

Callum Carter-Dawles

2008376

ACHA4000

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Q1)

$$a) i) \frac{3x-1}{5} = \frac{6x+5}{9}$$

$$\frac{27x-9}{5} = 6x+5$$

$$27x-9 = 30x+25$$

$$27x-30x = 25+9$$

$$-3x = 34$$

$$x = -\frac{34}{3}$$

$$ii) 1 + \frac{5}{x} = \frac{-6}{x^2}$$

$$\times x^2 \quad \cancel{6x^2} + 5x = -6$$

$$x^2 + 5x = -6$$

$$x = -2$$

$$\cancel{x^2 + 5x + 6 = 0}$$

$$(-2)^2 + 5(-2) = -6$$

$$\cancel{4 - 10} = -6$$

$$iii) 2 \log(3x+1) = \log(x+3) + \log(x+7)$$

$$2 \log(3x+1) = \log((x+3)(x+7))$$

$$\log(3x+1)^2 = \log(x^2+3x+7x+21)$$

$$\cancel{\log(3x+1)^2} = \cancel{\log(x^2+10x+21)}$$

$$(3x+1)^2 = x^2+10x+21$$

$$(3x+1)(3x+1) = x^2+10x+21$$

$$9x^2+3x+3x+1 = x^2+10x+21$$

$$9x^2-x^2+6x-10x = 21-1$$

$$8x^2-4x = 20$$

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iii) Continued:

$$8x^2 - 42x = 20$$

$$8x^2 - 42x - 20 = 0$$

$$(x \quad)(x \quad)$$

10x2
5x4

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i) $x^2 + 12x + 20 = 0$

$$(x + 10)(x + 2) = 0$$

$$x + 10 = 0$$

$$x = -10$$

$$x + 2 = 0$$

$$x = -2$$

ii) $2x^2 + x - 15 = 0$

$$(2x - 5)(x + 3) = 0$$

$$2x - 5 = 0$$

$$2x = 5$$

$$x = \frac{5}{2}$$

$$x = 2.5$$

$$x + 3 = 0$$

$$x = -3$$

iii) $x^2 - 400 = 0$

$$(x + 20)(x - 20) = 0$$

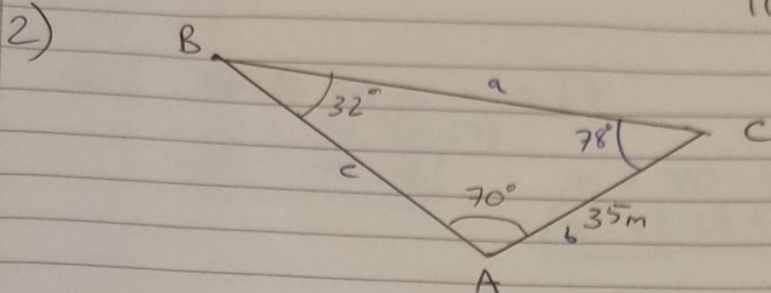
$$x + 20 = 0$$

$$x = -20$$

$$x = +20$$

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i) $180 - 32 - 70 = 78^\circ$

ii) $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$$\frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{35}{\sin 32} = \frac{c}{\sin 78}$$

$$\sin(78) \times \frac{35}{\sin 32} = c$$

$$c = 64.6 \text{ m}$$

iii) $\frac{a}{\sin A} = \frac{b}{\sin B}$ $a = \sin A \times \frac{b}{\sin B}$
 $= \sin(70) \times \frac{35}{\sin 32}$
 $= 62.06 \text{ m}$

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3)

a) i) ~~sin A~~ $\sin x = 0.67$

$$x = \sin^{-1}(0.67)$$

$$x = 42.06^\circ$$

$$x = 180 - 42.06$$

$$x = 137.94^\circ$$

ii) ~~tan 2x~~ ~~tan~~

$$\tan^2 x - 7 \tan x + 10 = 0$$

$$2 \tan x$$

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4) $y = 5x^2 + 12x - 14$

$$y = 4x^2 + 3x - 4$$

$$5x^2 + 12x - 14 = 4x^2 + 3x - 4$$

$$5x^2 - 4x^2 + 12x - 3x - 14 = -4$$

$$x^2 + 9x = 4 + 14$$

$$x^2 + 9x = 18$$

~~$$x^2 + 9x - 18 = 0$$~~

~~$$(x+4)(x-4.5)$$~~

$$\sqrt{x^2 + 9x} = \sqrt{18}$$

$$x + 3x = 3\sqrt{2}$$

~~$$4x = 3\sqrt{2}$$~~

$$x = \frac{3\sqrt{2}}{4} = 1.06$$

$$y = 4x^2 + 3x - 4$$

$$y = 4\left(\frac{3\sqrt{2}}{4}\right)^2 + 3\left(\frac{3\sqrt{2}}{4}\right) - 4$$

$$y = \frac{2 + 9\sqrt{2}}{4} = 3.68$$

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5) a) i)

$$z_1 = 3 + 7j$$

$$z_2 = 2 - 10j$$

i) $z_1 + z_2$

$$\begin{aligned} & 3 + 7j + 2 - 10j \\ &= \boxed{5 - 3j} \end{aligned}$$

ii) $z_1 - \bar{z}_2$

$$\bar{z}_2 = 2 + 10j$$

$$\begin{aligned} & 3 + 7j - (2 + 10j) \\ &= 3 + 7j - 2 - 10j \\ &= \boxed{1 - 3j} \end{aligned}$$

$$\text{iii) } \frac{z_1}{z_2} = \frac{(3 + 7j)}{(2 - 10j)} \times \frac{(2 + 10j)}{(2 + 10j)}$$

$$\begin{aligned} &= \frac{6 + 30j + 14j + 70j^2}{4 + 20j - 20j - 100j^2} \\ &= \frac{6 + 44j + 70(-1)}{4 - 100(-1)} \\ &= \frac{6 + 44j - 70}{-96} \\ &= \frac{44j - 64}{-96} \\ &= \frac{22j - 32}{-48} \\ &= \frac{11j - 16}{-24} \\ &= -\frac{11j}{24} - \frac{16}{24} = \boxed{-\frac{11j}{24} - \frac{2}{3}} \end{aligned}$$

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5b) ~~$7x^2 + 2x + 11 = 0$~~ $7x^2 = -2x - 11$

$7x^2 + 2x + 11 = 0$

~~$7x^2 + 2x + 11 = 0$~~

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-2 \pm \sqrt{4(2)^2 - 4(7 \times 11)}}{2 \times 7}$$

$$x = \frac{-2 \pm \sqrt{4 - 308}}{2 \cdot 14}$$

$$x = \frac{-2 \pm \sqrt{-304}}{14}$$

$$x = \frac{-2 \pm \sqrt{304} \sqrt{-1}}{14}$$

$$x = \frac{-2 \pm \sqrt{304} \times j}{14}$$

$$x = \frac{-2 \pm 4\sqrt{19} \times j}{14}$$

~~$x = \frac{-2 \pm 4\sqrt{19} \times j}{14}$~~

~~+~~

~~-~~

$$x = 1.10$$

$$x = -1.39$$

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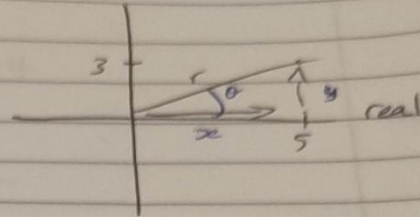
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5)c)

$$(5+3j)^4$$

$$n=4$$

$$\cancel{r=2.34} \quad r = \sqrt{34} \quad \theta = 31^\circ$$



$$[\sqrt{34} \angle 31] ^4$$

$$r^2 = x^2 + y^2$$

$$r = \sqrt{5^2 + 3^2}$$

$$r = \sqrt{34}$$

$$\tan \theta = \frac{y}{x}$$

$$\theta = \tan^{-1} \frac{y}{x}$$

$$\theta = \tan^{-1} \frac{3}{5}$$

$$\theta = 31.0^\circ$$

$$[r \angle \theta]^n \cancel{= r^n \angle n\theta} = r^n (\cos n\theta + j \sin n\theta)$$

$$[\sqrt{34} \angle 31] ^4 = \sqrt{34}^4 (\cos(4(31)) + j \sin(4(31)))$$

$$\cancel{[r \angle \theta]^n = r^n \angle n\theta} \quad \cancel{= r^n \left(\frac{1}{2} (z + \frac{1}{z}) + j \frac{1}{2} (z - \frac{1}{z}) \right)}$$

6) a) i)

$$y = 9x^3 + 7x^2 - 2x + 4$$

$$\frac{dy}{dx} = \boxed{27x^2 + 14x - 2}$$

$$\text{ii) } y = (2x + 3)(3x - 5)$$

$$y = \cancel{6x^2} - 10x + 9x - \cancel{15} = 6x^2 - x - 15$$

$$\frac{dy}{dx} = \boxed{12x - 1}$$

$$\text{iii) } y = 2 \ln x + \frac{5}{x} = 2 \ln x^2 + 5x^{-1}$$

$$\frac{dy}{dx} = 2 \times \frac{1}{x} - 5x^{-2}$$

$$= \boxed{\frac{2}{x} - \frac{5}{x^2}}$$

$$\text{iv) } y = 7 \sin(x) + 3 \cos(2x) + \tan\left(\frac{x}{2}\right)$$

$$y = 7 \sin(x) + 3 \cos(2x) + \tan(0.5x)$$

$$\frac{dy}{dx} = \boxed{7 \cos x - 6 \sin 2x + 0.5 \sec^2(0.5x)}$$

$$\text{v) } \cancel{y = x^5 \sin(2x)} \quad y = x^5 \sin(2x)$$

$$\frac{dy}{dx} = 2 \times 5x^4 \cos(2x)$$

$$= \boxed{10x^4 \cos(2x)}$$

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6) vi) $y = \frac{e^{3x}}{x^3}$ $y = e^{3x} \times x^{-3}$

$$\frac{dy}{dx} = 3e^{3x} \times -3x^{-4} = \boxed{\frac{3e^{3x}}{x^4}}$$

vii) $y = (5x^2 - 2x + 1)^5$

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7) i) $\int x^5 + 2x^2 + x - 10 \, dx$

$$y = \frac{x^6}{6} + \frac{2x^3}{3} + \frac{x^2}{2} - 10x + C$$

ii) $\int (2x+5)(x^2-1) \, dx$

$$\int 2x^3 + 5x^2 - 2x - 5 \, dx$$

$$y = \frac{2x^4}{4} + \frac{5x^3}{3} - \frac{2x^2}{2} - 5x + C$$

$$y = \frac{1}{2}x^4 + \frac{5}{3}x^3 - x^2 - 5x + C$$

iii) $\int_2^5 2x^2(x^2+3) \, dx$

$$\int_2^5 2x^4 + 6x^2 \, dx$$

$$y = \left[\frac{2}{5}x^5 + \frac{6}{3}x^3 + C \right]_2^5$$

$$y = \left[\frac{2}{5}(5)^5 + 2(5)^3 + C \right] - \left[\frac{2}{5}(2)^5 + 2(2)^3 + C \right]$$

$$y = 1250 + 250 - \frac{64}{5} - 16$$

$$y = 1484 - \frac{64}{5} = 1471.2$$

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$$\frac{x}{3} = \frac{1}{3}x$$

$$7) \text{ iv) } \int 5 \sin 3x - \cos \frac{x}{3} \, dx$$

$$y = 5\left(-\frac{1}{3} \cos(3x)\right) - \left(\frac{1}{\frac{1}{3}} \sin\left(\frac{1}{3}x\right)\right) + C$$

$$y = -\frac{5}{3} \cos 3x - 3 \sin \frac{x}{3} + C$$