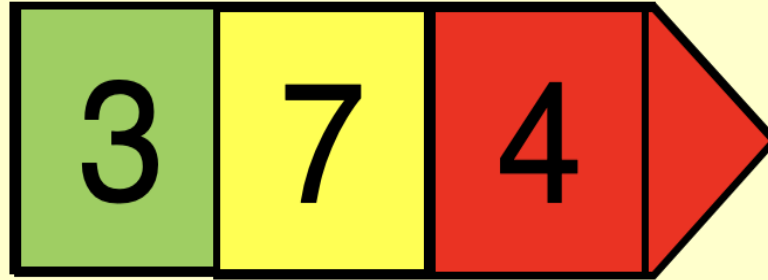
Gwerth Lle - Place value

23

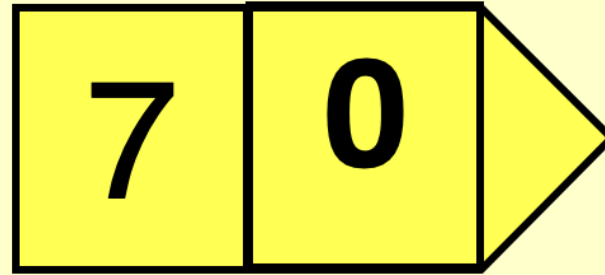
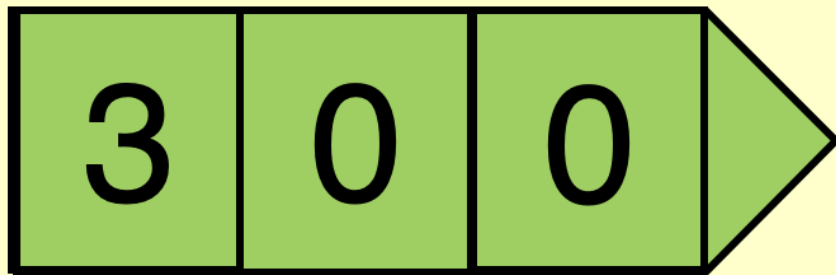
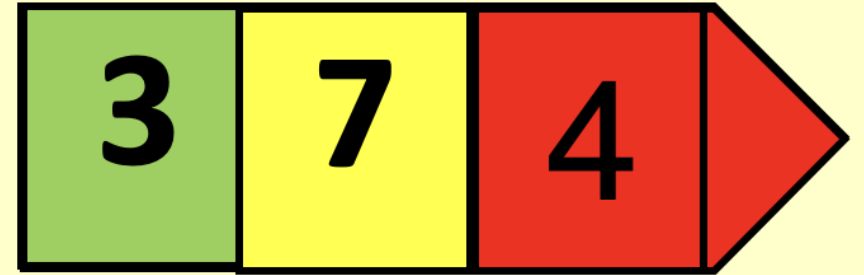


Gwerth Lle - Place value

374



Hundreds tens and units

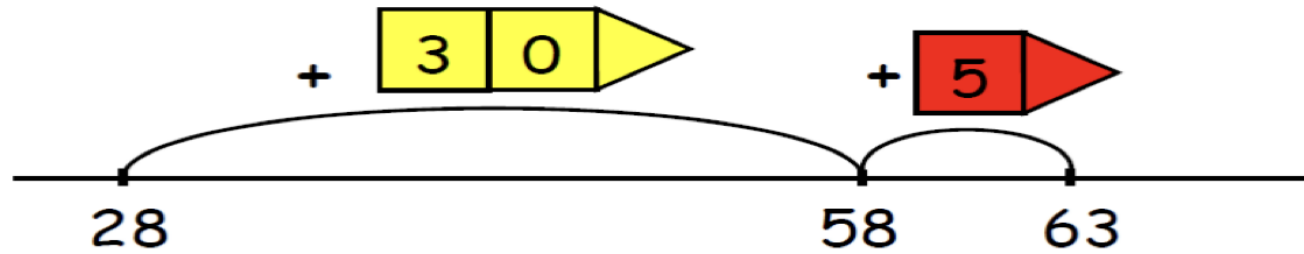


C	D	U
300	70	4

$$28 + 35 = \begin{array}{|c|c|} \hline 2 & 8 \\ \hline \end{array} \rightarrow + \begin{array}{|c|c|} \hline 3 & 5 \\ \hline \end{array} \rightarrow$$

4

$$= \begin{array}{|c|c|} \hline 2 & 8 \\ \hline \end{array} \rightarrow + \begin{array}{|c|c|} \hline 3 & 0 \\ \hline \end{array} \rightarrow + \begin{array}{|c|} \hline 5 \\ \hline \end{array} \rightarrow$$



$$28 + 35 = 63$$

$$28 + 35 = 63$$

$$35 + 28 = 63$$

$$63 - 35 = 28$$

$$63 - 28 = 35$$

$$28 + 35 = \begin{array}{|c|c|} \hline 2 & 8 \\ \hline \end{array} \rightarrow + \begin{array}{|c|c|} \hline 3 & 5 \\ \hline \end{array} \rightarrow$$

5

$$= \begin{array}{|c|c|} \hline 2 & 0 \\ \hline \end{array} \rightarrow + \begin{array}{|c|c|} \hline 3 & 0 \\ \hline \end{array} \rightarrow + \begin{array}{|c|} \hline 8 \\ \hline \end{array} \rightarrow + \begin{array}{|c|} \hline 5 \\ \hline \end{array} \rightarrow$$

$$= \begin{array}{|c|c|} \hline 5 & 0 \\ \hline \end{array} \rightarrow + \begin{array}{|c|c|} \hline 1 & 3 \\ \hline \end{array} \rightarrow$$

$$= \begin{array}{|c|c|} \hline 5 & 0 \\ \hline \end{array} \rightarrow + \begin{array}{|c|c|} \hline 1 & 0 \\ \hline \end{array} \rightarrow + \begin{array}{|c|} \hline 3 \\ \hline \end{array} \rightarrow$$

$$= \begin{array}{|c|c|} \hline 6 & 0 \\ \hline \end{array} \rightarrow + \begin{array}{|c|} \hline 3 \\ \hline \end{array} \rightarrow$$

$$= \begin{array}{|c|c|} \hline 6 & 3 \\ \hline \end{array} \rightarrow$$

$$28 + 35 = 63$$

$$35 + 28 = 63$$

$$63 - 35 = 28$$

$$63 - 28 = 35$$

$$28 + 35$$

8

20 adio 30 yw 50.
50 adio'r 10 wedi
ei 'gario' yw 60

3

+

2	8
3	5
6	3
1	

1

8 adio 5 yw 13
(10 + 3)

3 o'r 13

2a

10 wedi ei
'gario' o'r 13

2b

$$28 + 35 = 63$$

$$35 + 28 = 63$$

$$63 - 35 = 28$$

$$63 - 28 = 35$$

$$368 + 494$$

3a

60 plus 90 is 150
150 add the
'carried' 10 is 160

300 plus 400 is 700
700 add the
'carried' 100 is 800

4

3b

'carried' 100
from the 160

$$\begin{array}{r}
 368 \\
 + 494 \\
 \hline
 862 \\
 \hline
 11
 \end{array}$$

1

8 plus 4 is 12
(10 + 2)

11

2 from the 12

2a

2b

'carried' 10
from the 12

$$368 + 494 = 862$$

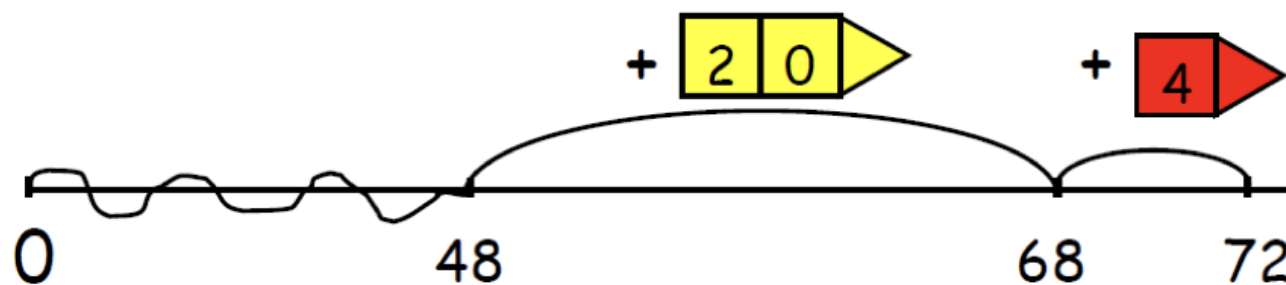
$$494 + 368 = 862$$

$$862 - 494 = 368$$

$$862 - 368 = 494$$

$$72 - 48 = \boxed{7} \boxed{2} \blacktriangleright - \boxed{4} \boxed{8} \blacktriangleright$$

4



$$= \boxed{2} \boxed{0} \blacktriangleright + \boxed{4} \blacktriangleright$$

$$= \boxed{2} \boxed{4} \blacktriangleright$$

$$72 - 48 = 24$$

$$72 - 24 = 48$$

$$24 + 48 = 72$$

$$48 + 24 = 72$$

$$72 - 48$$

1

2 tynnu 8

9

2a

Newid 70 i
60 + 10

3

60 tynnu
40 yw 20

$$\begin{array}{r} 61 \\ - 48 \\ \hline 24 \end{array}$$

2b

12 tynnu 8
yw 4

$$72 - 48 = 24$$

$$72 - 24 = 48$$

$$24 + 48 = 72$$

$$48 + 24 = 72$$

$$563 - 241$$

14

2

60 minus 40
is 20

3

500 minus 200
is 300

1

3 minus 1
is 2

	5	6	3
-	2	4	1
	3	2	2

$$563 - 241 = 322$$

$$563 - 322 = 241$$

$$322 + 241 = 563$$

$$241 + 322 = 563$$

$$842 - 276$$

12

4b

130 minus 70 is 60

3

30 minus 70

1

2 minus 6

4a

Change 800
to 700 + 100

Change 40 to
30 + 10

2a

5

700 minus 200
is 500

-

7	3	1
~~8~~	~~4~~	2
2	7	6
5	6	6

12 minus 6
is 6

2b

$$842 - 276 = 566$$

$$842 - 566 = 276$$

$$566 + 276 = 842$$

$$276 + 566 = 842$$

$$38 \times 7$$

$$3 \times 10$$

$\times$	30	8
7	210	56

$$7 \times 3 \times 10$$

$$7 \times 8$$

$$\begin{array}{r} 210 \\ + 56 \\ \hline 266 \end{array}$$

$$38 \times 7 = 266 \quad 7 \times 38 = 266$$

$$266 \div 38 = 7 \quad 266 \div 7 = 38$$

$$38 \times 7$$

3

3a

30 times 7 is 210,
add the 'carried'
50 is 260

1

8 times 7 is 56

3b

60 from the 260

x

4

200 from the
260

2 6 6

5

2a

6 from the 56

2b

'carried' 50 from
the 56

$$56 \times 27$$

$$2 \times 10$$

$$5 \times 10$$

$\times$	20	7
50	1000	350
6	120	42

$$5 \times 10 \times 7$$

or

$$5 \times 7 \times 10$$

$$6 \times 7$$

$$5 \times 10 \times 2 \times 10$$

or

$$5 \times 2 \times 10 \times 10$$

$$6 \times 2 \times 10$$

$$\begin{array}{r}
 56 \\
 \times 27 \\
 \hline
 42 \\
 350 \\
 120 \\
 1000 \\
 \hline
 1512
 \end{array}$$

$$56 \times 27 = 1512 \quad 27 \times 56 = 1512$$

$$1512 \div 56 = 27 \quad 1512 \div 27 = 56$$