

Examination Answer Books

12 Pages

Instructions to Candidates

1. Please give the information requested in the book.
2. Enter your name in the space provided to the right and seal down (using the label provided) to hide your name.
3. The answers must be legibly written. The Examiners can either accept answers that have been typed and the expense of typing charged to the candidate.
4. Write your answers in ink.
5. Begin each answer on a fresh page. Do all rough work in the answer book and cross it through.
6. No part of this book is to be torn out; it must be handed in complete.
7. If more books than one are used, fasten them together using the tags available from the invigilator(s).
8. **Every book must be handed in. In no circumstances may answer books, used or unused, be taken out of the examination venue.**

1	Student Number	2410225
2	Title of Examination Paper	Math Paper 2
3	Module Code	ACDA4008
4	Date and Time of Examination	07/1/25 0900
5	Examination Venue	Iq 307
6	Seat Number	
7	Number of Answer Books handed in	1

List the numbers of the questions you have attempted	Office Use
1	29
2	20
3	19
4	5
Total	73

73%
good

✓ GT Feb 25

1)

a)

i)

$$y = x + 3$$

$$\frac{dy}{dx} = 1x^{1-1} = 1x^0$$

$$\frac{dy}{dx} = 1 \quad \checkmark$$

(2)

$$ii) y = 4x^3 + 10x^{1/2}$$

$$\frac{dy}{dx} = (4 \times 3)x^{3-1} + (10 \times \frac{1}{2})x^{1/2-1}$$

$$\frac{dy}{dx} = 12x^2 + 5x^{-1/2}$$

(3)

$$iii) y = \frac{1}{5x^5} + x^3$$

$$y = x^{-5} + x^3$$

$$\frac{dy}{dx} = -5x^{-6} + 3x^2$$

(3)

(2)

$$iv) y = \sin(8x)$$

$$\frac{dy}{dx} = 8\cos(8x)$$

$$v) y = x \cdot \cos(x) \quad \text{Product Rule: } \frac{dy}{dx} = u \cdot \frac{dv}{dx} + v \cdot \frac{du}{dx}$$

$$u = x \quad \frac{du}{dx} = 1 \quad v = \cos(x) \quad \frac{dv}{dx} = -\sin(x)$$

$$\frac{dy}{dx} = x \cdot (-\sin(x)) + \cos(x) \cdot 1$$

$$\frac{dy}{dx} = -x \sin(x) + \cos(x)$$

(5)

vi) $y = (2x - 3)^{10}$

$$\frac{dy}{dx} = 10(2x - 3)^9 \cdot 2$$

$$\frac{dy}{dx} = 20(2x - 3)^9$$

3

vii) $y = (3x + \tan(x))$

$$\frac{dy}{dx} = 1 \cdot (3 + \sec^2(x))$$

$$\frac{dy}{dx} = 3 + \sec^2(x)$$

2

b) $y = \frac{1}{x} + \cos(2x)$, Find $\frac{d^3y}{dx^3}$

$$y = x^{-1} + \cos(2x)$$

$$\frac{dy}{dx} = -1x^{-2} - 2\sin(2x)$$

$$\frac{d^2y}{dx^2} = 2x^{-3} - 4\cos(2x)$$

$$\frac{d^3y}{dx^3} = -6x^{-4} + 8\sin(2x)$$

4

c) $y = \frac{x-1}{4+x^2}$

quotient rule: $\frac{dy}{dx} = \frac{v \cdot \frac{du}{dx} - u \cdot \frac{dv}{dx}}{v^2} = \frac{dy}{dx}$

$u = x-1$ $v = 4+x^2$

$\frac{du}{dx} = 1$ $\frac{dv}{dx} = 2x$

$$\frac{(4+x^2)(1) - (x-1)(2x)}{(4+x^2)^2} = \frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{4+x^2 - (2x^2-2x)}{16+4x^2+4x^2+x^4}$$

$$\frac{dy}{dx} = \frac{4+x^2-2x^2+2x}{x^4+8x^2+16}$$

$$\frac{dy}{dx} = \frac{x^2-2x^2+2x+4}{x^4+8x^2+16}$$

$$\frac{dy}{dx} = \frac{-x^2+2x+4}{x^4+8x^2+16}$$

$$x=0$$

$$\frac{dy}{dx} = \frac{-(0)^2+2(0)+4}{(0)^4+8(0)+16} = \frac{1}{4}$$

5

29

$$2) \quad i) \int (10x + x^3) dx$$

$$= 5x^2 + \frac{x^4}{4} + C$$

(3)

$$ii) \int (x - 3e^x) dx$$

$$= \frac{x^2}{2} - 3e^x + C$$

(3)

$$iii) \int \left(\frac{1}{x} + 4 \right) dx$$

$$\int (x^{-1} + 4) dx$$

$$= \ln x + 4x + C$$

(3)

$$iv) \int 2x \cdot (3x + 4) dx$$

$$\int (6x^2 + 8x) dx$$

$$= 2x^3 + 4x^2 + C$$

(3)

$$v) \int_0^2 (3x^2 + 2x) dx$$

$$= x^3 + x^2 + C \quad \text{no 'C' as definite integral}$$

$$((2)^3 + (2)^2) - (0^3 - 0^2)$$

$$= 8 + 4$$

$$= 12$$

(2)

b) $\int_0^1 \frac{x}{\sqrt{x^2+1}} dx$

$$\int_0^1 x \cdot (x^2+1)^{-\frac{1}{2}} dx$$

$$u = x^2 + 1$$

$$x = \sqrt{u} - 1$$

$$\frac{du}{dx} = 2x$$

$$du = 2x dx$$

$$dx = \frac{2x}{du}$$

$$\int_0^1$$

(1)

✓

2 C

$$i) \int x \cdot \sin(2x) dx \quad uv - \int v \cdot \frac{du}{dx}$$

$$u \cdot \frac{dv}{dx}$$

$$u = x \quad \frac{du}{dx} = 1$$

$$v = \frac{-\cos(2x)}{2} \quad \frac{dv}{dx} = \sin(2x)$$

$$x \left(\frac{-\cos(2x)}{2} \right) - \int \left(\frac{-\cos(2x)}{2} \right) \cdot \sin(2x) dx$$

$$= \frac{-x \cos(2x)}{2} - \left(\frac{-\sin(2x)}{4} \cdot \frac{-\cos(2x)}{2} \right)$$

$$= \frac{-x \cos(2x)}{2} + \left(\frac{\sin(2x)}{4} \cdot \frac{\cos(2x)}{2} \right) + C$$

$$ii) \int_1^e x \cdot \ln(x) dx \quad uv - \int v \cdot \frac{du}{dx}$$

$$\ln(x) = u \quad \frac{du}{dx} = \frac{1}{x}$$

$$v = \frac{x^2}{2} \quad \frac{dv}{dx} = x$$

Other way around

$$\left[\ln(x) \left(\frac{x^2}{2} \right) \right]_1^e - \int_1^e \left(\frac{x^2}{2} \right) \cdot \left(\frac{1}{x} \right) dx$$

$$\ln(x) \left(\frac{x^2}{2} \right) - \int \frac{1}{2} x^2 \cdot x^{-1}$$

$$\ln(x) \left(\frac{x^2}{2} \right) - \int \frac{1}{2} x^1 dx$$

$$\ln(x) \left(\frac{x^2}{2} \right) - \left[\frac{x^2}{4} \right]_1^e = \left(\frac{e^2 + 1}{4} \right)$$

$$\ln(e) \left(\frac{e^2}{2} \right) - \frac{e^2}{4} = 1.847 - 1.847 = -2.097$$

$$\ln(1) \left(\frac{1^2}{2} \right) - \frac{1^2}{4} = 0 - 0.25 = -0.25$$

$$= 2.097$$

3)

a)

$$i (3 + 2i)(1 - i)$$

$$3 + 2i - 3i - 2i^2$$

$$3 - i - 2i^2$$

$$= 3 - i - 2(-1)$$

$$= 5 - i$$

(2)

ii)

$$\frac{3}{2+i}$$

$$= \frac{3(2-i)}{(2+i)(2-i)}$$

$$(2+i)(2-i)$$

$$= \frac{6-3i}{4+2i-2i-i^2}$$

$$4+2i-2i-i^2$$

$$= \frac{6-3i}{4-(-1)}$$

$$4-(-1)$$

$$= \frac{6-3i}{5}$$

$$= \frac{6}{5} - \frac{3i}{5}$$

(2)

iii)

$$\frac{4}{i^5}$$

=

$$\frac{4}{i^2 \times i^2 \times i}$$

=

$$\frac{4}{-1 \times -1 \times i}$$

=

$$\frac{4}{i}$$

$$= \frac{4}{i} \times \frac{i}{i}$$

$$\frac{4i}{i^2}$$

=

$$\frac{4i}{-1}$$

$$= 0 - 4i$$

(2)

$$b) \quad z_1 = \frac{2-j}{2+j} \quad z_2 = \frac{3+j}{3-j}$$

$$z_1 = \frac{2-j}{2+j} \times \frac{(2-j)}{(2-j)}$$

$$= \frac{(2-j)(2-j)}{(2+j)(2-j)}$$

$$= \frac{4 - 2j - 2j + j^2}{4 - 2j + 2j - j^2}$$

$$= \frac{4 - 4j + (-1)}{4 - (-1)}$$

$$z_1 = \frac{3-4j}{5}$$

3

$$z_2 = \frac{3+j}{3-j} \times \frac{(3+j)}{(3+j)}$$

$$z_2 = \frac{(3+j)(3+j)}{(3-j)(3+j)} = \frac{9 + 3j + 3j + j^2}{9 - 3j + 3j - j^2}$$

$$z_2 = \frac{9 + 6j + j^2}{9 - j^2} = \frac{9 + 6j + (-1)}{9 - (-1)}$$

$$z_2 = \frac{8+6j}{10} = \frac{2(4+3j)}{10}$$

$$= \frac{4}{5} + \frac{3}{5}j$$

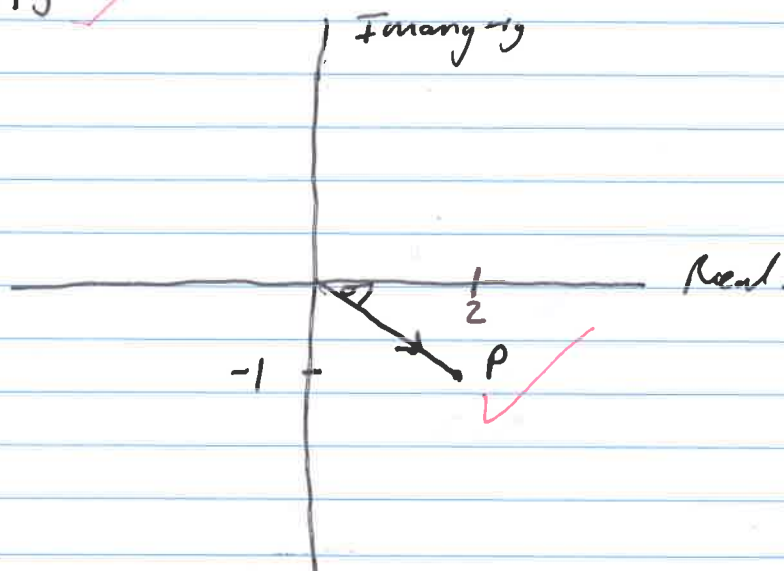
2

✓

$$c) p = 2 \left(\frac{3-4j}{5} \right) + \frac{8+6j}{10}$$

$$p = \frac{6-8j}{5} + \frac{8+6j}{10}$$

$$p = 2 - 1j$$



3

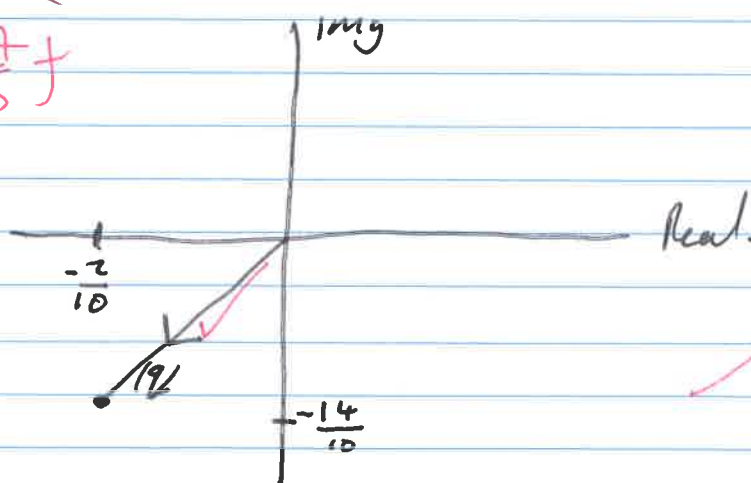
$$ii) q = z_1 - z_2$$

$$q = \frac{3-4j}{5} - \frac{8+6j}{10}$$

$$q = \left(\frac{3}{5} - \frac{8}{10} \right) + \left(\frac{-4j}{5} - \frac{6j}{10} \right)$$

$$q = -\frac{2}{10} - \frac{14}{10}j$$

$$= -\frac{1}{5} - \frac{7}{5}j$$



3

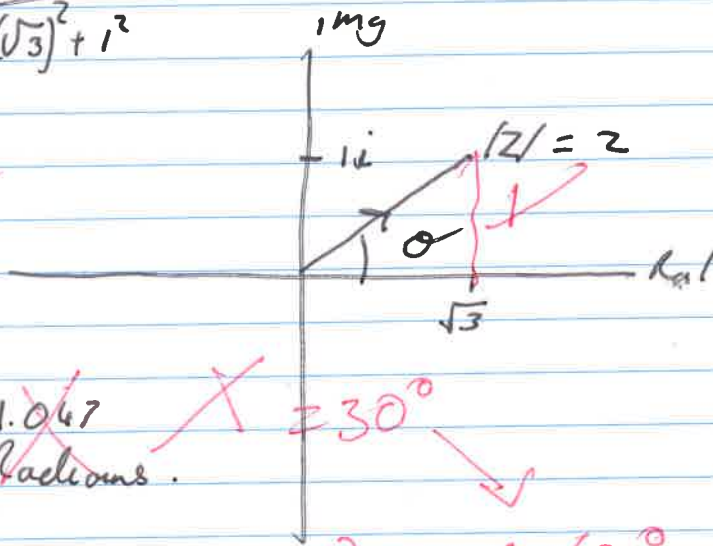
d) $z = \sqrt{3} + i$

~~$\sqrt{(\sqrt{3})^2 + 1^2}$~~

$r = |z| = \sqrt{3+1}$

$|z| = 2$

$\tan^{-1}\left(\frac{\sqrt{3}}{1}\right) = 1.047$
Radians.



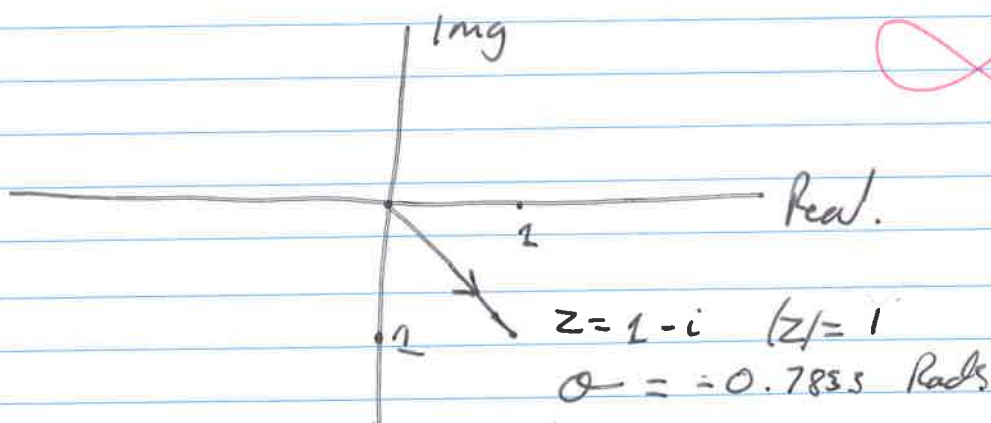
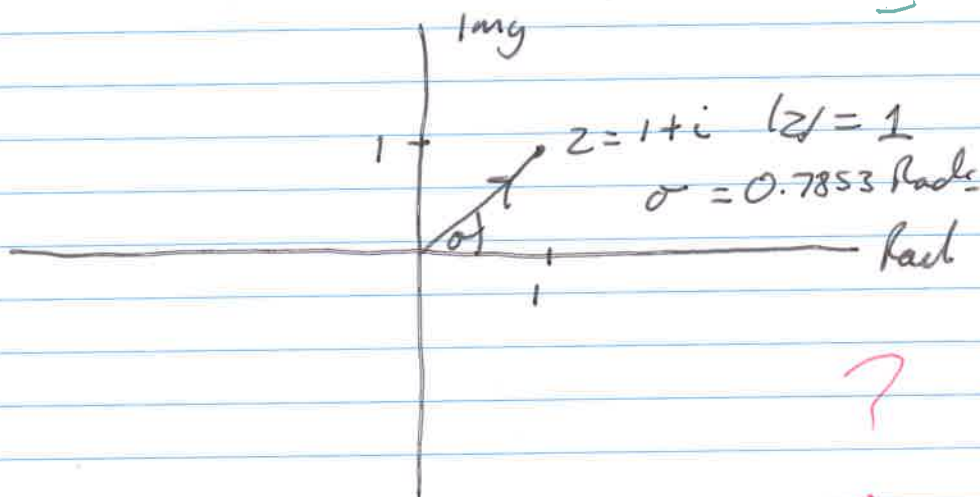
$z = 2 \cos(30^\circ) + i \sin(30^\circ)$
 $= 2 \left(\cos \frac{\pi}{6} + i \sin \frac{\pi}{6} \right)$

e) $\tan^{-1}\left(\frac{1}{1}\right) = 0.7853$ Rads

$\tan^{-1}\left(\frac{1}{-1}\right) = -0.7853$ Rads

↑ Polar form

②



4)

a

$$i \quad 3^{ex} = 2^{x+1} \quad ?$$

$$ex \log(3) = x+1 \log(2)$$

$$x = \frac{(ex \log(3)) - \log(2)}{e}$$

$$= e$$

$$ii \quad 2 \ln(x) = \ln(5x-6) \quad \times e$$

$$2(x) = (5x-6) \quad x^2 = 5x-6$$

$$2x = 5x-6$$

$$3x = -6$$

$$x = -3$$

$$x^2 - 5x + 6 = 0$$

$$(x-2)(x-3) = 0$$

$$x = 2, x = 3$$

b

$$i \quad 6 \log(x) + \log(y) + \log(z)$$

$$ii \quad 3 \log(x) + \log(y) + \log(z)$$