

Q1)

a) i)

$$y = \frac{x^2}{\sqrt{2x+1}}$$

$$y = x^2 (2x+1)^{-1/2}$$

$$u = 2x+1$$

$$u' = 2$$

$$y = x^2 u^{-1/2} \quad \frac{dy}{dx} = \frac{du}{2}$$

$$\frac{du}{2} = dx$$

$$y = x^2 (2x+1)^{-1/2}$$

$$u = x^2$$

$$du = 2x$$

$$v = (2x+1)^{-1/2}$$

$$v' = (-1/2)(2x+1)^{-3/2}$$

$$v' = u^{-1/2}$$

$$2x^{-3/2} - 1/2 (2x+1)^{3/2}$$

$$- (2x+1)^{3/2}$$

$$v \frac{du}{dx} + u \frac{dv}{dx}$$

$$= (2x+1)^{-1/2} \times 2x + x^2 \times (-1/2)(2x+1)^{-3/2}$$

$$2x(2x+1)^{-1/2} - x^2(2x+1)^{-3/2}$$

$$\frac{2x}{\sqrt{2x+1}} - \frac{x^2}{3\sqrt{2x+1}}$$

ii)

$$y = \sqrt{x^2+1}$$

$$y = (x^2+1)^{1/2}$$

$$u = x^2+1$$

$$\frac{dy}{dx} = \frac{1}{2} u^{-1/2} \quad \frac{du}{dx} = 2x$$

$$x^2 = u = x^2+1$$

$$x = \sqrt{u-1}$$

$$\frac{dy}{dx} = \frac{1}{2} u^{-1/2} \quad \frac{du}{dx} = 2x$$

$$\frac{1}{2} (x^2+1)^{-1/2} \times \frac{1}{2x} dx$$

$$(x^2+1)^{-1/2}$$

$$\frac{1}{4x\sqrt{x^2+1}}$$

$$\text{iii) } y = 3x^{3/2} + 3x^{-2} - 6x$$

$$\frac{dy}{dx} = \frac{9}{2}x^{1/2} - 6x^{-3} - 6$$

$$\text{iv) } y = \sec^3 4x$$

$$u = 4x$$

$$u = \sec \frac{4x}{1}$$

$$y = \sec^2 4x$$

$$\frac{dy}{dx}$$

$$\frac{du}{dx} = 4$$

$$\frac{du}{dx} = \sec x \tan x$$

$$\tan^2 4x$$

$$\sec^2 4x \tan 4x$$

$$\frac{du}{dx}$$

$$\sec^2 4x \tan 4x$$

$$\text{v) } y = x + \sin 2x$$

$$\frac{dy}{dx} = 1 + 2\cos 2x$$

$$u = \sec 4x$$

$$du = 4 \sec 4x \tan 4x$$

$$v = \tan 4x$$

$$dv = 4 \sec^2 4x$$

$$4 \sec^2 4x$$

$$\text{vi) } y = \frac{1 - 10x}{(2x-1)^2}$$

$$\frac{y}{v}$$

$$\frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

$$\frac{[(2x-1)^2 \times -10] - [(1-10x)(2x-1)]}{((2x-1)^2)^2}$$

$$u = 1 - 10x$$

$$du = -10$$

$$v = (2x-1)^2$$

$$\frac{dv}{dx} = 2 \times (2x-1) \times 2$$

$$dv = 4(2x-1)$$

$$da = 2$$

$$-10x$$

$$1$$

$$2x$$

$$-20x$$

$$2x$$

$$-1$$

$$-10x$$

$$-1$$

$$2(2x-1) \times 2$$

$$4(2x-1)$$

$$8x-4$$

$$\frac{1}{2} u^2 \frac{dy}{dx}$$

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$$\frac{-10(2x-1)^2 - [-20x^2 - 8x - 1]}{(2x-1)^4}$$

$$(2x-1)^4$$

$$-10(2x-1)^2 + 20x^2 + 8x + 1$$

$$\begin{array}{r} 2x \quad -1 \\ 2x \quad 4x^2 \quad -2x \\ -1 \quad -2x \quad +1 \end{array}$$

$$-10(4x^2 - 4x + 1) + 20x^2 + 8x + 1$$

$$-40x^2 - 40x - 10 + 20x^2 + 8x + 1$$

$$(2x-1)^4$$

$$\frac{dy}{dx} = \frac{-20x^2 - 32x - 9}{(2x-1)^4}$$

b) $y = \frac{1}{x^2} + \cos 2x$ $\frac{d^5 y}{dx^5}$

$$y = x^{-1} + \cos 2x$$

$$\frac{d^4 y}{dx^4} = +24x^{-5} + 16 \sin 2x$$

$$\frac{dy}{dx} = -x^{-2} - 2 \sin 2x$$

$$\frac{d^5 y}{dx^5} = -120x^{-6} - 32 \sin 2x$$

$$\frac{d^2 y}{dx^2} = +2x^{-3} - 4 \cos 2x$$

$$-8(15x^{-6} - \sin 2x)$$

$$\frac{d^3 y}{dx^3} = -6x^{-4} + 8 \sin 2x$$

$$f'(x) = \operatorname{cosec} x$$

$$c) f(x) = \ln(\operatorname{cosec} x - \cot x)$$

$$u = \operatorname{cosec} x - \cot x$$

$$f(x) \ln u \quad \frac{du}{dx} = \frac{-\operatorname{csc} x}{-\operatorname{cosec} x \cot x - \operatorname{cosec}^2 x}$$

$$f(x) = \frac{1}{u}$$

$$f'(x) = \frac{1}{\operatorname{cosec} x - \cot x}$$

$$\frac{1}{4} \quad \frac{1}{8} \quad \frac{1}{12} \quad \frac{1}{24} \quad \frac{1}{32}$$

$$\frac{1}{-\operatorname{cosec} x \cot x - \operatorname{cosec}^2 x} \rightarrow \frac{1}{\operatorname{cosec} x - \cot x}$$

$$\begin{array}{rcl} & -\operatorname{cosec} x \cot x & -\operatorname{cosec}^2 x \\ \operatorname{cosec} x & -\operatorname{cosec}^2 x \cot x & -\operatorname{cosec}^3 x \\ -\cot & -\operatorname{cosec} x \cot^2 x & +\operatorname{cosec}^2 x \cot \end{array}$$

$$\frac{\operatorname{cosec} x - \cot x}{-\operatorname{cosec} x \cot x - \operatorname{cosec}^2 x}$$

$$\frac{1}{-\operatorname{cosec} x \cot x^2 - \operatorname{cosec}^3 x}$$

$$f(x) \ln(\operatorname{cosec} x - \cot x)$$

$$u = \operatorname{cosec} x^2 - \cot x$$

$$= -\operatorname{cosec} x \cot x - \operatorname{cosec}^2 x$$

$$\ln u \quad \frac{1}{u}$$

$$\frac{du}{dx} = \frac{-\operatorname{cosec} x \cot x - \operatorname{cosec}^2 x}{-\operatorname{cosec} x \cot x - \operatorname{cosec}^2 x}$$

$$\frac{1}{\operatorname{cosec} x - \cot x} \times \frac{1}{-\operatorname{cosec} x \cot x - \operatorname{cosec}^2 x}$$

Q2)

a) i) $\int \frac{2x}{(4+3x^2)} dx$

$$\int 2x(4+3x^2)^{-1}$$

$$u = 4+3x^2 \quad \frac{du}{dx} = 6x$$

$$\frac{du}{dx} = 6x$$

$$\int 2x u^{-1} \quad \frac{du}{6x} = dx$$

$$\frac{2}{6} \int u^{-1} du$$

$$\frac{2}{6} \int \frac{1}{u} du$$

$$\frac{2}{6} [\ln u] + c$$

$$\frac{2}{6} \ln(4+3x^2) + c$$

ii) $\int_0^4 e^{\frac{1}{2}x} dx$

$$[2e^{\frac{1}{2}x}]_0^4$$

$$2e^{\frac{1}{2}(4)} - 2e^{\frac{1}{2}(0)} = \frac{12.478}{3}$$

$$2e^2 - 2e^0$$

$$2e^2 - 2$$

$$\text{iii)} \int_0^4 \frac{5x+13}{(2x+1)(x+4)} dx$$

$$\begin{array}{r} 2x^2 + 1 \\ x \\ 2x^2 + 4x \\ \hline 8x + 4 \end{array}$$

$$\int_0^4 \frac{5x+13}{2x^2+9x+4} dx$$

$$\int_0^4 (5x+13)(2x^2+9x+4)^{-1} dx$$

$$u = 2x^2 + 9x + 4$$

$$\frac{du}{dx} = 4x + 9$$

$$\int_0^4 (5x+13) u^{-1} dx$$

$$\int_0^4 5x u^{-1} + 13 u^{-1} \frac{du}{4x+9}$$

$$\int uv' = uv - \int u'v$$

$$\int_0^4 (5x+13)(2x^2+9x+4)^{-1} dx \quad \begin{array}{l} u' = 5x+13 \\ u = \end{array}$$

$$v = \frac{5x^2}{2} + 13x$$

$$u = (2x^2+9x+4)^{-1} \quad u' = -1$$

$$du = -1 du$$

$$\frac{2x^2+9x+4}{4x+9} \ln(2x^2+9x+4)$$

$$\int uv' = (2x^2+9x+4)^{-1} \times \left(\frac{5x^2}{2} + 13x \right) - \int (4x+9) \ln(2x^2+9x+4) \times \left(\frac{5x^2}{2} + 13x \right)$$

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$$f(x) = \frac{4}{x(1+4\ln x)^2} dx$$

$$\frac{4}{(16\ln^2 x + 8\ln x + 1)x}$$

$$f(x) = 16\ln^3 x + 8\ln^2 x + 8x$$

$$f(x) = 4(16\ln^3 x + 8\ln^2 x + 8x)^{-1} dx$$

$$\frac{4}{x(1+4\ln x)} dx$$

$$u = 1 + 4\ln x$$

$$\frac{du}{dx} = \frac{4}{x}$$

$$\int \frac{4}{x} \frac{1}{u} = \frac{4}{4} \frac{1}{u}$$

$$\int \frac{1}{u}$$

$$\int \ln u$$

$$\ln(1+4\ln x) + C$$

$$c) i) \int_{\ln 2}^{\ln 5} \frac{3e^{2x}}{\sqrt{e^x - 1}} dx$$

$$\int 3e^{2x} (e^x - 1)^{-1}$$

$$\int uv' = uv - \int u'v$$

$$u = \frac{3e^{2x}}{(e^x - 1)^{-1}} \quad v' = e^x$$

$$u' = e^x \ln e^x - 1 \quad v = \frac{1}{2} e^{2x}$$

$$(e^x - 1) 3e^{2x}$$

$$e^x \quad a = a^{-1} \\ da = \ln a$$

$$\int uv' = (e^x - 1)^{-1} \times \frac{3}{2} e^{2x} - \int e^x \ln e^x - 1 \times \frac{3}{2} e^{2x}$$

$$\int 3e^{2x} (e^x - 1) = \frac{3}{2} e^{2x} (e^x - 1)^{-1} - \int 3e^{2x} (e^x \ln e^x - 1)$$

$$\int 3e^{2x} (e^x - 1) = \frac{3}{2} e^{2x} (e^x - 1)^{-1} - \int -3e^{2x}$$

$$\left[\frac{\frac{3}{2} e^{2x}}{e^x - 1} + \frac{3}{2} e^{2x} \right]_{\ln 2}^{\ln 5}$$

$$\left[\frac{\frac{3}{2} e^{2 \ln 5}}{e^{\ln 5} - 1} + \frac{3}{2} e^{2 \ln 5} \right] - \left[\frac{\frac{3}{2} e^{2 \ln 2}}{e^{\ln 2} - 1} + \frac{3}{2} e^{2 \ln 2} \right]$$

$$\left[\frac{\frac{3}{2} \times 25}{4} + \frac{3}{2} \times 25 \right] - \left[\frac{\frac{3}{2} \times 4}{1} + \frac{3}{2} \times 4 \right]$$

$$\left[\frac{15}{8} + \frac{15}{4} \right] - [6 + 6]$$

$$\frac{45}{8} - 12 = -\frac{51}{8}$$

$$\frac{-i - 3 + 3i + 1}{-i + 3 - 3i + 1}$$

$$\frac{-i + 3 - 3i + 1}{-i + 3 - 3i + 1}$$

$$\frac{-4i + 4}{-4i + 4}$$

$$\frac{2i - 2}{-4i + 4}$$

$$\underline{\underline{-\frac{1}{2}i - \frac{1}{2}}}$$

$$\underline{\underline{-\frac{1}{2} - \frac{1}{2}i}}$$

$$\text{ii) } \frac{3}{(1+i)(2+i)} = \frac{3}{i^2 + 3i + 2}$$

$$\frac{3}{-1 - 3i + 2}$$

$$\frac{3}{3i + 1}$$

$$\frac{3 - 3i}{3 + i}$$

$$\text{iii) } \frac{1+i}{1-i} = (1+2i)(2+i) + \frac{3-i}{1+i}$$

$$\frac{1+i}{1-i} \times \frac{1+i}{1+i} = \frac{1+i^2}{1-i^2} = \frac{1-1}{1-(-1)} = \frac{0}{2} = 0$$

$$\frac{1+i}{1-i} \times \frac{1+i}{1+i} = \frac{1+i^2}{1-i^2} = \frac{1-1}{1-(-1)} = \frac{0}{2} = 0$$

$$\frac{1-i}{1+i} \times \frac{1-i}{1-i} = \frac{1-i^2}{1-i^2} = \frac{1-(-1)}{1-(-1)} = \frac{2}{2} = 1$$

$$\frac{1-i}{1+i} \times \frac{1-i}{1-i} = \frac{1-i^2}{1-i^2} = \frac{1-(-1)}{1-(-1)} = \frac{2}{2} = 1$$

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$$\begin{array}{r} 1 - i \\ 1 - i \\ \hline +i - i - i^2 \end{array}$$

$$\cancel{1+1}$$

$$\begin{array}{r} 3 - i \\ 1 - 3 - i \\ \hline -i - i + i^2 \end{array}$$

$$3 - 2i - 1$$

$$2 - 2i$$

$$2i - [2 + 6i] + 2 - 2i$$

$$\cancel{2i} - 2 - 6i + 2 - \cancel{2i}$$

$$\underline{-6i}$$

$$\text{iv) } \frac{2i(i-1)}{2i^2 - 2i} + \frac{(\sqrt{3} - i)}{\sqrt{3} - i} + \frac{(1+i)(1+i)}{2i}$$

$$-2$$

$$\begin{array}{r} 1 + i \\ 1 - i \\ \hline +i - i - i^2 \end{array}$$

$$2i$$

$$-2 - \cancel{2i} + \sqrt{3} - i + \cancel{2i}$$

$$\underline{-2 + \sqrt{3} - i}$$

$$\frac{-2 + \sqrt{3}}{-2 + \sqrt{3}} \times \frac{-2 - \sqrt{3}}{-2 - \sqrt{3}}$$

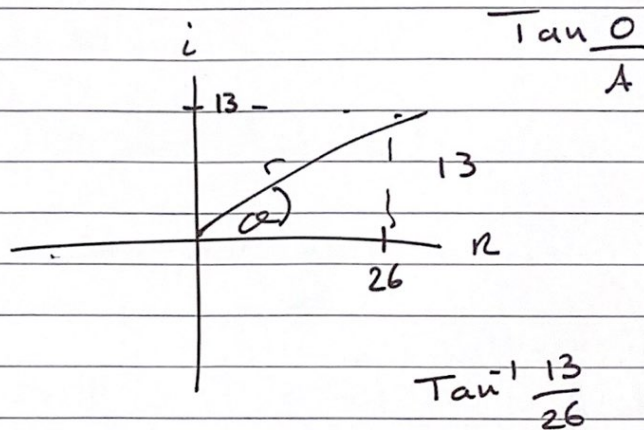
$$\begin{array}{r} -2 + \sqrt{3} \\ -2 - \sqrt{3} \\ \hline 4 - 2\sqrt{3} \\ -\sqrt{3} - 2\sqrt{3} - 3 \end{array}$$

$$b) (4+7i)(3-2i)$$

$$\begin{array}{r} 4 \quad 7i \\ 3 \quad 12 \quad 21i \end{array}$$

$$26 + 13i$$

$$\begin{array}{r} -2i \quad -8i \quad -14i^2 \\ 14 \end{array}$$



$$\angle 26.6^\circ$$

$$r = \sqrt{13^2 + 26^2}$$

$$r = \underline{\underline{13\sqrt{5}}}$$

$$c) 1 + 3i \quad 1 + i$$

1) a) $y = \frac{x^2}{(2x+1)^{1/2}} \frac{u}{v}$

$\frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$

$u = x^2$

$v = (2x+1)^{1/2}$

$du = 2x$

$dv =$

$\frac{(2x+1)^{1/2} \times 2x - x^2 \times \frac{1}{4} (2x+1)^{-1/2}}{(2x+1)}$

$\frac{2x(2x+1)^{1/2} - \frac{x^2}{4} (2x+1)^{-1/2}}{2x+1}$

$2x+1$

$\frac{1}{2} a^{1/2} \frac{1}{2} a^{-1/2} \frac{dx}{2}$

$\frac{1}{4} a^{-1/2}$

$\frac{1}{4} (2x+1)^{-1/2}$

1) iv) $y = \sec^3 4x$

$u = \sec 4x$

$du = 4 \sec 4x \tan 4x$

$y = \sec^3 4x \frac{du}{4 \sec 4x \tan 4x}$

$y = \frac{\sec 4x}{16 \tan 4x}$

$\frac{dy}{dx} = \frac{\sec 4x}{16 \tan 4x} \frac{u}{v}$

$du = 4 \sec 4x \tan 4x$

$dv = 32 \sec^2 4x$