

DATE: 11 May 2020

TIME: 1:00 PM

MODULE: Advanced Stress Analysis & FEA (M6X00420)

STUDENT NAME: António Maria Paquete Januário

STUDENT NUMBER: 1805566

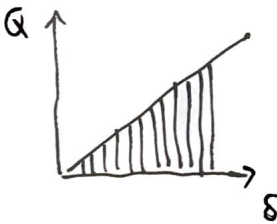
QUESTION 2

a) (i) ~~scribbles~~

$$\tau = G\gamma, \quad \tau = \text{SHEAR STRESS} = \frac{Q}{A}$$

$$\Leftrightarrow \frac{Q}{A} = G\gamma \quad \Leftrightarrow \gamma = \frac{Q}{AG}$$

(ii)

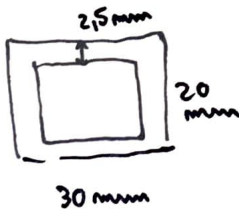


$$U = \frac{1}{2} Q \cdot \delta, \quad \text{WHERE } \delta = \gamma \cdot L, \text{ because } \gamma = \frac{\delta}{L}$$

$$U = \frac{1}{2} Q \cdot \gamma \cdot L = \frac{1}{2} Q \frac{Q}{AG} \cdot L \quad \Leftrightarrow$$

$$\Leftrightarrow U = \frac{Q^2 L}{2AG}$$

b) (i)



$$L = 2,4 \text{ m}$$

$$\sigma = 480 \text{ MPa}$$

$$E = 190 \text{ GPa}$$

$$U = ?$$

$$U = \frac{P^2 L}{2AE} = \frac{6^2 AL}{2E}$$

$$A = (0,03 \times 0,02) - (0,025 \times 0,015) = 2,25 \times 10^{-4} \text{ m}^2$$

$$U = \frac{6^2 AL}{2E} = \frac{(480 \times 10^6)^2 \times 2,25 \times 10^{-4} \times 2,4}{2 \times 190 \times 10^9} \approx 327,4 \text{ N}$$

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$$b)_{(ii)} \quad P.E. = mgh \quad (1) \quad , \quad mg = W$$

$$U = \frac{\sigma^2 AL}{2E} \quad (2) \quad , \quad AL = V$$

WHEN (1) = (2)

$$mgh = \frac{\sigma^2 AL}{2E} \quad (\Rightarrow) \quad Wh = \frac{\sigma^2 V}{2E} \quad (\Rightarrow)$$

$$(\Rightarrow) \quad h = \frac{\sigma^2 V}{2EW}$$

$$(iii) \quad m = 20 \text{ kg} \\ h = 0,45 \text{ m}$$

$$\sigma = \sqrt{\frac{h \cdot 2EW}{V}}$$

$$W = mg = 20 \times 9,81 = 196,2 \text{ N}$$

$$V = AL = 2,25 \times 10^{-4} \times 2,4 = 5,4 \times 10^{-4} \text{ m}^3$$

$$\sigma = \sqrt{\frac{0,45 \times 2 \times 190 \times 10^9 \times 196,2}{5,4 \times 10^{-4}}} \approx 249,26 \text{ MPa}$$

iv)

$$\sigma = 249,26 \text{ MPa} \\ L = 2,4 \text{ m}$$

$$\text{IF, } h = \frac{\sigma^2 AL}{2EW} \quad (\Rightarrow) \quad A = \frac{h \cdot 2EW}{\sigma^2 L} \quad (\Rightarrow) \quad \pi(R^2 - r^2) = \frac{h \cdot 2EW}{\sigma^2 L} \quad (\Rightarrow)$$

$$(\Rightarrow) \quad R^2 - r^2 = \frac{h \cdot 2EW}{\sigma^2 L \pi} \quad (\Rightarrow) \quad r = \sqrt{R^2 - \frac{h \cdot 2EW}{\sigma^2 L \pi}}$$

$$(\Rightarrow) \quad r = \sqrt{0,01^2 - \frac{0,45 \times 2 \times 190 \times 10^9 \times 196,2}{(249,26)^2 \times 2,4 \times \pi}}$$

iv) IN
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b)
(iv)

$$\phi = 20 \text{ mm} = 0,02 \text{ m}$$

$$L = 2,4 \text{ m}$$

$$\sigma = 249,26 \text{ MPa}$$

$$E = 190 \text{ GPa}$$

$$W = 196,2$$

$$\text{If, } h = \frac{\sigma^2 AL}{2EW} \Leftrightarrow A = \frac{h 2EW}{\sigma^2 L} \Leftrightarrow \pi(R^2 - r^2) = \frac{h 2EW}{\sigma^2 L} \Leftrightarrow$$

$$\Leftrightarrow R^2 - r^2 = \frac{h 2EW}{\sigma^2 L \pi} \Leftrightarrow r = \sqrt{R^2 - \frac{h 2EW}{\sigma^2 L \pi}}$$

$$r = \sqrt{0,01^2 - \frac{0,45 \times 2 \times 190 \times 10^9 \times 196,2}{(249,26 \times 10^6)^2 \times 2,4 \times \pi}} = 5,3 \times 10^{-3} \text{ m} = 5,3 \text{ mm}$$

$$\text{THICKNESS} = R - r = 10 - 5,3 = 4,7 \text{ mm}$$

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QUESTION 3



$$\phi = 75 \text{ mm} = 0,075 \text{ m}$$

$$\text{THICK} = 5 \text{ mm} = 5 \times 10^{-3} \text{ m}$$

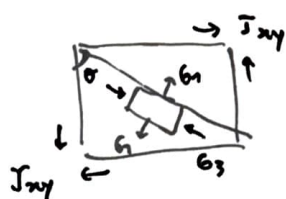
$$\sigma = 325 \text{ MN/m}^2 \quad \eta = 6$$

$$\nu = 0,3$$

$$\eta = \frac{\text{TENSILE STRENGTH}}{\text{ALLOWABLE WORKING STRESS}} \quad (\Rightarrow) \quad \text{ALLOWABLE STRESS} = \frac{325 \times 10^6}{6} = 54,17 \text{ MPa}$$

$$\sigma_\theta = \tau_{xy} \cdot \sin 2\theta, \text{ MAXIMUM FOR } \theta = 45^\circ$$

$$\text{THEREFORE, } \sigma_{1,3} = \pm \tau_{xy} \sin 90^\circ$$



$$\sigma_1 = \tau_{xy}$$

$$\sigma_3 = -\tau_{xy}$$

$$\tau / \eta = \frac{\gamma}{R} = \frac{G\theta}{L} \quad \therefore \gamma = \frac{T \cdot R}{G} \quad , \quad \gamma = \frac{\pi}{32} (D^4 - d^4) = 7,5 \times 10^{-7} \text{ m}^4$$

$$\gamma = \frac{0,075/2 \cdot T}{7,5 \times 10^{-7}} = 50000 T$$

$$a) \quad \sigma_1 - \sigma_3 = \sigma_A \quad , \quad \text{because } \eta = \frac{\sigma_y}{\sigma_A} \Rightarrow \sigma_A = \frac{\sigma_y}{\eta} \quad (\text{PREVIOUSLY CALCULATED})$$

$$\tau_{xy} - \tau_{xy} = 54,17 \times 10^6 \quad (\Rightarrow) \quad 2 \cdot (50000 \cdot T) = 54,17 \times 10^6$$

$$T = \frac{54,17 \times 10^6}{2 \cdot 50000} = 541,7 \text{ Nm}$$

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$$b) \left[\left(\frac{61}{0} - \frac{62}{0} \right)^2 + \left(\frac{62}{0} - 63 \right)^2 + (63 - 61)^2 \right] = 2 \sigma_a^2$$

$$\tau_{xy}^2 + (-(-\tau_{xy}))^2 + (-\tau_{xy} - \tau_{xy})^2 = 2 \sigma_a^2$$

$$\tau_{xy}^2 + \tau_{xy}^2 + (2\tau_{xy})^2 = 2 \sigma_a^2$$

$$2 \tau_{xy}^2 + (2\tau_{xy})^2 = 2 \sigma_a^2$$

~~$$2 \cdot (50000)^2 + (900000T)^2 = 2 \cdot (54,17 \times 10^6)^2$$~~

$$2 \tau_{xy}^2 + 4 \tau_{xy}^2 = 2 \sigma_a^2$$

$$6 \tau_{xy}^2 = 2 \sigma_a^2$$

$$\tau_{xy} = \sqrt{\frac{1}{3} \sigma_a^2}$$

$$50000T = \sqrt{\frac{1}{3} \cdot (54,17 \times 10^6)^2}$$

$$T = 625,5 \text{ Nm}$$

$$c) \phi_e = 50 \text{ mm}$$

$$\phi_i = 44 \text{ mm}$$

$$\eta = 2,75$$

$$\sigma_y = 880 \text{ MN/m}^2$$

$$\gamma = \frac{T \cdot R}{J_y}, \quad \gamma = \frac{\pi}{32} (0,05^4 - 0,044^4) = 2,46 \times 10^{-7} \text{ m}^4$$

$$\gamma = \frac{0,05/2 \cdot T}{2,46 \times 10^{-7}} = 101626 T$$

$$\tau_{xy} = \sqrt{\frac{1}{3} \left(\frac{\sigma_y}{\eta} \right)^2} \quad (\Rightarrow) 101626 \cdot T = \sqrt{\frac{1}{3} \left(\frac{880 \times 10^6}{2,75} \right)^2}$$

$$T = 1817,96 \text{ Nm}$$

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d)

LOW CARBON STEEL

$$T_{\text{STEEL}} = 625,5 \text{ Nm}$$

TITANIUM

$$T_{\text{TITA}} = 1817,96 \text{ Nm}$$

$$\% \text{ DIFFERENCE} = \frac{T_{\text{TITA}} - T_{\text{STEEL}}}{\frac{T_{\text{TITA}} + T_{\text{STEEL}}}{2}} \times 100 = \frac{1817,96 - 625,5}{\frac{1817,96 + 625,5}{2}} \times 100 = 97,6 \%$$

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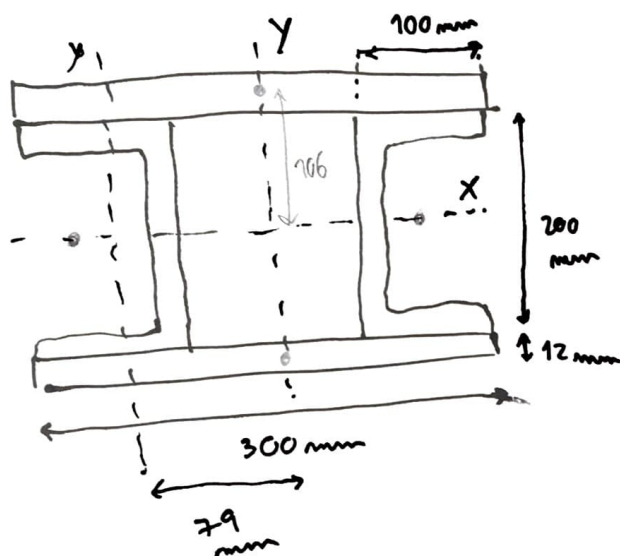
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QUESTION 4



$$I_{yy} = 26,2 \times 10^6 \text{ mm}^4$$

$$I_{xx} = 1,57 \times 10^6 \text{ mm}^4$$

$$A = 4400 \text{ mm}^2$$

$$L = 8 \text{ m}$$

$$\alpha = 1/7500$$

$$E = 200 \text{ GPa}$$

$$\sigma = 320 \text{ MPa}$$

$$a) I_{\text{TOTAL}} = 2 \times I_{\text{PLATES}} + 2 I_{\text{CHANNELS}}$$

$$I_{xx} = 2 \times \left(\frac{b d^3}{12} + A h^2 \right) + 2 I_{xx \text{ CHANNELS}}$$

$$I_{xx} = 2 \times \left(\frac{300 \times 12^3}{12} + (300 \times 12) \cdot 106^2 \right) + 2 \times (26,2 \times 10^6) = 13\,338\,5600 \text{ mm}^4$$

$$I_{yy} = 2 \times \left(\frac{b d^3}{12} \right) + 2 \times \left(I_{yy \text{ CHANNELS}} + A \frac{h^2}{4} \right)$$

$$I_{yy} = 2 \times \left(\frac{12 \times 300^3}{12} \right) + 2 \times \left(1,57 \times 10^6 + 4400 \times 79^2 \right) = 11\,206\,0800 \text{ mm}^4$$

$$I_{yy} < I_{xx}$$

IT WILL BUCKLE ABOUT THE Y AXIS !

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$$b) K = \sqrt{\frac{I}{A}} = \sqrt{\frac{112060800}{2 \times (300 \times 12) + 2 \times 4400}} = 83,69 \text{ mm}$$

$$c) \text{ SLENDERNESS RATIO} = \frac{L}{K} = \frac{8000}{83,69 \times 10^{-3}} = 95,59$$

d) PINNED - PINNED

$$P_e = \frac{\pi^2 E I}{L^2} = \frac{\pi^2 \times 200 \times 10^9 \times (112060800 \times 10^{-12})}{8^2} = 3456,2 \text{ kN}$$

$$e) P_R = \frac{G_y A}{1 + \alpha (L/K)^2} = \frac{320 \times 10^6 \times (2 \times (300 \times 12) + 2 \times 4400) \times 10^{-6}}{1 + \frac{1}{7500} (95,59)^2} = 2308 \text{ kN}$$

f) VERTICAL PLATES

$$I_{xx_{\text{new}}} = \frac{6 \times 224^3}{12} = 5619712 \text{ mm}^4$$



$$I_{yy_{\text{new}}} = \frac{224 \times 6^3}{12} + (224 \times 6) \cdot 153^2 = 31465728 \text{ mm}^4$$

NEW TOTAL MOMENT OF INERTIA:

$$I_{xx_{\text{total}}} = I_{xx} + 2 I_{xx_{\text{new}}} = 133385600 + 2 \times 5619712 = 144625024 \text{ mm}^4$$

$$I_{yy_{\text{total}}} = I_{yy} + 2 I_{yy_{\text{new}}} = 112060800 + 2 \times 31465728 = 174992256 \text{ mm}^4$$

NEW STRUCTURE WILL BUCKLE ABOUT X AXIS!

$$P_e = \frac{\pi^2 E I}{L^2} = \frac{\pi^2 \times 200 \times 10^9 \times (144625024 \times 10^{-12})}{8^2} = 4460,5 \text{ kN}$$

∴ THERE WAS AN INCREASE OF EULER BUCKLING LOAD FROM 3456,2 kN TO 4460,5 kN!