

Question 1

a) (i) $AB = \begin{pmatrix} 5 & -11 & 3 \\ 37 & 33 & 33 \\ 8 & 23 & 21 \end{pmatrix}$

5
(-1 per incorrect element)

(ii) $BA = \begin{pmatrix} 19 & 12 & 11 \\ 65 & 21 & -5 \\ -13 & 13 & 19 \end{pmatrix}$

5
(-1 per incorrect element)

(iii) $AC = \begin{pmatrix} -8 & -7 \\ 22 & 21 \\ 18 & 22 \end{pmatrix}$

3
(-1 per incorrect element)

(iv) Cannot calculate CA.

1

(v) $|A| = 54$

2

(vi) $|B| = 131$

2

(vii) $A+B = \begin{pmatrix} 5 & -1 & 1 \\ 13 & 11 & 4 \\ -3 & 3 & 5 \end{pmatrix}$

1

(viii) C+D cannot calculate.

1

$$b) \quad 3x + 2y = 10 \quad (1) \times 3$$

$$4x + 3y = 13 \quad (2) \times 2$$

$$9x + 6y = 30 \quad (3)$$

$$8x + 6y = 26 \quad (4) \quad -$$

$$(x = 4) \rightarrow \text{in } (1)$$

$$12 + 2y = 10$$

$$2y = -2$$

$$(y = -1)$$

$$c) \quad \begin{vmatrix} (x+1) & -5 & -6 \\ -1 & 4 & 2 \\ -3 & 2 & (x+1) \end{vmatrix} = 0$$

$$(x+1)(4(x+1) - 4) + 5 \overset{-(-5)}{\downarrow} (-x+1) \overset{-(-6)}{\downarrow} + 6(-2+12) = 0$$

$$(x+1)(4x+4-4) + 5(-x+1+6) - 6(10) = 0$$

$$(x+1)(4x) + 5(-x+7) - 60 = 0$$

$$4x^2 + 4x - 5x + 35 - 60 = 0$$

$$4x^2 - x - 25 = 0$$

using quadratic formula gives

$$x = 3.09, \quad x = -2.84$$

$$d) \quad \begin{aligned} 3x + 2y - z &= 17 \\ 2x - y + 4z &= -5 \\ 5x - y - z &= 6 \end{aligned}$$

$$\begin{pmatrix} 3 & 2 & -1 \\ 2 & -1 & 4 \\ 5 & -1 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 17 \\ -5 \\ 6 \end{pmatrix}$$

↑
A

$$|A| = 56$$

$$A_1 = \begin{pmatrix} 17 & 2 & -1 \\ -5 & -1 & 4 \\ 6 & -1 & -1 \end{pmatrix} \quad |A_1| = 112$$

$$A_2 = \begin{pmatrix} 3 & 17 & -1 \\ 2 & -5 & 4 \\ 5 & 6 & -1 \end{pmatrix} \quad |A_2| = 280$$

$$A_3 = \begin{pmatrix} 3 & 2 & 17 \\ 2 & -1 & -5 \\ 5 & -1 & 6 \end{pmatrix} \quad |A_3| = -56$$

$$x = \frac{112}{56}$$

$$y = \frac{280}{56}$$

$$z = \frac{-56}{56}$$

$$x = 2$$

$$y = 5$$

$$z = -1$$

Question 2

a) (i) $\int (x^2 + 4)^3 + 2 \sec^2 3x \, dx$

$$\int (x^2 + 4)(x^2 + 4)(x^2 + 4) + 2 \sec^2 3x \, dx$$

$$\int (x^4 + 8x^2 + 16)(x^2 + 4) + 2 \sec^2 3x \, dx$$

$$\int x^6 + 4x^4 + 8x^4 + 32x^2 + 16x^2 + 64 + 2 \sec^2 3x \, dx$$

$$\int x^6 + 12x^4 + 48x^2 + 64 + 2 \sec^2 3x \, dx$$

$$= \frac{x^7}{7} + \frac{12x^5}{5} + \frac{48x^3}{3} + 64x + \frac{2}{3} \tan 3x + C$$

~~111~~
(-1 incorrect)
5

(ii) $\int_2^6 4x^2(3x-1) \, dx$

$$\int_2^6 12x^3 - 4x^2 \, dx$$

$$\left[\frac{12x^4}{4} - \frac{4x^3}{3} \right]_2^6$$

$$= \left[3(6)^4 - \frac{4(6)^3}{3} \right] - \left[3(2)^4 - \frac{4(2)^3}{3} \right]$$

$$= [3888 - 288] - [48 - 10.67]$$

$$= [3600] - [37.33]$$

$$= 3562.67$$

$$(iii) \int_0^3 e^{2x}(e^{2x}-1) dx$$

$$\int_0^3 e^{4x} - e^{2x} dx$$

$$= \left(\frac{1}{4} e^{4x} - \frac{1}{2} e^{2x} \right)_0^3$$

$$= \left[\frac{1}{4} e^{12} - \frac{1}{2} e^6 \right] - \left[\frac{1}{4} e^0 - \frac{1}{2} e^0 \right]$$

$$= [40688.70 - 201.71] - [0.25 - 0.5]$$

$$= [40486.99] - [-0.25]$$

$$= 40487.24$$

⑤

$$b) \quad (i) \frac{1+x}{(1-x)(1+2x)} = \frac{A}{(1-x)} + \frac{B}{(1+2x)}$$

$$1+x = A(1+2x) + B(1-x)$$

$$\underline{x=1} \quad 2=3A$$

$$\frac{2}{3} = A$$

$$x = -1/2$$

$$\frac{1}{2} = \frac{3}{2} B$$

$$\frac{1}{3} = B$$

$$\therefore \text{Pfs are } \frac{\frac{2}{3}}{(1-x)} + \frac{\frac{1}{3}}{(1+2x)}$$

$$(ii) \int \frac{\frac{2}{3}}{(1-x)} + \int \frac{\frac{1}{3}}{(1+2x)} dx$$

$$= \frac{2}{3} \int \frac{1}{(1-x)} + \frac{1}{3} \int \frac{1}{(1+2x)} dx$$

$$= -\frac{2}{3} \ln(\cancel{1-x}) + \frac{1}{6} \ln(1+2x)$$

$$= \frac{1}{6} \ln(1+2x) - \frac{2}{3} \ln(1-x)$$

$$= \ln \sqrt[6]{(1+2x)} - \ln \sqrt[3]{(1-x)^2} \quad \text{using laws of logs.}$$

$$= \ln \frac{\sqrt[6]{(1+2x)}}{\sqrt[3]{(1-x)^2}} + C \quad \text{as required.}$$

$$c) \int 2x \sin 3x \, dx \quad = uv - \int v \frac{du}{dx} \, dx$$

$\uparrow$ $\uparrow$
 u $\frac{dv}{dx}$

$$\frac{du}{dx} = 2 \quad v = -\frac{1}{3} \cos 3x$$

$$= -\frac{2x}{3} \cos 3x - \int -\frac{2}{3} \cos 3x \, dx$$

$$= -\frac{2x}{3} \cos 3x + \frac{2}{3} \int \cos 3x \, dx$$

$$= -\frac{2x}{3} \cos 3x + \frac{2}{3} \left(\frac{1}{3} \sin 3x \right)$$

$$= -\frac{2x}{3} \cos 3x + \frac{2}{9} \sin 3x + C$$

$$d) (i) \int \frac{15x^2 + 4x}{5x^3 + 2x^2 - 1} \, dx$$

$$= \ln(5x^3 + 2x^2 - 1) + C$$

$$(ii) \int \frac{2}{3x+1} \, dx$$

$$2 \int \frac{1}{3x+1}$$

$$= \frac{2}{3} \ln(3x+1) + C$$

e) $\int \frac{\sin x}{\cos x} dx$ (diff bottom & get $-\sin x$)

$$= \int \frac{-\sin x}{\cos x} dx$$

$$= -\ln(\cos x) + C$$

f) $\int \frac{4}{5} \sin^3 x \cos x dx$ $u = \sin x$

$$\frac{du}{dx} = \cos x$$

$$\int \frac{4}{5} u^3 \cancel{\cos x} \frac{du}{\cancel{\cos x}} \leftarrow \frac{du}{\cos x} = dx$$

$$\int \frac{4}{5} u^3 du$$

$$= \frac{4u^4}{20}$$

$$= \frac{u^4}{5}$$

$$= \frac{\sin^4 x}{5} + C$$

Question 3

a) $z_1 = 5+3j$ $z_2 = 2-7j$

(i) $z_1 + z_2 = 7-4j$

(ii) $z_1 - z_2 = 3+10j$

(iii) $\frac{z_1}{z_2} \Rightarrow \frac{5+3j}{2-7j} \times \frac{2+7j}{2+7j}$

$$= \frac{10+35j+6j+21j^2}{4+14j-14j-49j^2}$$

$$= \frac{-11+41j}{53}$$

$$= \frac{-11}{53} + \frac{41}{53}j$$

(iv) $(z_1)^2 \times (z_2)$

$$= (5+3j)(5+3j)(2-7j)$$

$$= (25+30j+9j^2)(2-7j)$$

$$= (16+30j)(2-7j)$$

$$= 32-112j+60j-210j^2$$

$$= 242-52j$$

$$b) \quad 12x^2 + 5x + 7 = 0$$

$$x = \frac{-5 \pm \sqrt{(5)^2 - 4(12)(7)}}{2(12)}$$

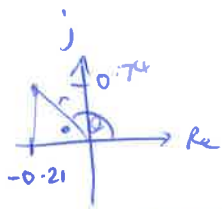
$$x = \frac{-5 \pm \sqrt{-311}}{24}$$

$$x = \frac{-5 \pm \sqrt{311} \sqrt{-1}}{24}$$

$$x = \frac{-5 \pm 17.64j}{24}$$

$$x = \frac{-5}{24} + \frac{17.64}{24}j$$

$$x = -0.21 + 0.74j$$



$$r = \sqrt{0.21^2 + 0.74^2}$$

$$r = \sqrt{0.59}$$

$$r = 0.77$$

$$\tan \theta = \frac{0.74}{0.21}$$

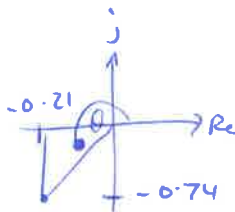
$$\theta = 74.16$$

$$\theta = 105.84^\circ$$

$$0.77(\cos 105.84^\circ + j \sin 105.84^\circ)$$

$$x = \frac{-5}{24} - \frac{17.64}{24}j$$

$$x = -0.21 - 0.74j$$



$$r = 0.77$$

$$\tan \theta = \frac{0.74}{0.21}$$

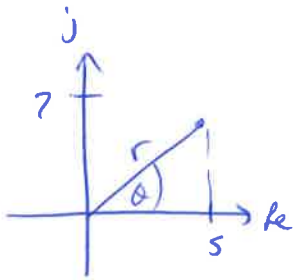
$$\theta = 74.16$$

$$\theta = 254.16^\circ$$

$$0.77(\cos 254.16^\circ + j \sin 254.16^\circ)$$

c) $(5 + 7j)^6$

(i)



$$r = \sqrt{7^2 + 5^2}$$

$$r = \sqrt{74}$$

$$r = 8.60$$

$$\tan \theta = 7/5$$

$$\theta = 54.46^\circ$$

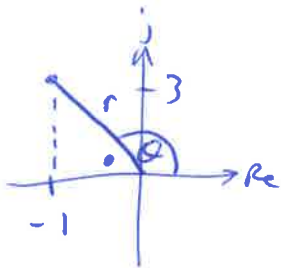
$$\text{DMT} = (8.60)^6 (\cos 6 \times 54.46^\circ + j \sin 6 \times 54.46^\circ)$$

$$= 404567.24 (\cos 326.76 + j \sin 326.76)$$

$$= 338372.69 - 221762.42j$$

⑤

(ii) $(-1 + 3j)^5$



$$r = \sqrt{1^2 + 3^2}$$

$$r = \sqrt{10}$$

$$r = 3.16$$

$$\tan \theta = \frac{3}{1}$$

$$\theta = 71.57^\circ$$

$$\theta = 108.43^\circ$$

$$\text{DMT} = (3.16)^5 (\cos 5 \times 108.43^\circ + j \sin 5 \times 108.43^\circ)$$

$$= 315.09 (\cos 542.15 + j \sin 542.15)$$

$$= -314.87 - 11.82j$$

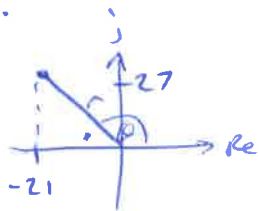
⑤

d) 'Cartesian' form

e) $5 + 3j$

f) $(2+3j)(3-6j)(-1+j)$
 $(6-12j+9j-18j^2)(-1+j)$
 $(\cancel{24}-3j)(-1+j)$
 $-24+24j+3j-3j^2$
 $\underline{-21+27j}$

in PF:



$$r = \sqrt{21^2 + 27^2}$$

$$r = \sqrt{1170}$$

$$\underline{r = 34.21}$$

$$\tan \theta = \frac{27}{21}$$

$$\theta = 52.13^\circ$$

$$\theta = 127.87^\circ$$

$$\underline{34.21 (\cos 127.87^\circ + j \sin 127.87^\circ)}$$