

question one.

$L = 1.5m$

$P = 3500N$

$w = 1500Nm^{-1}$

$EI = 2.2 \times 10^6 Nm^2$

a)

$$\text{deflection} = \frac{FL^3}{3EI} + \frac{wL^4}{8EI} = \frac{3500 \times 1.5^3}{3 \times (2.2 \times 10^6)} + \frac{-1500 \times 1.5^4}{8 \times (2.2 \times 10^6)}$$

$$y = -2.22 \times 10^{-3} mm$$

b) $5mm = \frac{F \times 1.5^3}{3(2.2 \times 10^6)} + \frac{1500 \times 1.5^4}{8(2.2 \times 10^6)}$

$$F = \frac{5 \times 3(2.2 \times 10^6)}{1.5^3} - \frac{-1500 \times 1.5^4}{8(2.2 \times 10^6)}$$

$F = +97.78 \times 10^5 N$

question two

a) $P=0$, When atmospheric pressure acts on one side of the wall, make this 0.

$r = r_i = 0.05m$
 $\sigma_r = -p = -100 \times 10^6 Nm^{-2}$

b) $D = 45mm$
 $35mm$
 $p = 115 \times 10^5 Nm^{-2}$

inner surface
 $\sigma_r = \frac{a-b}{r^2}$

$r = r_o = 0.07m$
 $\sigma_r = 0$

inner
 $r = r_i = d/2$
 $\sigma_r = -p$

outer
 $r = r_o = d/2$
 $\sigma_r = 0$
 $p = 0$

b) $r_i = 0.0175$

$r_o = 0.0225$

inner

$-r = r_i = 0.0175$

$\sigma_R = -P = -115 \times 10^3$

outer

$-r = r_o = 0.0225$

$\sigma_R = 0$

$\sigma_R = a - \frac{b}{r^2}$

$-115 \times 10^3 = a - \frac{b}{0.0175^2}$

$-115 \times 10^3 \times 0.0175^2 = b - 0.0175^2 a$

$-115 \times 10^3 \times 0.0175^2 = 0.0225^2 a - 0.0175^2 a$

$-115 \times 10^3 \times 0.0175^2 = a (0.0225^2 - 0.0175^2)$

$a = \frac{115 \times 10^3 \times 0.0175^2}{(0.0225^2 - 0.0175^2)}$

$a = 17.6 \times 10^6$

$\sigma_R = a - \frac{b}{r^2}$

$0 = a - \frac{b}{0.0225^2}$

$b = 0.0225^2 a$

$b = 0.0225^2 a$

$b = 0.0225^2 \times 17.6 \times 10^6$

$b = 89.1 \times 10^2$

c) $\sigma_{ci} = a + \frac{b}{r^2} = 17.6 \times 10^6 + \frac{89.1 \times 10^2}{0.0175^2}$
 $= 46.7 \times 10^6 \text{ Nm}^{-2}$

$\sigma_{co} = a + \frac{b}{r^2} = 17.6 \times 10^6 + \frac{89.1 \times 10^2}{0.0225^2}$
 $= 35.2 \times 10^6 \text{ Nm}^{-2}$

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Question three

$$\epsilon(x) = -10 \times 10^{-6}$$

$$\epsilon(y) = 13 \times 10^{-6}$$

$$\nu = 0.2$$

$$E = 165 \times 10^9 \text{ Pa}$$

$$a) \sigma_1 = \frac{E}{1 - \nu^2} (\epsilon_1 + \nu \epsilon_2) = \frac{165 \times 10^9}{1 - 0.2^2} \times (-10 \times 10^{-6} + 0.2 \times 13 \times 10^{-6})$$
$$\sigma_1 = -1271875 \text{ Pa (compressive)}$$

$$\sigma_2 = \frac{E}{1 - \nu^2} (\epsilon_2 + \nu \epsilon_1) = \frac{165 \times 10^9}{1 - 0.2^2} \times (13 \times 10^{-6} + 0.2 \times -10 \times 10^{-6})$$
$$\sigma_2 = 1890825 \text{ Pa (tensile)}$$

$$b) \sigma_x = 20 \times 10^6 \text{ Pa}$$

$$\sigma_y = -15 \times 10^6 \text{ Pa}$$

$$\epsilon_1 = \frac{1}{E} (\sigma_1 - \nu \sigma_2) = \frac{1}{165 \times 10^9} \times (20 \times 10^6 - (0.2 \times -15 \times 10^6))$$
$$\epsilon_1 = 1.39 \times 10^{-4}$$

$$\epsilon_2 = \frac{1}{E} (\sigma_2 - \nu \sigma_1) = \frac{1}{165 \times 10^9} \times (-15 \times 10^6 - (0.2 \times 20 \times 10^6))$$
$$\epsilon_2 = -1.15 \times 10^{-4}$$

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question four.

$$d = 200\text{mm} \rightarrow 0.2\text{m}$$

$$L = ~~115\text{mm}~~ 390\text{mm} \rightarrow 0.39\text{m}$$

$$t = 1.5\text{mm} \rightarrow 0.0015$$

$$\sigma_{\max} = 100\text{MPa} \rightarrow 100 \times 10^6\text{Pa}$$

$$\eta_L = 0.9$$

$$\eta_c = 0.6$$

$$\sigma_c = \frac{pD}{2t\eta_c} \rightarrow p = \frac{\sigma_c \times 2t\eta_c}{D}$$

$$P = \frac{100 \times 10^6 \times (2 \times 0.0015 \times 0.6)}{0.2}$$

$$P = 450000\text{ Pa}$$

$$\sigma_L = \frac{pD}{4t\eta_L} \rightarrow p = \frac{\sigma_L \times 4t\eta_L}{D}$$

$$P = \frac{100 \times 10^6 \times (4 \times 0.0015 \times 0.9)}{0.2}$$

$$P = 2700000\text{ Pa}$$

question five

$$L = 0.35 \text{ m}$$

$$p = 50 \times 10^6 \text{ Pa}$$

$$v = 0.3$$

$$E = 165 \times 10^9 \text{ Pa}$$

$$\begin{aligned} a) k &= \frac{E}{3(1-2v)} = \frac{165 \times 10^9}{3(1-2 \times 0.3)} \\ &= 13.75 \times 10^{11} \text{ Nm}^2 \end{aligned}$$

$$\begin{aligned} k &= \frac{\sigma}{\epsilon_v} \rightarrow \frac{-p}{\epsilon_v} \rightarrow \epsilon_v = \frac{-p}{k} = \frac{-50 \times 10^6}{13.75 \times 10^{11}} \\ \epsilon_v &= -3.64 \times 10^{-5} \end{aligned}$$

$$\begin{aligned} \text{volume} &= 0.35^3 \\ &= 0.042875 \text{ m}^3 \end{aligned}$$

$$\begin{aligned} \epsilon_v &= \frac{\Delta v}{v} \rightarrow \Delta v = v \epsilon_v = 0.042875 \times -3.64 \times 10^{-5} \\ &= -1.56 \times 10^{-6} \text{ m}^3 \end{aligned}$$

b) an example of orthotropic materials is wood, this means that the elastic property is different in all three directions.