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Investigation of Three Forces in Equilibrium

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Contents

1	Introduction	2
2	Methodology	2
2.1	Equipment	2
2.2	Experimental Procedure	3
3	Results	3
3.1	Uncertainty and error	5
4	Discussion	5
5	Conclusions	8

List of Figures

1	The apparatus setup [1]	2
2	A graph of average spring length L/m plotted against spring tension/ N with a line of best fit to the origin.	4
3	A graph of average spring length L/m plotted against the spring constant k	6
4	An overlay of the graph from Figure 4 onto the expected behaviour of a spring obeying Hookes Law.	7
5	An overlay of the graph from Figure 4 onto the graph showing the change in spring constant over spring length.	8

List of Tables

1	Data recorded from experiment.	3
2	Derived experimental results.	4
3	Results showing mass, spring length, and calculated spring constant	6

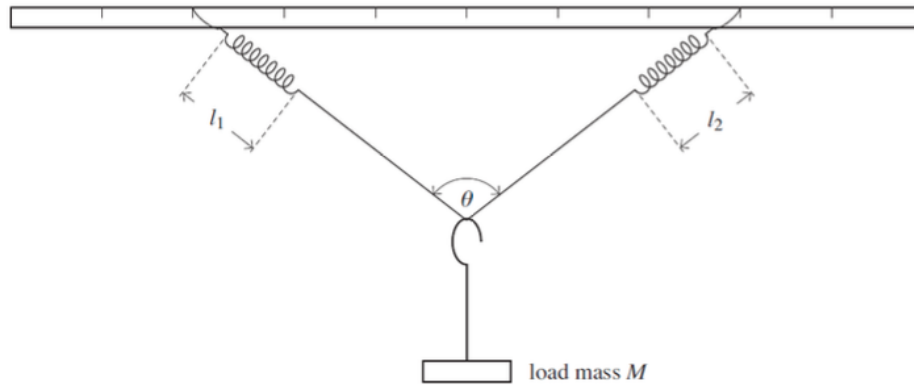


Figure 1: The apparatus setup [1]

1 Introduction

The aim of this experiment was to investigate the relationship between the angle of two strings and the weight that these strings were supporting.

The purpose of this was to allow the identification of physical principles and relations in the solution of engineering problems.

After the experiment was complete the results were used in the manipulation of equations to allow the production of numerical values in engineering units.

2 Methodology

2.1 Equipment

The experiment was carried out with four core pieces of apparatus:

1. A central supporting beam.
2. Two identical springs of equal length and material.
3. A set of weights.

The apparatus was set up as shown in Figure 2.1. Length measurements were taken by eye with a supplied $0.3m$ rule. The rule was un-supported and handheld. Angle measurements were taken with a hand held protractor with precision to one degree.

Table 1: Data recorded from experiment.

Mass/kg	Experiment 1			Experiment 2		
	l1/m	l2/m	θ /degree	l1/m	l2/m	θ /degree
0.1	0.080	0.075	120	0.080	0.076	118
0.2	0.105	0.100	110	0.105	0.100	106
0.3	0.132	0.128	100	0.130	0.122	97
0.4	0.158	0.146	92	0.152	0.146	92
0.5	0.178	0.169	87	0.176	0.164	87
0.6	0.190	0.187	83	0.196	0.184	84

Prior to beginning the experiment several equipment checks were carried out. The relaxed length of each spring was measured twice. Spring 1 had an average relaxed length of $0.0495m$. Spring 2 also had an average relaxed length of $0.0495m$. These lengths support the uncertainty estimate. The weights were also split and individually weighed. There were eleven weights each weighing $0.05kg$ and one weight hanger, also weighed at $0.05kg$.

2.2 Experimental Procedure

For each load of mass M (see Figure ??) two measurements of $l1$ and $l2$ were taken. These measurements were the length of the springs including end loops. An additional measurement of angle θ was also taken. These measurements were recorded in a table.

The starting mass M was $0.1kg$ and the mass was increased by a further $0.1kg$ for each successive test until a total mass M of $0.6kg$ was reached. The experiment was carried out twice.

3 Results

Data recorded from the experiment was used to derive a further three measurements, shown in Table 2. Experiment measurements are shown in Table 1.

Equations in Table 2 were derived in several ways. For every value of M , L was calculated as the mean of the two $l1$ and $l2$ values. For every value of M , θ_m is the mean value of θ .

For every value of M , the final calculation of $\frac{M}{\cos(\frac{\theta_m}{2})}$ was also carried out. This is the tension in each spring.

Table 2: Derived experimental results.

Mass/kg	L/m	θ_m/degree	$M/\cos(\theta_m/2)$
0.1	0.080	119	0.197
0.2	0.105	108	0.340
0.3	0.131	98.5	0.460
0.4	0.155	92	0.576
0.5	0.177	87	0.689
0.6	0.193	83.5	0.804

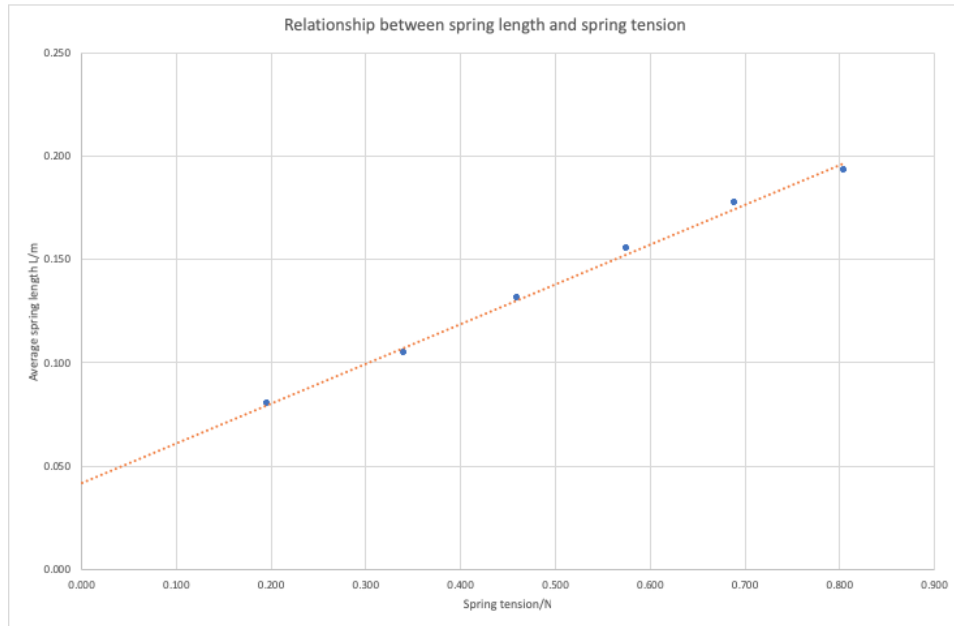


Figure 2: A graph of average spring length L/m plotted against spring tension/N with a line of best fit to the origin.

3.1 Uncertainty and error

The uncertainty of a piece of equipment is generally considered to be half of the smallest measurement [2]. Therefore the uncertainty of all measurements made with the rule should be taken as $\pm 0.5mm$ or $\pm 0.0005m$. Angle measurements were taken with a hand held protractor with precision to one degree. Uncertainty for degree measurements should be considered $\pm 0.5^\circ$.

4 Discussion

If an object is at rest and is in a state of equilibrium it is at static equilibrium. If an object is at equilibrium then the net force acting on the object is 0N. Therefore, if all forces are added together as vectors the resultant force should be 0N.

Figure 3 shows that there is a linear relationship between spring length and spring tension. The gradient of the graph can be calculated using

$$\frac{\Delta y}{\Delta x}$$
$$\frac{0.113}{0.607} = 0.186$$

Using the gradient it is then possible to calculate the y-intercept of the graph.

$$c = y - mx$$

$$c = 0.08 - (0.186)(0.197)$$

$$c = 0.043$$

From the results of this experiment it is also possible to calculate the spring constant using Hookes Law [3]. Hookes Law takes the form of

$$F = kx$$

where F is the force applied to the spring, k is the spring constant, and x is the distance that the spring is stretched or compressed from its equilibrium or rest position.

From the procedure detailed above the values of F and x are known for every value of m . The equation can be rearranged as

$$k = \frac{F}{x}$$

Table 3: Results showing mass, spring length, and calculated spring constant

Mass/kg	L/m	k
0.1	0.080	15.823
0.2	0.105	17.518
0.3	0.131	17.945
0.4	0.155	18.509
0.5	0.177	19.160
0.6	0.193	20.438

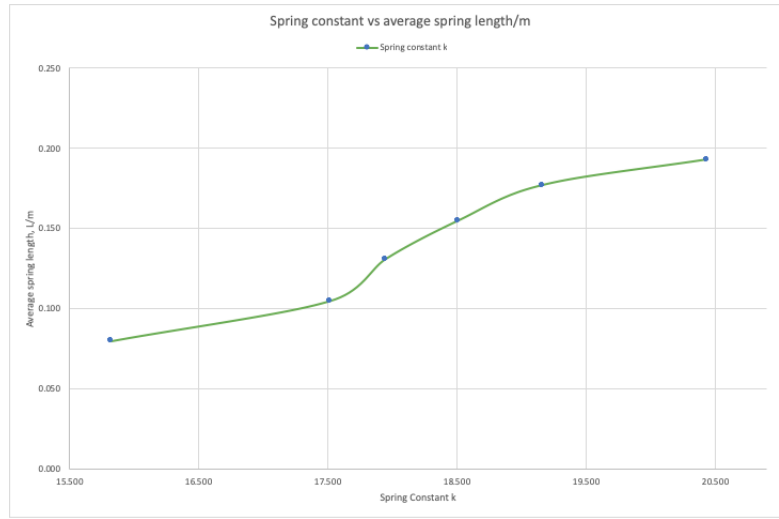


Figure 3: A graph of average spring length L/m plotted against the spring constant k .

The total F for each mass is calculated by

$$F = mg$$

However, as two springs are used in parallel the overall value of F is divided by 2 to give F for a single spring. x is calculated by subtracting each value of L from the average starting length of each spring - $0.049m$ in both cases.

Figure 4 shows that the spring constant does not increase in a linear fashion with weight. This is to be expected as Hooke's Law is very general in its application. Almost any object that can be distorted pushes or pulls with a restoring force proportional to the displacement from equilibrium towards the equilibrium position, for very small displacements.

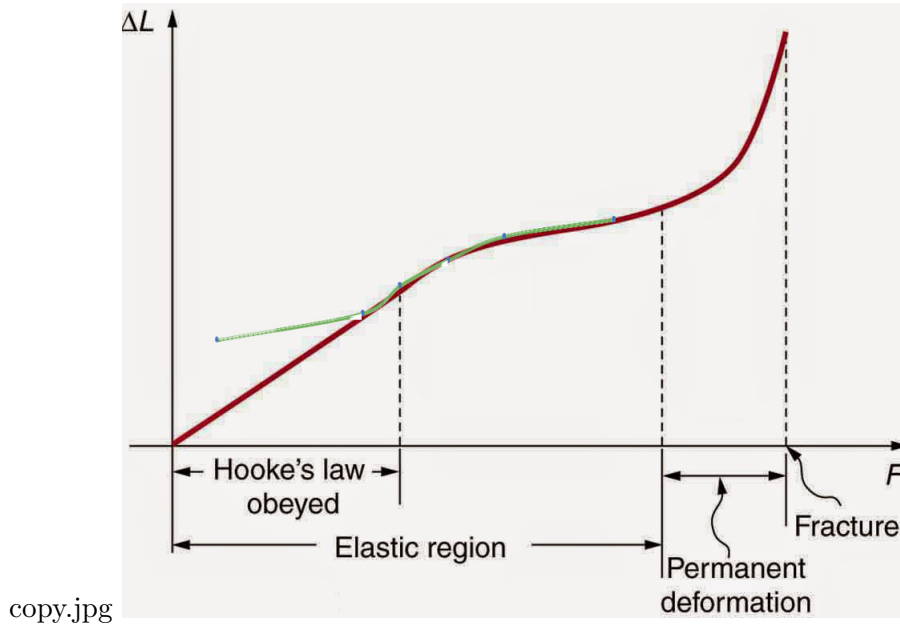


Figure 4: An overlay of the graph from Figure 4 onto the expected behaviour of a spring obeying Hookes Law.

However, when the displacements become large, the elastic limit is reached. The stiffer the object, the smaller the displacement it can tolerate before the elastic limit is reached. If you distort an object beyond the elastic limit, you are likely to cause permanent distortion or to break the object. The calculated spring constant approximately fits this behaviour as shown in Figure 4. From this it is possible to conclude that the weights used were within the elastic regions of the springs.

In order to observe any nonlinearity of spring behaviour over the given weights a further analysis was carried out. For this analysis each previous spring length was considered to be a new equilibrium position, giving a Δx for each corresponding added load of $\Delta m = 0.1kg$. Using these values the Δk was calculated in the form

$$\Delta k = \frac{\frac{(\Delta mg)}{2}}{\Delta x}$$

These values of Δk were then plotted in Figure 4. It is possible to observe in this figure that the change in the spring constant for each weight is not linear. Returning to Figure 4 it is possible to observe that the third point,

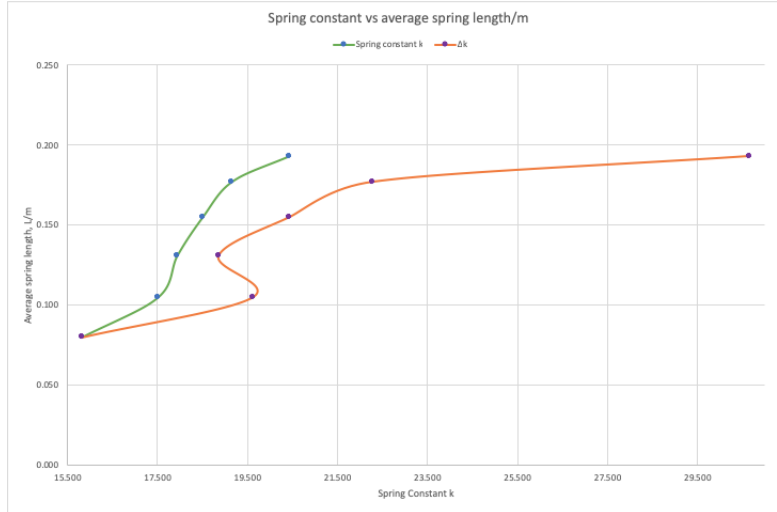


Figure 5: An overlay of the graph from Figure 4 onto the graph showing the change in spring constant over spring length.

which appears to move ‘backwards’, is approximately equal to the point at which the spring enters the elastic region.

5 Conclusions

An experiment was carried out to investigate the relationship between the angle of two strings and the weight that these strings were supporting.

From the results of this experiment it was possible to calculate the average spring constant and comment on the likely elastic region that the spring was in.

The uncertainty of recorded measurements is generally small and so conclusions can be considered relatively reliable.

References

- [1] G. Owen, “Investigation of three forces in equilibrium - assignment specification.”
- [2] “Uncertainty and Error - UBC Wiki.” [Online]. Available: https://wiki.ubc.ca/Uncertainty_and_Error
- [3] “What is Hooke’s Law?” [Online]. Available: <https://phys.org/news/2015-02-law.html>