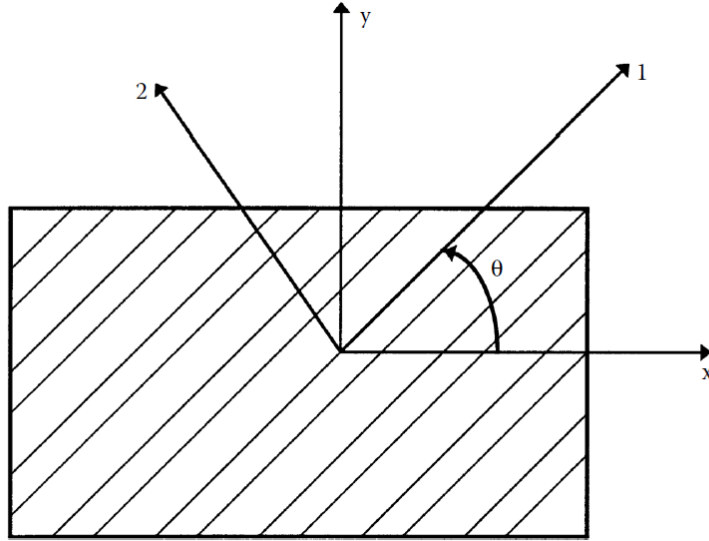


Macromechanical analysis of a lamina – Angle Plys

We have looked at calculating stress and strain for a uni-directional laminate. However due to the fact that the transverse stiffness is generally much lower in most laminates, some of the lamina are placed at an angle. As such it is necessary to develop the stress-strain relationship for an angle lamina. Consider the below lamina where the global axis are in x and y and the fibers are aligned at an angle theta such that the 1 direction is in the fiber orientation and the 2 direction is perpendicular to the fibers.



It can be shown that the stresses in the local axis relative to the global axis are given by;

$$\sigma_1 = \sigma_x \cos^2 \theta + \sigma_y \sin^2 \theta + 2\tau_{xy} \sin \theta \cos \theta \quad [1]$$

$$\sigma_2 = \sigma_x \sin^2 \theta + \sigma_y \cos^2 \theta - 2\tau_{xy} \sin \theta \cos \theta \quad [2]$$

$$\tau_{12} = -\sigma_x \cos \theta \sin \theta + \sigma_y \cos \theta \sin \theta + \tau_{xy} (\cos^2 \theta - \sin^2 \theta) \quad [3]$$

If equations 1-3 are combined and written in matrix for we get the following;

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix} = \begin{bmatrix} c^2 & s^2 & 2sc \\ s^2 & c^2 & -2sc \\ -sc & sc & c^2 - s^2 \end{bmatrix} \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix}$$

Where $c = \cos \theta$ and $s = \sin \theta$

This can also be written as;

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} = [T]^{-1} \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix},$$

Where $[T]$ is called the transformation matrix and is defined as;

$$[T] = \begin{bmatrix} c^2 & s^2 & 2sc \\ s^2 & c^2 & -2sc \\ -sc & sc & c^2 - s^2 \end{bmatrix},$$

Since

$$\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & 0 \\ S_{12} & S_{22} & 0 \\ 0 & 0 & S_{66} \end{bmatrix} \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix}$$

If we want to convert this to a local co-ordinate axis, we simply include the transformation matrix;

$$\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & 0 \\ S_{12} & S_{22} & 0 \\ 0 & 0 & S_{66} \end{bmatrix} \begin{bmatrix} c^2 & s^2 & 2sc \\ s^2 & c^2 & -2sc \\ -sc & sc & c^2 - s^2 \end{bmatrix} \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix}$$

Examples

For a 60° angle lamina of Graphite/Epoxy find

- a) The reduced compliance matrix
- b) The local stresses
- c) The local strains

The material properties and the loading conditions are as follows;

$$E_1 = 181 \text{ GPa}$$

$$E_2 = 10.3 \text{ GPa}$$

$$\nu_{12} = 0.28$$

$$G_{12} = 7.17 \text{ GPa}$$

$$\sigma_x = 2 \text{ MPa}$$

$$\sigma_y = -3 \text{ MPa}$$

$$\tau_{xy} = 4 \text{ MPa.}$$

Notes adapted from/further reading:

Mechanics of composite materials, 2nd edition, Autar K Kaw