

Recap Suspension

Ride Frequency $f = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$

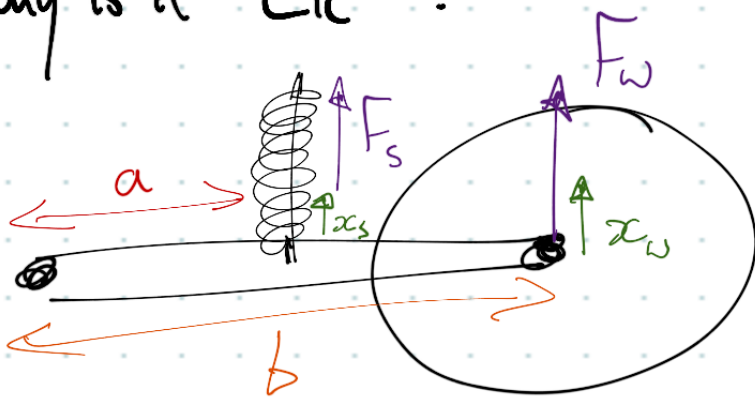
Critical Damping $C_{crit} = 2\sqrt{kM}$

Damping Ratio $\zeta = \frac{c}{C_{crit}}$

Zeta

Motion Ratio (Link Ratio)

Why is it LR^2 ?



$$F_w = F_s \times \left(\frac{a}{b}\right)$$

force at wheel is less

$$x_w = x_s \left(\frac{b}{a}\right)$$

Travel at the wheel is more

Now, Spring stiffness is K

$$F = Kx$$

So $F_w = K_w x_w$

We can describe the stiffness at the wheel as

$$K_w = \frac{F_w}{x_w} = \frac{F_s \left(\frac{a}{b}\right)}{x_s \left(\frac{b}{a}\right)} = \frac{F_s \left(\frac{a}{b}\right) \left(\frac{a}{b}\right)}{x_s}$$

$$\Rightarrow \frac{F_s \left(\frac{a}{b}\right)^2}{x_s} \quad \text{Since } F_s = K_s x_s$$

$$K_w = \frac{F_s}{x_s} \left(\frac{a}{b}\right)^2 = K_s \left(\frac{a}{b}\right)^2$$

So $K_w = K_s (\text{Link Ratio})^2$

for link ratios < 1 or $K_s = K_w \times LR^2$

So far looks ...

For real forks (k)
displacement (x)



$$F \approx K\pi$$

①

$$N_g \cos \epsilon = K x$$

vertical spring rate (k')
|| displacement (?)



$$F = kx$$

②

$$N_f = K' x \cos \varepsilon$$

So put $N_f =$ into (1)

~~$K' \approx \cos \epsilon \cos \epsilon = K \approx$~~

So

$$K = \cos^2(\epsilon) K'$$

$$K = K' \cos^2(\epsilon)$$

Example Suspension Analysis

Mass of bike + rider - 215 kg

Weight Distribution - 60% front

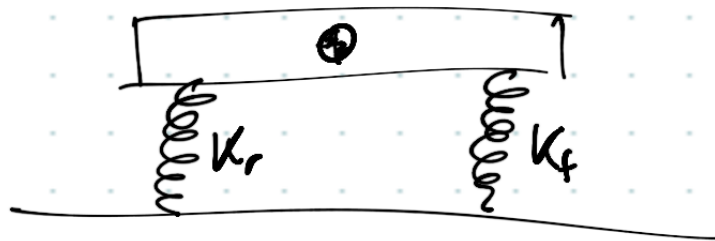
Front unsprung mass - 8 kg

Rear unsprung mass - 12 kg

So front sprung mass = $(0.6 \times 215) - 8 = 121 \text{ kg}$

Rear Sprung mass = $(0.4 \times 215) - 12 = 74 \text{ kg}$

Consider



So we want a specific 'Ride frequency'

Say $f \sim 2.1 \text{ Hz}$

Now we use

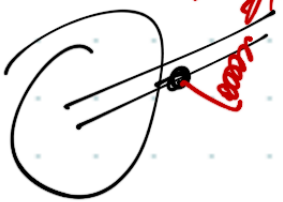
$$f = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$$

$$\begin{aligned}
 \text{To find } K_f &= (2\pi f)^2 \times M_{sf} \\
 &= (2\pi \times 2.1)^2 \times 121 \\
 &= 21.066 \text{ N/mm (Vertically)}
 \end{aligned}$$

$$\begin{aligned}
 \text{To find } K_e &= (2\pi f)^2 \times M_{se} \\
 &= (2\pi \times 2.1)^2 \times 74 \\
 &= \underline{12.883 \text{ N/mm}}
 \end{aligned}$$

These are the vertical stiffnesses

link Ratio system



Lets say we have a link Ratio of 2.4 (average). So if shock moves 10mm, wheel moves 24mm

we want K_s , so $K_s = K_w \times 2.4^2$

$$K_s = 12.88 \times 2.4^2 = \underline{74.2 \text{ N/mm}}$$

Now for the front:

Assuming a 21° rake angle

$$K_s = K_w \times \cos^2 \epsilon$$

$$(K = K' \cos^2 \epsilon)$$

$$\text{So } K_s = 21.066 \times \cos^2(21)$$

$$K_s = \underline{18.36 \text{ N/mm}}$$

Remember that the front generally has 2 springs

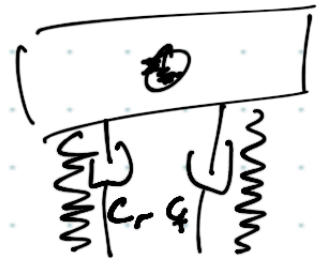
$$\text{Fork Spring stiffness} = 18.36 / 2$$

$$\Rightarrow \underline{9.18 \text{ N/mm}}$$

So what about damping?

We can start again with vertical stiffness
(there are other ways, but easy to get errors)

We first look at critically damping
the sprung mass.



$$\text{Mass (Rear)} = 74 \text{ kg}$$

$$\text{Mass (Front)} = 121 \text{ kg}$$

So

$$\text{The Rear Critical Damping (Vertical)} = 2\sqrt{KM}$$

$$C_{crit_R} = 2\sqrt{12,883 \times 74}$$

$$C_{crit_R} = 1952.78 \text{ Ns/m}$$

what is this at the shock

$$C_{crit_{R,S}} = 1952.78 \times 2.4^2 = 11248 \text{ Ns/m}$$

Now for the front ---

$$\text{front Critical Damping} = 2\sqrt{KM}$$

$$C_{crit,f} = 2\sqrt{18360 \times 121}$$

$$C_{crit,f} = 2980.9 \text{ Ns/m}$$

which at the fork, relates to

$$C_{crit,f,f} = 2980.9 \times \cos^2(21)$$

$$= 2598 \text{ Ns/m}$$

So for the front

$$\text{each fork spring} \sim 9.2 \text{ N/mm}$$

$$\text{Critical damping for this spring} \sim C_{crit}/2$$

$$\sim \underline{1299 \text{ Ns/m}}$$