

Matus Exam

Differentiate with respect to x

$$\frac{(u \times v)' - (v \times u')}{v^2}$$

ai

$$y = \frac{x^2}{2x+1}$$

$$u = x^2$$

$$u' = 2x$$

$$v = 2x+1$$

$$v' = 2$$

$$= \frac{2x(2x+1) - x^2(2)}{(2x+1)^2}$$

$$= \frac{4x^2 + 2x - 2x^2}{(2x+1)^2}$$

$$= \frac{2x^2 + 2x}{(2x+1)^2}$$

$$\text{aii } y = \sqrt{x^2+1} = (x^2+1)^{1/2}$$

$$u = x^2+1$$

$$du/dx = 2x$$

$$= \frac{1}{2} u^{-1/2} \times 2x$$

$$y = u^{1/2}$$

$$dy/du = \frac{1}{2} u^{-1/2}$$

$$= \frac{1}{2} (x^2+1)^{-1/2} \times 2x$$

$$= x(x^2+1)^{-1/2}$$

$$\text{aiii } y = 3x^{3/2} + \frac{3}{x^2} - 6x$$

$$\frac{dy}{dx} = 3 \times \frac{3}{2} x^{1/2} - 6 \times \frac{1}{x^3} - 6$$

$$= 3 \frac{d}{dx} x^{3/2} - 6 \frac{d}{dx} x + 3 \frac{d}{dx} \frac{1}{x^2}$$

$$= 3 \times \frac{3}{2} x^{3/2-1} - 6 \times 1 + 3(-2)x^{-3} = \frac{9\sqrt{x}}{2} = \frac{6}{x^3} - 6$$

b $y = 1/x + \cos 2x$ Find $\frac{d^5 y}{dx^5}$

$$y = x^{-1} + \cos 2x \quad \frac{dy}{dx} = \frac{d}{dx}(x^{-1}) + \frac{d}{dx}(\cos 2x)$$

① $= -x^{-2}$	and	~~$-2 \sin 2x$~~	$-2 \sin 2x$
② $= +2x^{-3}$	and	~~$-4 \cos 2x$~~	$-4 \cos 2x$
③ $= -6x^{-4}$	and	~~$8 \sin 2x$~~	$8 \sin 2x$
④ $= 24x^{-5}$	and	~~$16 \cos 2x$~~	$16 \cos 2x$
⑤ $= -120x^{-6}$	and	~~$-32 \sin 2x$~~	$-32 \sin 2x$

$$\frac{d^5 y}{dx^5} = -120x^{-6} + 32 \sin 2x$$

c $f(x) = \ln(\sec x - \cot x)$ $f'(x) = \sec x$

+C

$$\textcircled{2} \text{ai} \int \frac{2x}{(4+3x^2)} dx = \frac{\ln(3x^2+4)}{3} + C$$

$$\text{aii} \int_0^4 e^{1/2x} dx = \left[2e^{1/2x} \right]_0^4 = 12.778$$

$$\text{iii} \int_0^4 \frac{5x+13}{(2x+1)(x+4)} dx = \ln 54 = 3.989$$

$$b) f(x) = \frac{4}{x(1+4\ln x)^2} dx$$

$$u = 1 + 4\ln x + 1$$

$$\frac{du}{dx} = \frac{4}{x}$$

$$= \int \frac{1}{u^2} \quad \text{So } = -\frac{1}{u}$$

$$dx = \frac{x}{4} du$$

$$= \frac{-1}{4\ln x + 1} + C$$

$$y = \frac{1}{u^2}$$

$$\frac{dy}{du} = -\frac{1}{u^3}$$

$$c) \int_{\ln^2}^{\ln^5} \frac{3e^{2x}}{\sqrt{e^x - 1}} dx$$

$$u = e^x - 1 \quad dx = e^x du$$

$$\frac{du}{dx} = e^x$$

$$3 \int \frac{e^{2x}}{\sqrt{e^x - 1}} dx = 3 \int \frac{u+1}{\sqrt{u}} du = \int \sqrt{u} du$$

$$= \frac{u^{3/2}}{3/2} = \frac{2}{3} u^{3/2}$$

$$ii \int_1^e \ln x dx = (e \ln e - e) - (1 \ln 1 - 1) = 1$$

+C

$$3ai \quad \frac{(1+i)^3}{1-i} = \left(\frac{1+i}{1-i} \times \frac{1+i}{1-i} \right) \times \frac{1+i}{1-i}$$

$$\frac{1+i}{1-i} \times \frac{1+i}{1-i} = -1$$

$$-1 \times \frac{1+i}{1-i} = -1$$

$$ii \quad \frac{3}{(1+i)(2+i)} = \frac{3}{1+3i} = \frac{3}{1+3i} \times \frac{1-3i}{1-3i} = \frac{3-9i}{10}$$

$$= \frac{3}{10} - \frac{9}{10}i$$

$$iii \quad \frac{1+i}{1-i} - \frac{(1+2i)(2+2i) + 3-1}{1+i} = \underline{3-7i}$$

$$iv \quad 2i(i-1) + (\sqrt{3}-i) + (1+i)(1+i)$$

$$= \underline{\sqrt{3}-3i}$$

$$b \quad 2 = (4+7i)(3-2i) = 26+13i$$

$$R = \sqrt{26^2 + 13^2} = \underline{\sqrt{676 + 169} = 29.1}$$

$$\tan \theta = \frac{0}{9} \quad \tan \theta = \frac{13}{26} \quad \theta = 26.6^\circ$$

~~scribbled out text~~

C Quotient of $1+3i$ and $1+i$

$$\frac{1+3i}{1+i} \times \frac{1-i}{1-i} = \frac{4+2i}{2} = \underline{2+i}$$

$$\frac{d}{dx} \frac{y}{\sec 4x} = \sec^3 4x \quad = 3 \sec^2(4x) \times \frac{d}{dx} \sec(4x)$$

$$\sec 4x \tan 4x \quad = 3 \sec^2(4x) \sec(4x) \tan(4x) \times \frac{d}{dx} 4x$$

$$= 3 \tan(4x) \sec^3(4x) \times 4 \times \frac{d}{dx} x$$

$$= 12 \tan(4x) \sec^3(4x) \times 1$$

$$= 12 \sec^3(4x) \tan(4x)$$

$$\frac{d}{dx} y = (x + \sin 2x) \quad x, 1$$

$$= 2 \cos 2x + 1 \quad \sin 2x, 2 \cos 2x$$

$$u \cdot y = \frac{1-10x}{(2x-1)^2} \quad \frac{u}{v} = \frac{(u' \cdot v) - (u \cdot v')}{v^2} \quad u = 1-10x$$

$$v = (2x-1)^2$$

~~$$\frac{d}{dx} \frac{1-10x}{(2x-1)^2} = \frac{(1-10x)' \cdot (2x-1)^2 - (1-10x) \cdot (2x-1)^2'}{(2x-1)^4}$$~~

$$= \frac{2 \times (10 \times x + 3)}{(2 \times x - 1)^3}$$