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LOCATION: SWANSEA

LLEOLIAD: ABERTAWE

LEVEL 4

LEFEL 4

**EXAMINATION PERIOD
JAN 2021**

**MODULE CODE: ACHA4000
ANALYTICAL METHODS EXAM**

TIME: 1 ½ HOURS

(PLUS HALF AN HOUR TO SUBMIT SOLUTIONS – 2 HOURS IN TOTAL)

ANSWER TWO OUT OF THREE QUESTIONS

Equipment or materials required –

Calculators allowed

Formula booklet provided

SHOW ALL WORKINGS

IMPORTANT INSTRUCTIONS FOR ONLINE EXAMINATIONS:

- *You will need to submit a digital photograph or scan of each page of workings*
- *Please submit these to Moodle (underneath where you accessed the exam paper)*
- *Put the time, date, module code and your student number at the top of each page*
- *Number each page ‘x of y’*
- *If you have any issues, email richard.sutton@uwtsd.ac.uk with a contact number and they will call you with advice*

QUESTION ONE – MATRICES

a) Consider the following matrices:

$$A = \begin{pmatrix} 8 & -1 & 7 \\ -5 & 2 & 5 \\ 3 & 0 & 2 \end{pmatrix} \quad B = \begin{pmatrix} -2 & 8 & 0.5 \\ 2 & 5 & 2 \\ 0 & -9 & 1 \end{pmatrix} \quad C = \begin{pmatrix} 2 & 5 \\ -6 & 0 \\ 1 & 12 \end{pmatrix} \quad D = \begin{pmatrix} 8 & 14 \\ 0.25 & 21 \end{pmatrix}$$

Calculate the following, if mathematically possible:

(i) AB

(ii) BA

(iii) AC

(iv) CA

(v) |A|

(vi) |B|

(vii) A+B

(viii) AB+D

(20 marks)

b) If the following is true:

$$\begin{pmatrix} 15x^2 + 4x - 17 & 1 & 11 \\ 2 & 0 & -2 \\ 2 & -2 & x^4 \end{pmatrix} = \begin{pmatrix} y & 1 & 11 \\ 2 & 5 & -2 \\ 2 & -2 & y^2 \end{pmatrix}$$

Solve for x .

(7 marks)

c) Given the following matrix:

$$\omega = \begin{pmatrix} 2 & 4x & -5 \\ 4 & x & 0 \\ 1 & (x+1) & 3 \end{pmatrix}$$

Solve for x if the determinant of $\omega = 2$.

(5 marks)

d) Use Cramer's rule to solve the following simultaneous equations:

$$2x + 3y + 2z = 14$$

$$x + 2y + 5z = 4$$

$$3x + y - 2z = 19$$

(8 marks)

QUESTION TWO – COMPLEX NUMBERS

a) Given the following complex numbers, $z_1 = 3 + 7j$, $z_2 = 2 - 10j$ and $z_3 = -4 - j$, calculate the following, expressing your answers in rectangular form:

(i) $z_1 + z_3$ (ii) $z_1 - z_3$ (iii) $\frac{z_1}{z_2}$ (iv) $z_1 - \bar{z}_2$ (v) $(z_3)^3 - \bar{z}_1$

(11 marks)

b) Solve the following quadratic equation, expressing your answers in Polar Form:

$$8x^2 = -3x - 18$$

(10 marks)

c) Use de Moivre's' theorem to calculate the following, giving your answers in rectangular form:

(i) $(3 - 10j)^6$

(ii) $(-3 + 8j)^5$

(12 marks)

d) Write down the value of $j^5 \times j^3$.

(1 mark)

e) Given the following complex numbers, $z_1 = 9 + 4j$, $z_2 = -7 - 5j$, $z_3 = 2 - 5j$, calculate the following:

$$\bar{z}_1 z_2 \bar{z}_3$$

Expressing your answer in Polar Form.

(5 marks)

f) If a complex number is denoted by X , what is denoted by $\bar{\bar{X}}$?

(1 marks)

QUESTION THREE – INTEGRATION

a) Integrate the following:

(i) $\int (x^5 - 2x)(5x^3 + 6x) \, dx$

(ii) $\int (e^{5x} + 1)(e^{3x} + 2) \, dx$

(iii) $\int_0^2 (3x^3 + 2x)^3 + 4x + 3 \, dx$

(iv) $\int_2^4 (2x + 3)(x + 5) \, dx$

(18 marks)

b) Use integration by parts to integrate the following:

(i) $\int 4x \cos 3x \, dx$

(ii) $\int_1^2 3x e^{2x} \, dx$

(9 marks)

c) Integrate the following:

(i) $\int \frac{21x^2 + 10x - 3}{7x^3 + 5x^2 - 3x - 1} \, dx$

(ii) $\int \frac{8}{3x + 10} \, dx$

(iii) $\int \frac{1}{x^2 + 9} \, dx$

(4 marks)

d) Integrate $\int \frac{7}{9} \sin x \cos x \, dx$ between the limits $\frac{\pi}{2}$ (upper limit) and $\frac{\pi}{5}$ (lower limit), using the substitution $u = \sin x$.

(4 marks)

e) Integrate $\int \frac{3x}{\sqrt{2x-1}} \, dx$ using the substitution $u^2 = 2x - 1$.

(5 marks)