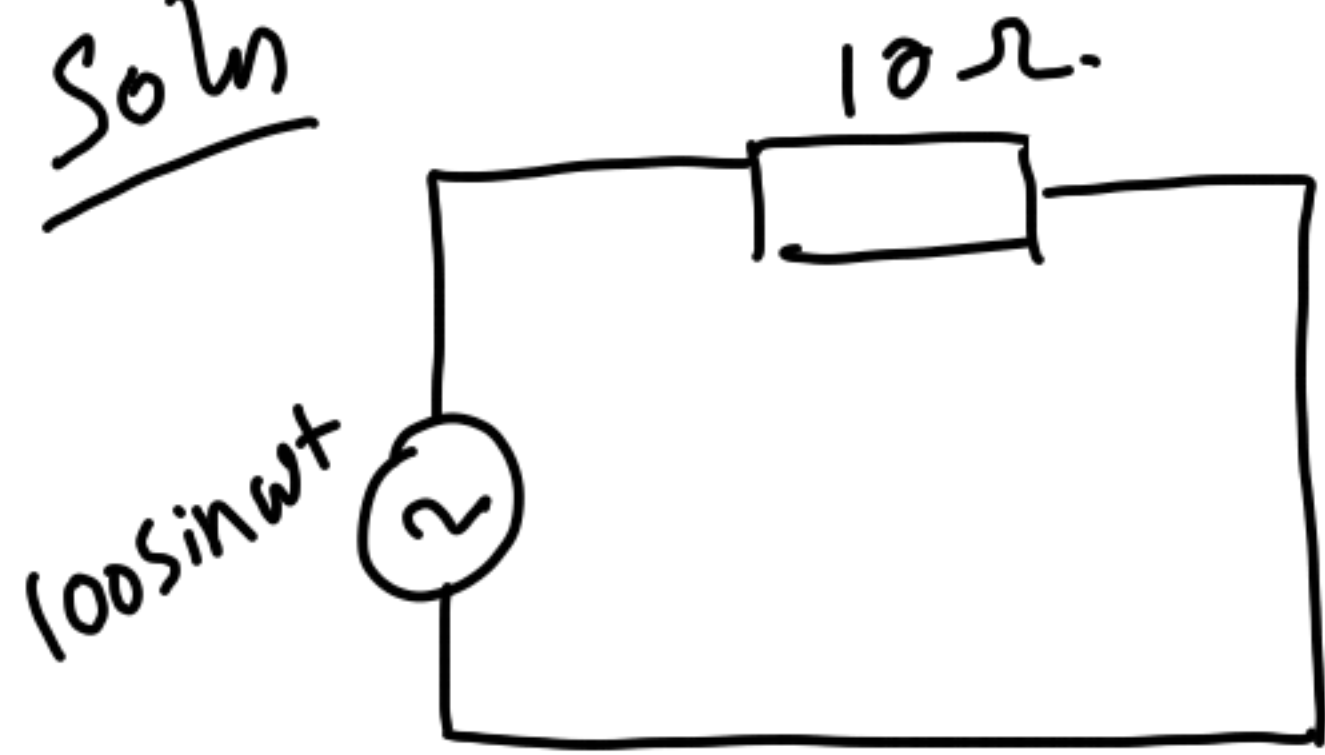


1) A Voltage of $100\sin\omega t$ is applied to a 10Ω resistor. Find the current, the instantaneous and the average power.

Soln



$$\text{Current } \hat{I} = \frac{V}{R} = \frac{100\sin\omega t}{10}$$

$$= 10\sin\omega t$$

$$\text{Instantaneous power } \tilde{P} = V \hat{I}$$

$$= 100\sin\omega t \times 10\sin\omega t$$

$$= 1000\sin^2\omega t \text{ W}$$

$$\begin{aligned} \text{Average Power } (P_{avg}) &= V \times \hat{I} \\ &= \frac{V_m}{\sqrt{2}} \times \frac{\hat{I}_m}{\sqrt{2}} \\ &= \frac{100}{\sqrt{2}} \times \frac{10}{\sqrt{2}} \\ &= 500 \text{ W} \end{aligned}$$

2) A voltage $V = 340 \sin 314t$ is applied to a circuit and the resulting current is $i = 42.5 \sin 314t$. Identify and hence find the value of the component. Find the value of average power consumed.

Soln. Given $V = 340 \sin 314t$
 $i = 42.5 \sin 314t$

From the equation: $\omega = 314$ for both i & V . As current and voltage are in phase $\phi = 0$, Resistive circuit.

$$R = ?$$

$$P_{avg}$$

$$R = \frac{V}{I} = \frac{340 \cancel{\sin 314t}}{42.5 \cancel{\sin 314t}} = \frac{340}{42.5}$$

$$R = 8 \Omega.$$

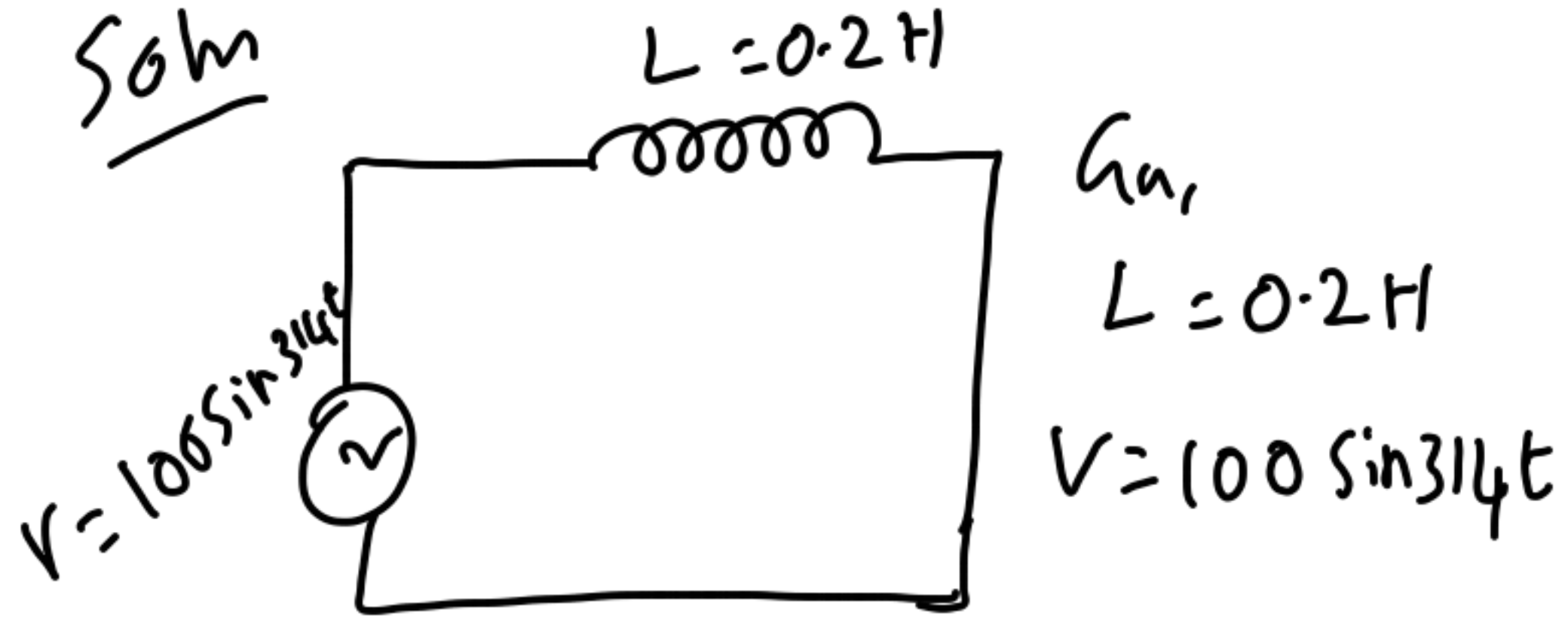
$$P_{avg} = \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}}$$

$$V_m = 340, \quad I_m = 42.5$$

$$= \frac{340}{\sqrt{2}} \cdot \frac{42.5}{\sqrt{2}} = 7225 \text{ W}$$

3) A pure inductance $L = 0.2 \text{ H}$ has an applied voltage of $v = 100 \sin 314t$. Find the instantaneous current, instantaneous and average power, inductive reactance and the RMS current.

Soln



Instantaneous current

$$i = \frac{1}{L} \int v dt = \frac{1}{0.2} \int 100 \sin 314t$$

$$= \frac{100}{0.2} \left[\frac{-1 \cos 314t}{314} \right]$$

Instantaneous power

$$P = V \cdot i = 100 \sin 314t \times (-) 1.592 \cos 314t$$

$$= -159.2 \sin 314t \cdot \cos 314t$$

$$= -159.2 \left(\frac{\sin 2 \times 314t}{2} \right)$$

$$\left[\sin \theta \cdot \cos \theta = \frac{\sin 2\theta}{2} \right]$$

$$P = -79.6 \sin 628t$$

Average power. $P_{avg} = 0$

Inductive Reactance. (X_L)

$$X_L = \omega L = 2\pi fL$$

Ans, $\sin 314t$, comparing with $\sin \omega t$.

$$\omega = 314$$

$$X_L = 314 \times 0.2 = 62.8 \Omega$$

$$\hat{I}_{RMS} = \frac{\hat{I}_m}{\sqrt{2}} = \frac{1.592}{\sqrt{2}} = 1.125 A$$

$$\hat{I}_{Rm} = \frac{\textcircled{V}}{X_L} = \frac{V_m / \sqrt{2}}{X_L} = \frac{100 \sin 314t / \sqrt{2}}{62.8} = 1.125 A$$

4) A coil of wire, which is considered as a pure inductance of 0.225H connected to a 120V , 50Hz source. Calculate (a) Inductive Reactance (b) Current (c) the instantaneous power delivered to the inductor (d) the average power. (e) Voltage and Current equation.

Soln G_n , $L = 0.225\text{H}$, $V = 120\text{V}$, $f = 50\text{Hz}$

a) Inductive Reactance (X_L)

$$X_L = 2\pi fL = 2\pi \times 50 \times 0.225 = 70.69 \sim$$

b) Current.

$$I = \frac{1}{L} \int V dt$$

w.k.t

$$V = V_m \sin \omega t$$

$$\omega = 2\pi f = 2\pi \times 50 = 314.2$$

$$V_m = V_{rms} \sqrt{2} = 120 \times \sqrt{2} = 169.7$$

$$V = 169.7 \sin 314.2 t$$

$$I = \frac{1}{L} \int 169.7 \sin 314.2 t dt$$

$$= \frac{169.7}{0.225} \int \sin 314.2 t dt$$

$$= \frac{169.7}{0.225} \left[-\frac{\cos 314.2 t}{314.2} \right]$$

$$\hat{I} = \frac{169.7}{0.225 \times 314.2} [\cos 314.2t]$$

$$\hat{I} = -2.4 \cos 314.2t$$

from the above equation.

$$\hat{I}_m = 2.4 \text{ A}$$

$$\text{So, } \hat{I} = \frac{\hat{I}_m}{\sqrt{2}} = \frac{2.4}{\sqrt{2}} = 1.7 \text{ A}$$

$$\hat{I} = \hat{I}_{\text{RMS}} = 1.7 \text{ A}$$

c) Instantaneous power

$$P = V \hat{I} = 169.7 \sin 314.2t \times (-2.4 \cos 314.2t)$$

$$= -407.28 \sin 314.2t \cdot \cos 314.2t$$

$$= -\frac{407.28}{2} \sin 2 \times 314.2t$$

$$P = 203.64 \sin 628.4t$$

$$P_{\text{avg}} = 0$$

Voltage equation

$$V = 169.7 \sin 314.2t$$

Current equation.

$$\hat{I} = -2.4 \cos 314.2t$$

5) A sinusoidal current 10 A , 60 Hz , is supplied to a pure capacitor of value $50\text{ }\mu\text{F}$. Write the equation for the voltage across the capacitor. Find the rms value of the voltage. Capacitive Reactance and the average power.

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi \times 60 \times 50 \times 10^{-6}}$$

$$X_C = 53\text{ }\Omega.$$

$$P_{\text{avg}} = 0$$

RMS Voltage

$$V_{\text{RMS}} = V = \hat{I} \cdot X_C = 10 \times 53$$

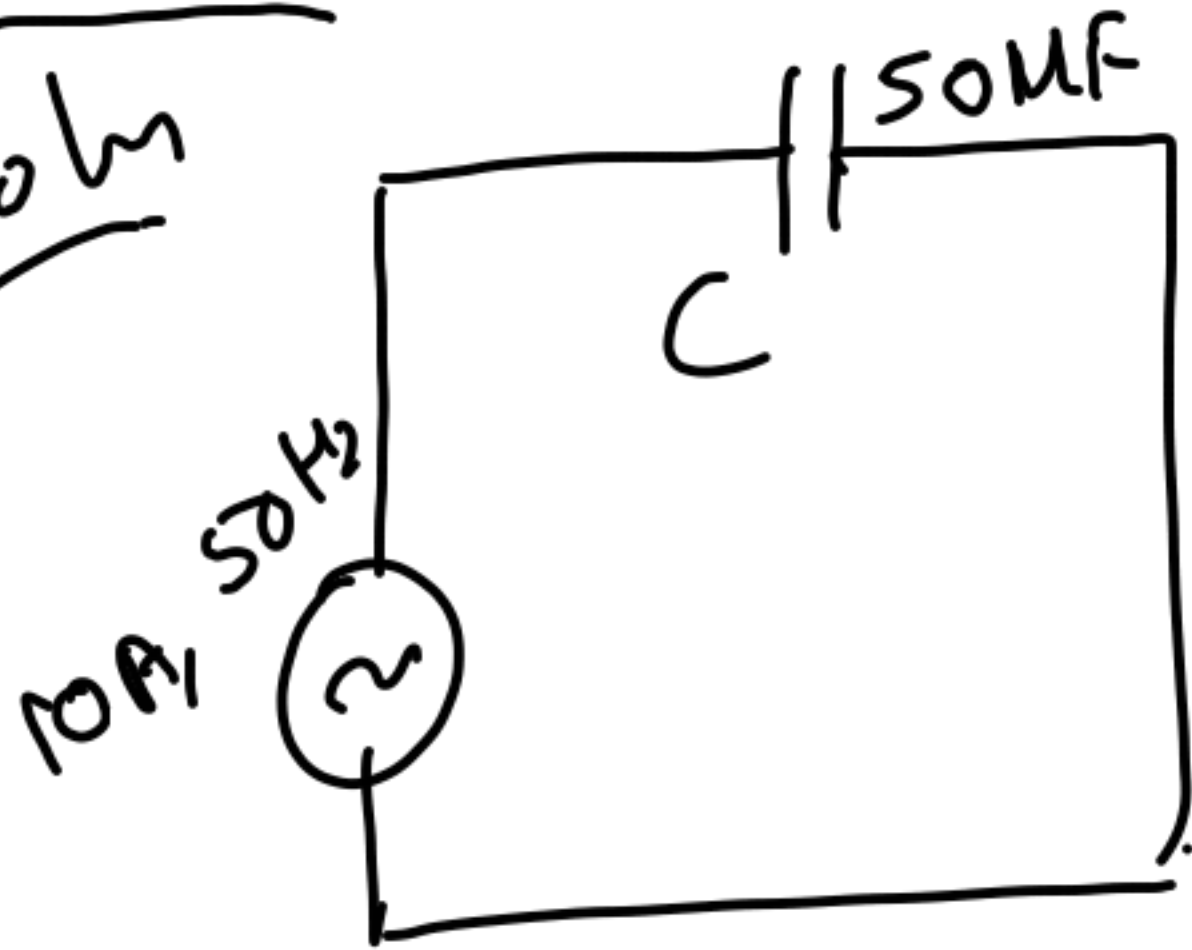
$$V_{\text{RMS}} = 530\text{ V}$$

$$V_m = V_{\text{RMS}} \sqrt{2} = 530 \sqrt{2} = 750\text{ V}$$

Voltage equation.

$$V = V_m \sin \omega t$$

Soln



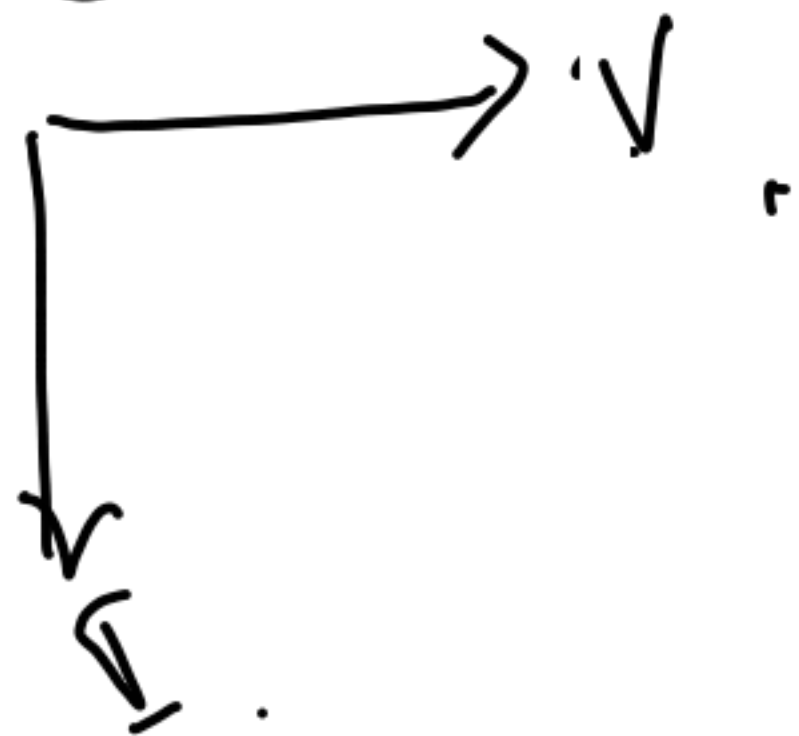
gm, $\hat{I} = 10\text{ A}$, $f = 60\text{ Hz}$,
 $C = 50\text{ }\mu\text{F}$

$$V = 750 \sin(2\pi f)t$$

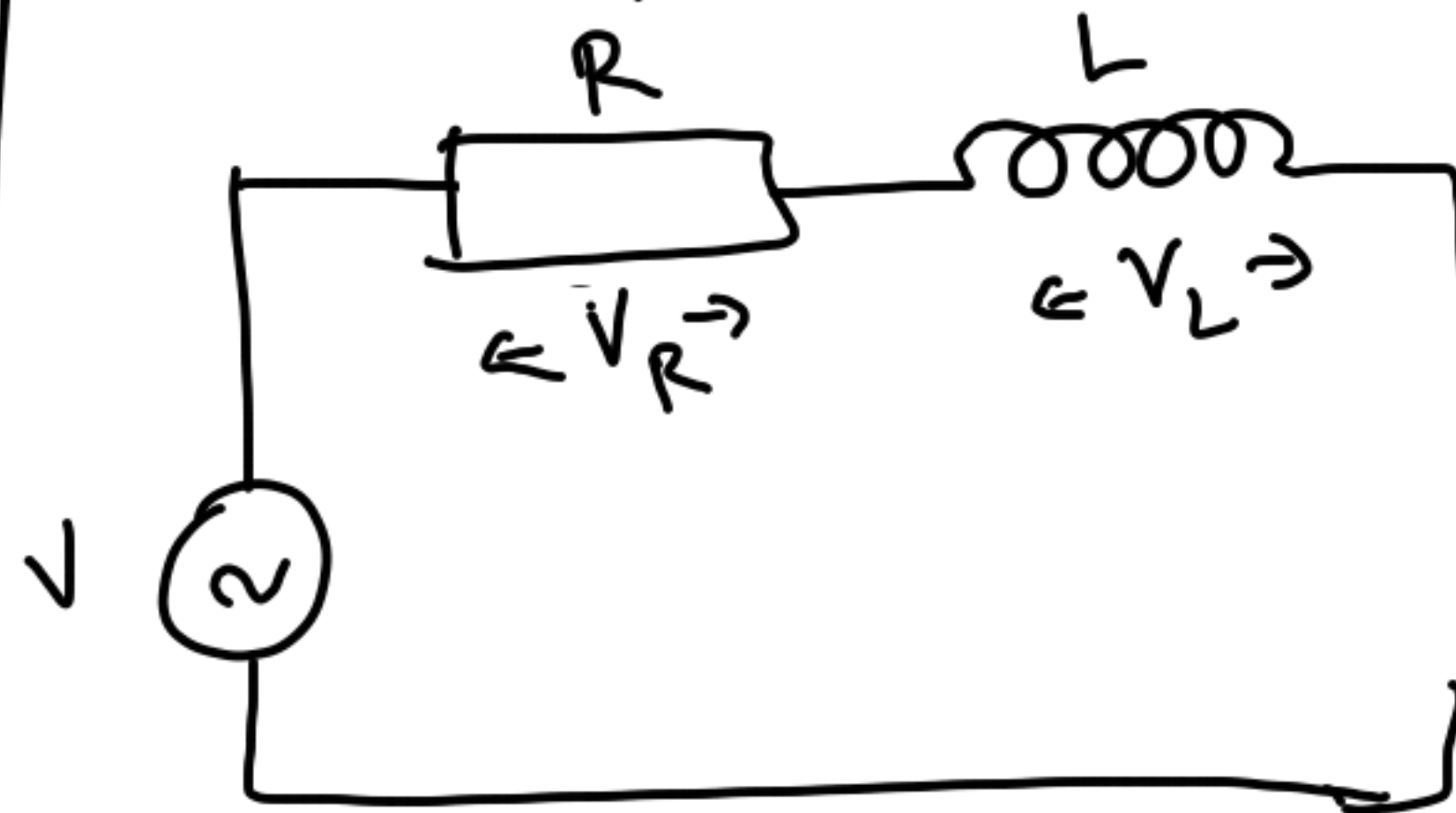
$$V = 750 \sin(2\pi \times 60)t$$

$$V = 750 \sin 377t$$

①



RL Series Circuit



Impedance $Z = R + jX_L$

$$\Rightarrow Z = \sqrt{R^2 + X_L^2}$$

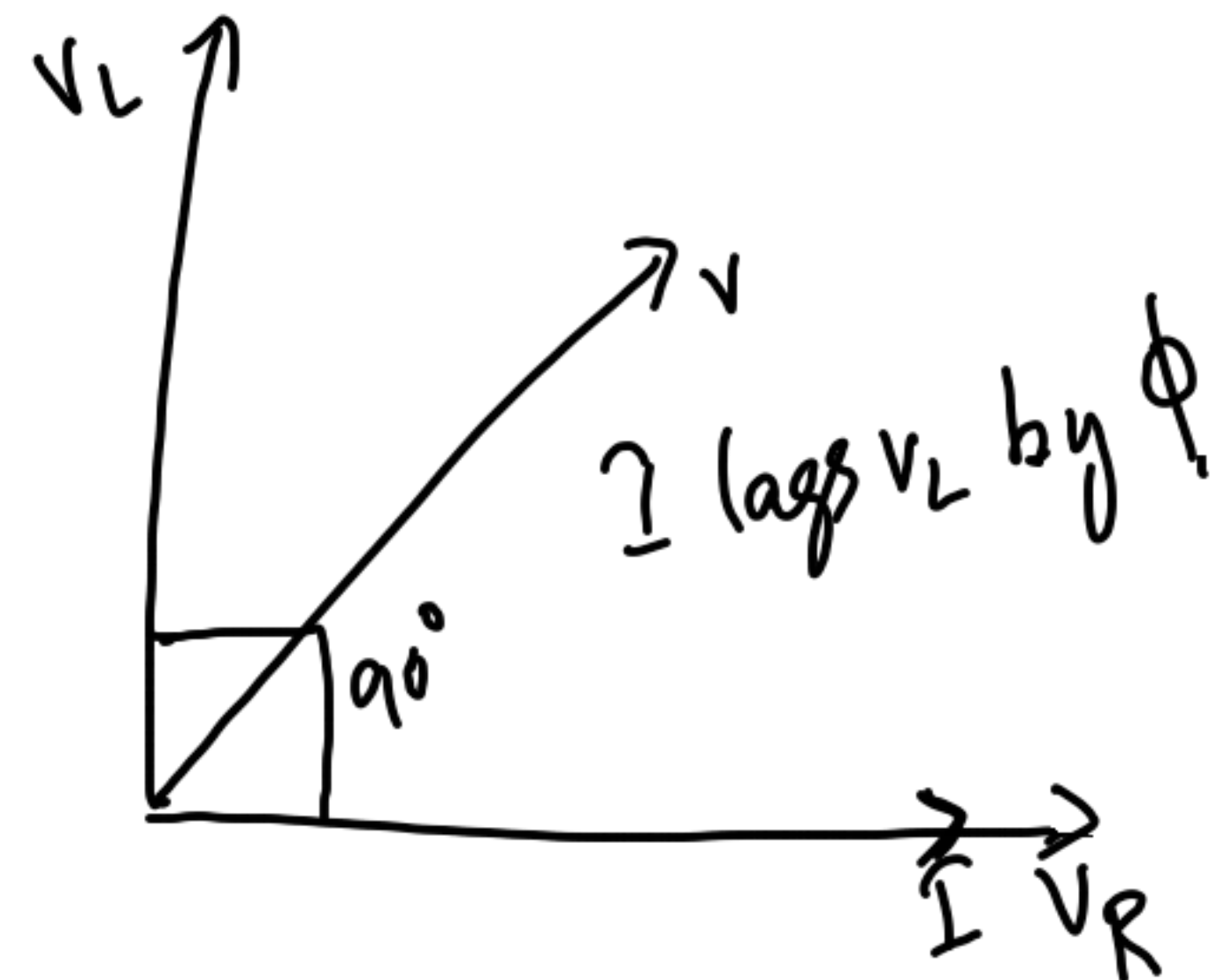
$$Z = \frac{V}{I}$$

Voltage equation

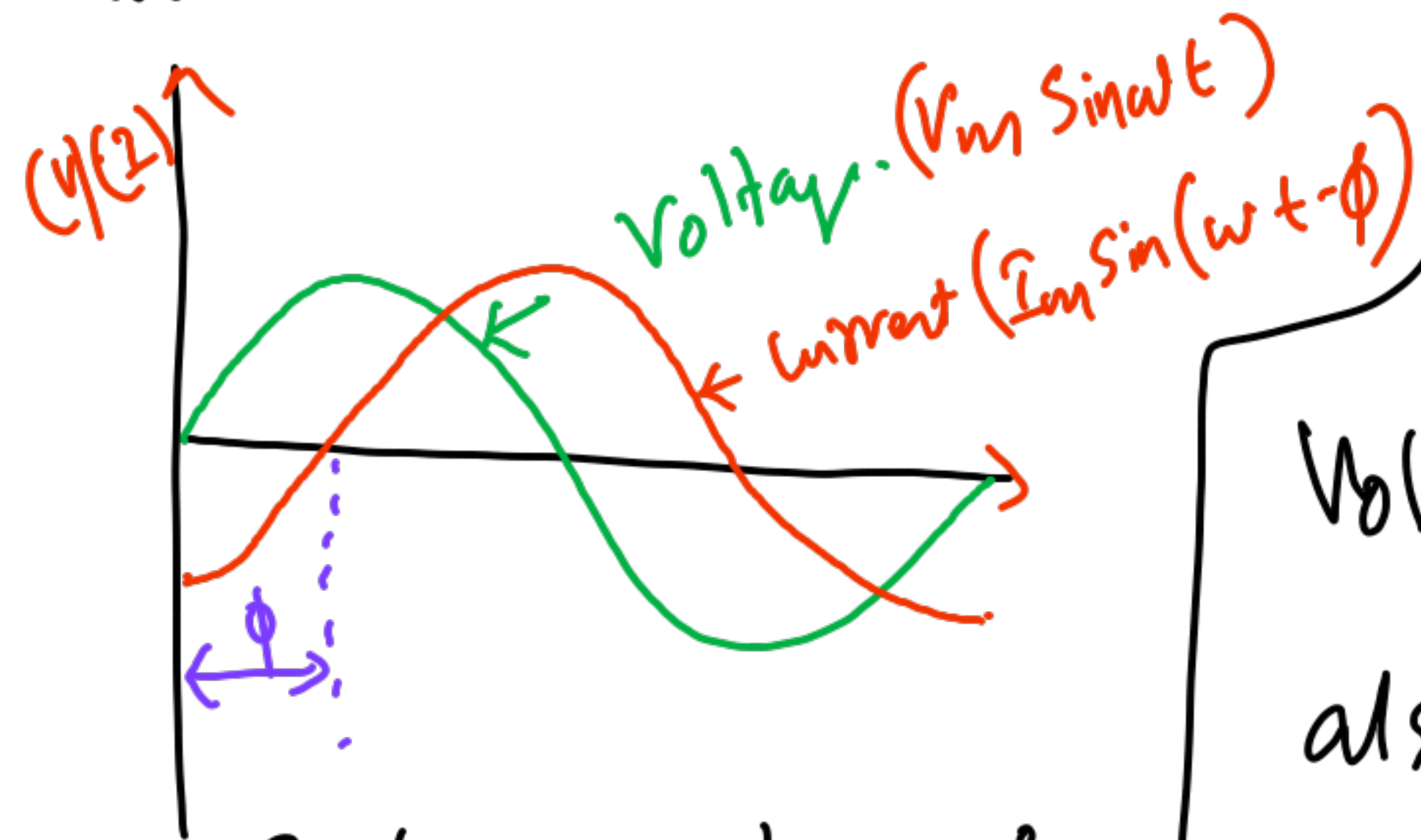
$$V = V_m \sin \omega t$$

$$I = I_m \sin(\omega t - \phi)$$

Phasor diagram



Wave form.



I lags V by 90°

Phase $\phi = \tan^{-1}\left(\frac{X_L}{R}\right)$

Power $P = \frac{V_m I_m}{2} \cos \phi$
 $P = V I \cos \phi$

Power factor: $\cos \phi$

$$\cos \phi = \frac{R}{Z}$$

$$\text{Voltage} = \sqrt{V_R^2 + V_L^2}$$

also

$$\tan \phi = \frac{V_L}{V_R}$$

$$\sin \phi = \frac{X_L}{Z}$$

Voltage drop across

Pure inductor-

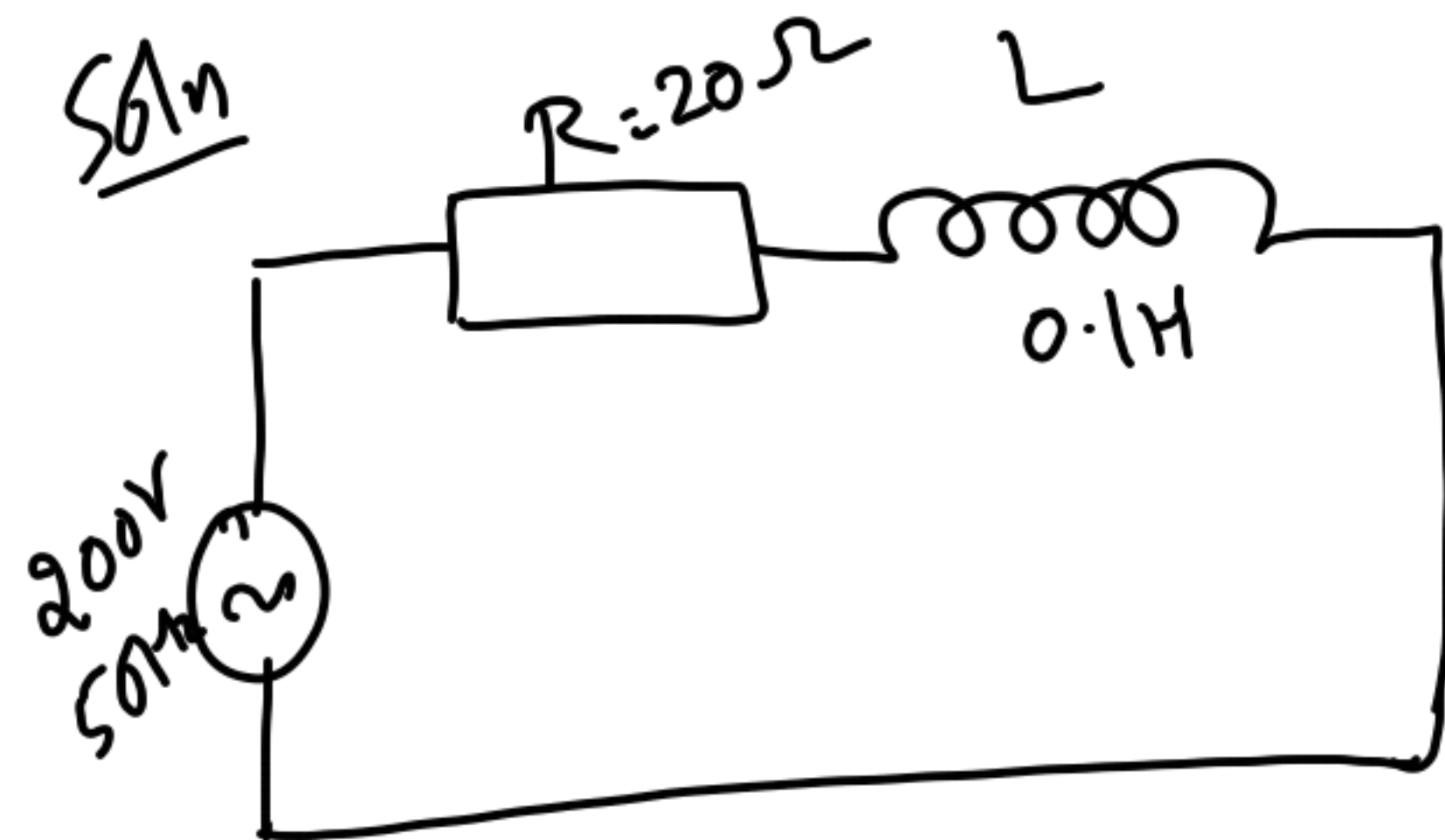
$$V_L = I X_L$$

1) A Circuit consisting of resistor of 20Ω and inductance of $0.1H$ is connected in series across a single phase $200V$, $50Hz$ supply

Calculate (a) Current drawn

(b) Power consumed.

(c) Phasor diagram.



a) to find Current

$$\hat{I} = \frac{V}{Z}$$

$$Z = \sqrt{R^2 + X_L^2}$$

find. $X_L = 2\pi fL$

$$= 2\pi \times 50 \times 0.1$$

$$X_L = 31.4\Omega$$

$$Z = \sqrt{(20)^2 + (31.4)^2}$$

$$Z = 37.2\Omega$$

Sub 'Z' in $\hat{I}$

$$Z = \frac{200}{37.2} = 5.4A$$

b) Power consumed.

$$P = V \hat{I} \cos \phi$$

$$P = 200 \times 5.4 \cos \phi$$

$$\phi = \tan^{-1} \left(\frac{X_L}{R} \right)$$

$$\phi = \tan^{-1} \left(\frac{31.4}{20} \right) = 1$$

$$P = 200 \times 5.4 \cos 1$$

$$P = 583.5 \text{ Watts}$$

Phasor diagram.

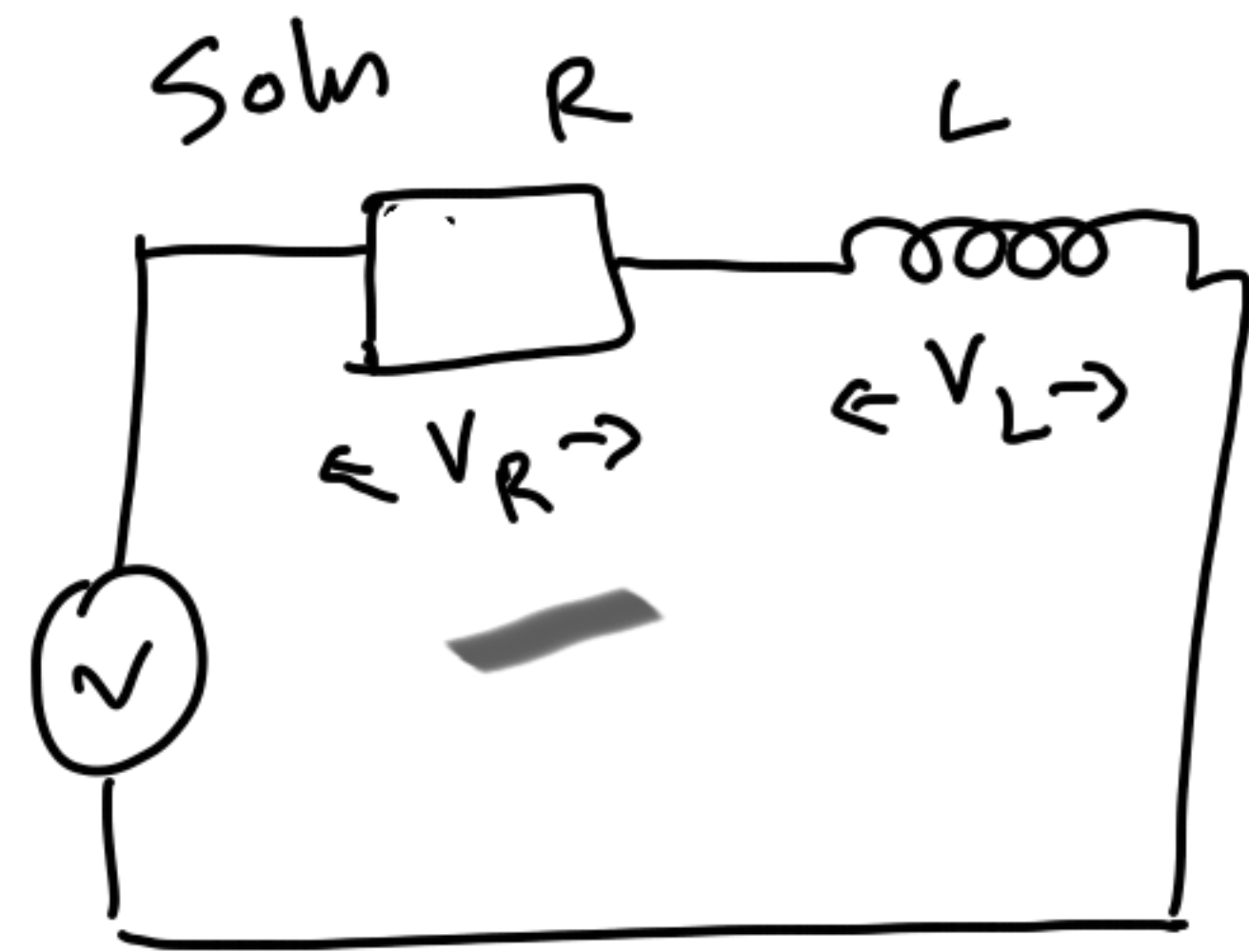


2) In a Series Circuit, containing Pure Resistor and pure inductor, the Current and Voltage expressed as

$$i(t) = 5 \sin\left(314t + \frac{2\pi}{3}\right) \text{ and}$$

$$v(t) = 2 \sin\left(314t + \frac{5\pi}{6}\right) \text{ Calculate.}$$

- i) the impedance
- ii) Value of Resistor, inductor
- iii) Power factor.
- iv) Average power.



Ans.

$$i(t) = 5 \sin\left(314t + \frac{2\pi}{3}\right)$$

$$v(t) = 2 \sin\left(314t + \frac{5\pi}{6}\right)$$

$$V_m = 2 \text{ V}, \quad I_m = 5 \text{ A}$$

$$\omega = 2\pi f$$

$$\omega = 2\pi f$$

$$f = 50 \text{ Hz}$$

$$V_{Rms} = \frac{V_m}{\sqrt{2}} = \frac{2}{\sqrt{2}} = 1.4 \text{ V}$$

$$\hat{I}_{Rms} = \frac{\hat{I}_m}{\sqrt{2}} = \frac{5}{\sqrt{2}} = 3.5 \text{ A}$$

$$Z = \frac{V}{\hat{I}} = \frac{1.4}{3.5} = 0.4 \Omega$$

$$Z = \sqrt{R^2 + X_L^2}$$

$$\underline{V_R} = \underline{\hat{I}} R$$

$$\underline{V_L} = \underline{\hat{I}} X_L$$

$$P.F. = \cos \phi$$

Phase for voltage.

$$\phi_v = \frac{5\pi}{3} = \frac{5 \times 180^\circ}{6} = 150^\circ$$

Phase for current

$$\phi_i = \frac{2\pi}{3} = \frac{2 \times 180}{3} = 120^\circ$$

$$\phi = 150 - 120 = 30^\circ$$

$$P.F. = \cos 30^\circ = 0.866$$

Average power:

$$= \frac{\hat{I}_m}{\sqrt{2}} \cdot \frac{V_m}{\sqrt{2}} \cos \phi$$

$$= \frac{5}{\sqrt{2}} \cdot \frac{2}{\sqrt{2}} \cos 30^\circ$$

$$= \frac{10}{2} \times 0.866$$

$$P_{Avg} = 4.3 \text{ W}$$

$$\cos \phi = \frac{R}{Z}$$

$$0.866 = \frac{R}{0.4}$$

$$R = 0.866 \times 0.4 = 0.34 \Omega$$

$$Z = \sqrt{R^2 + X_L^2}$$

Squaring both the sides.

$$Z^2 = R^2 + X_L^2$$

$$X_L^2 = Z^2 - R^2$$

$$X_L = \sqrt{Z^2 - R^2}$$

$$X_L = 0.21$$

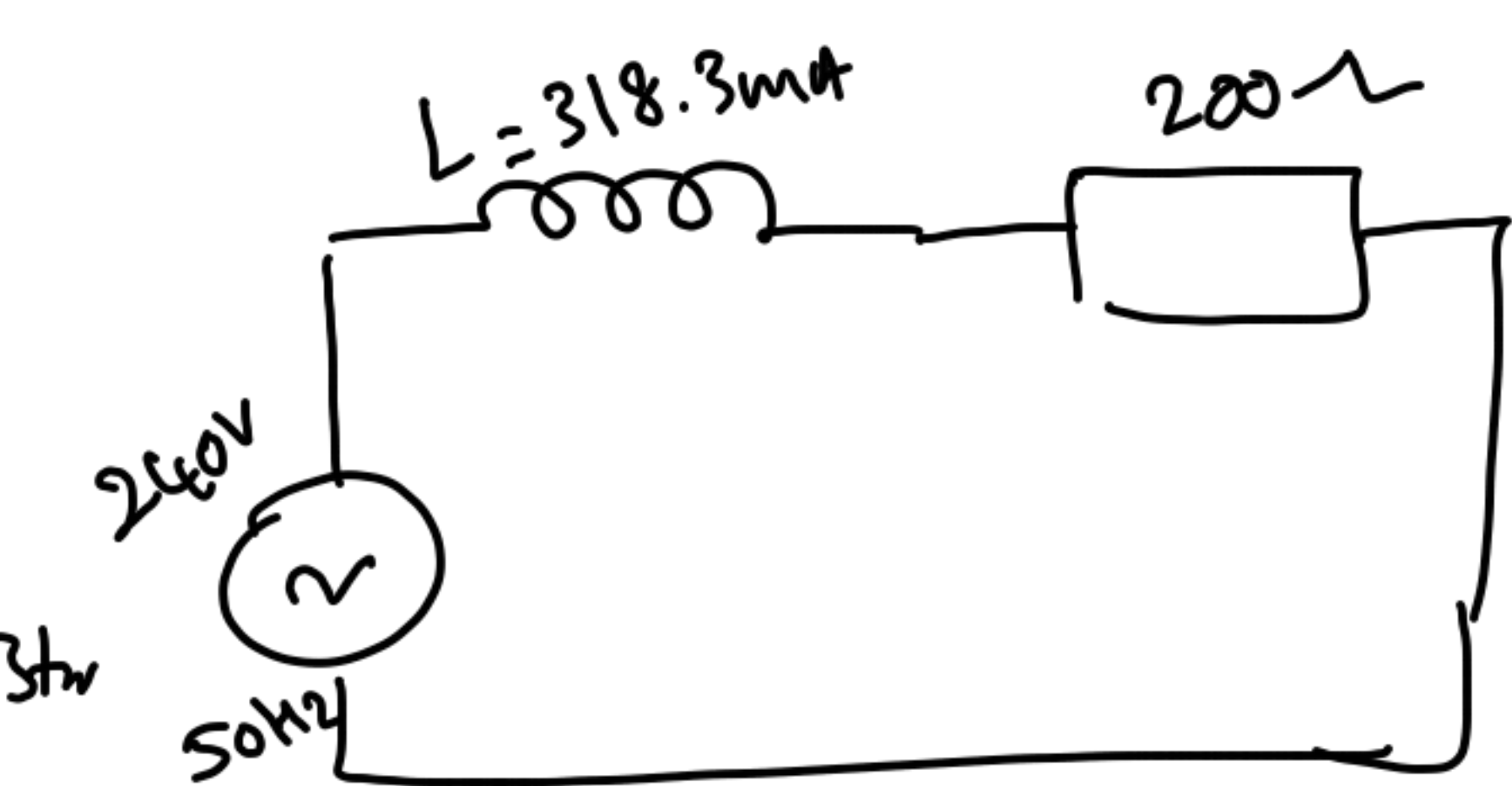
$$X_L = 2\pi fL$$

$$0.21 = 2 \times 3.14 \times 50 \times L$$

$$L = \frac{0.21}{314} = \underline{669 \text{ mH}}$$

3) A coil of inductance 318.3mH and negligible resistance is connected in series with a 200Ω resistor to a 240V, 50Hz Supply. Calculate

- the inductive reactance of the coil
- impedance
- Current in the circuit
- the potential difference across each component
- the circuit phase angle



soln

or

$$X_L = 2\pi f L$$

$$= 2 \times 3.14 \times 50 \times 318.3 \times 10^{-3}$$

$$X_L = 99.9 \Omega$$

$$Z = \sqrt{R^2 + X_L^2} = \sqrt{(200)^2 + (99.9)^2}$$

$$Z = 223.7 \Omega$$

$$\underline{I} = \frac{V}{Z} = \frac{240}{223.7}$$

$$\underline{I} = 1.07 \text{ A}$$

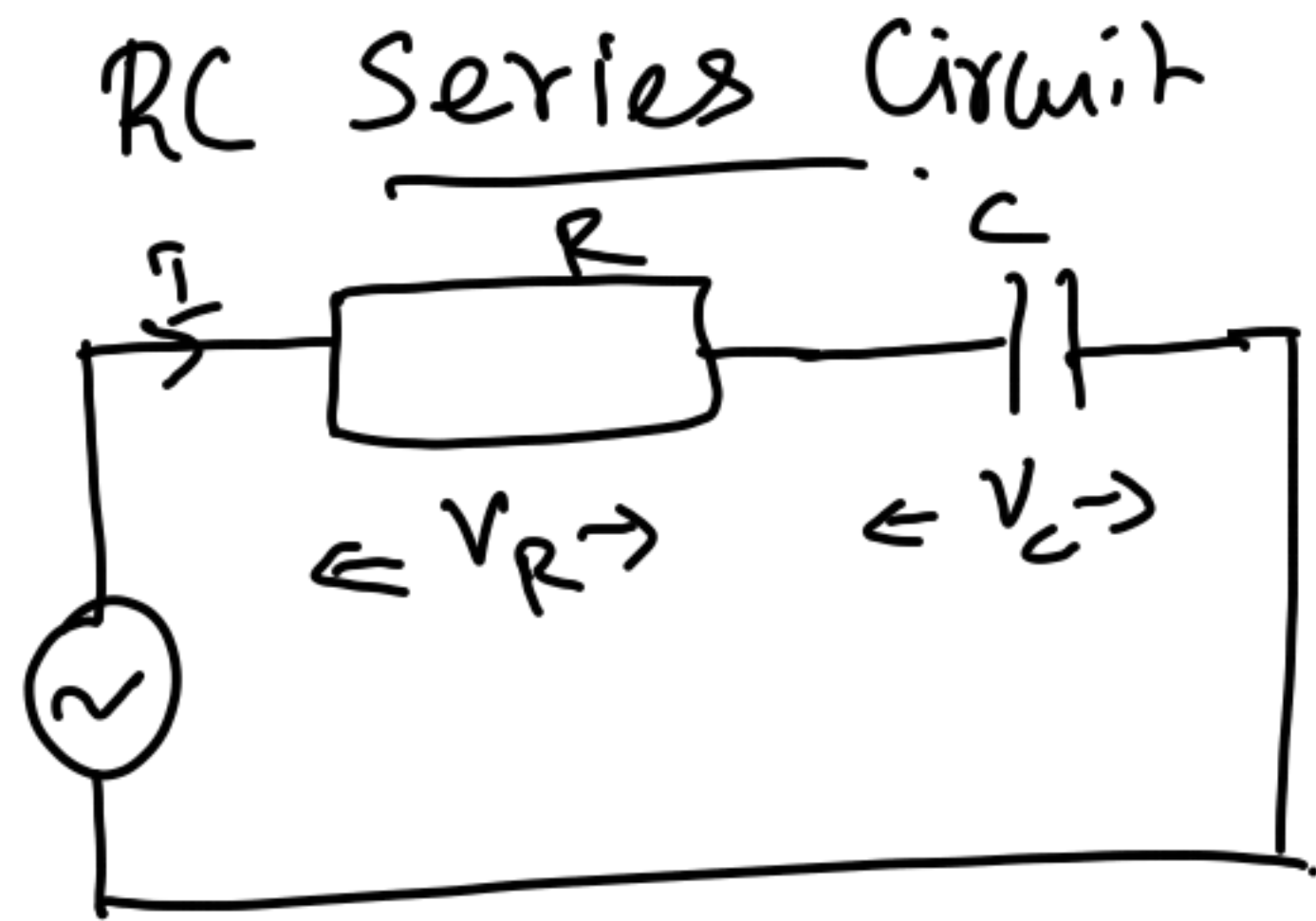
$$V_R = \underline{I} \cdot R = 1.07 \times 200 \\ = 214 \text{ V}$$

$$V_L = \underline{I} X_L \\ = 1.07 \times 99.9 \\ = 106.9 \text{ V}$$

$$V = \sqrt{V_R^2 + V_L^2} \\ V = \sqrt{(214)^2 + 106.9^2} \\ = 240 \text{ V}$$

$$\phi = \tan^{-1} \left(\frac{X_L}{R} \right) \\ \phi = 26.6^\circ \text{ [lag]}$$

ing



Resistor and Capacitor are connected in series across an AC source.

Impedance (Z)

$$Z = R + jX_C$$

$$Z = \sqrt{R^2 + X_C^2}$$

$$Z = \frac{V}{I}$$

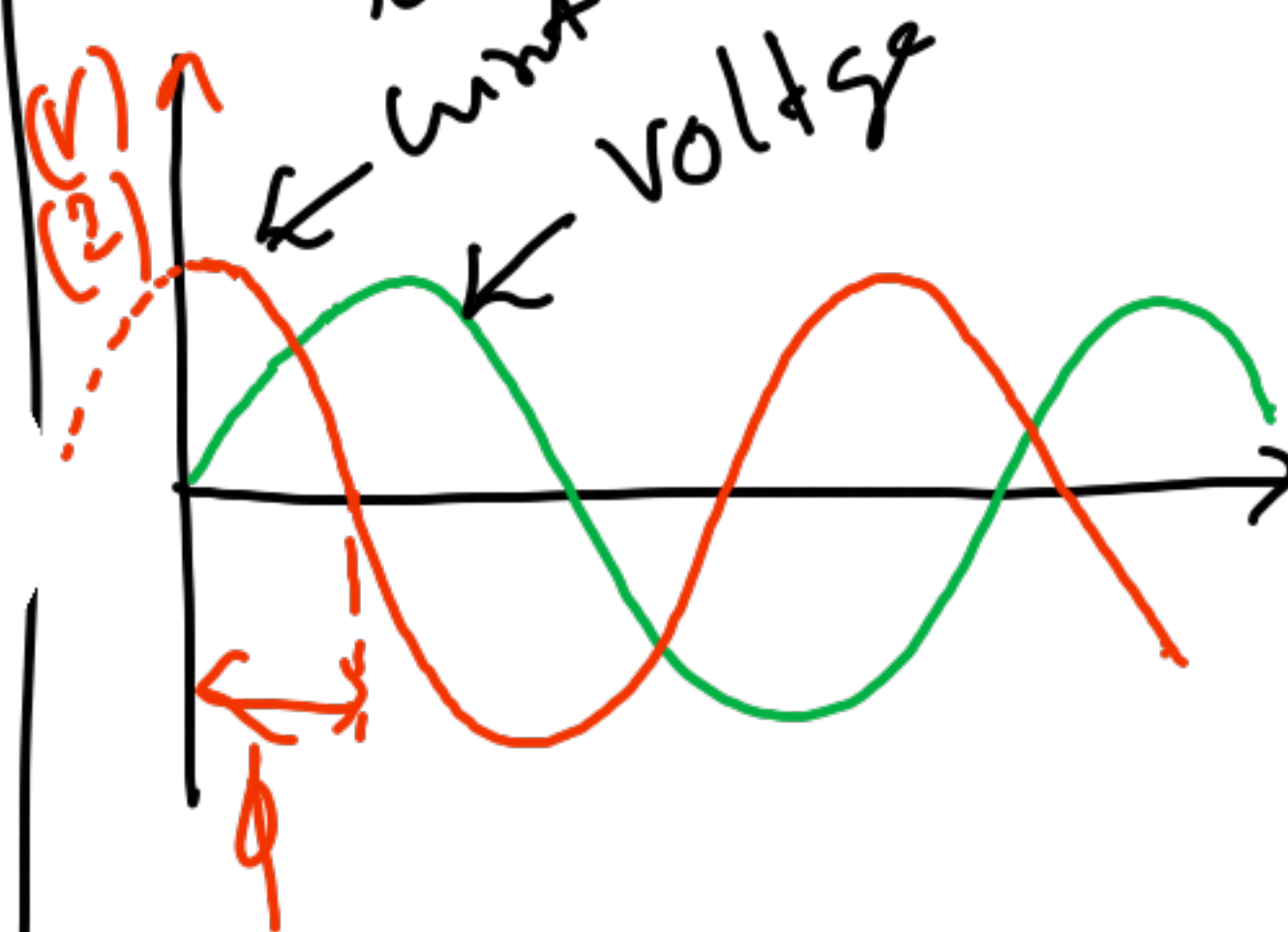
Current equation.

$$I = I_m \sin(\omega t + \phi)$$

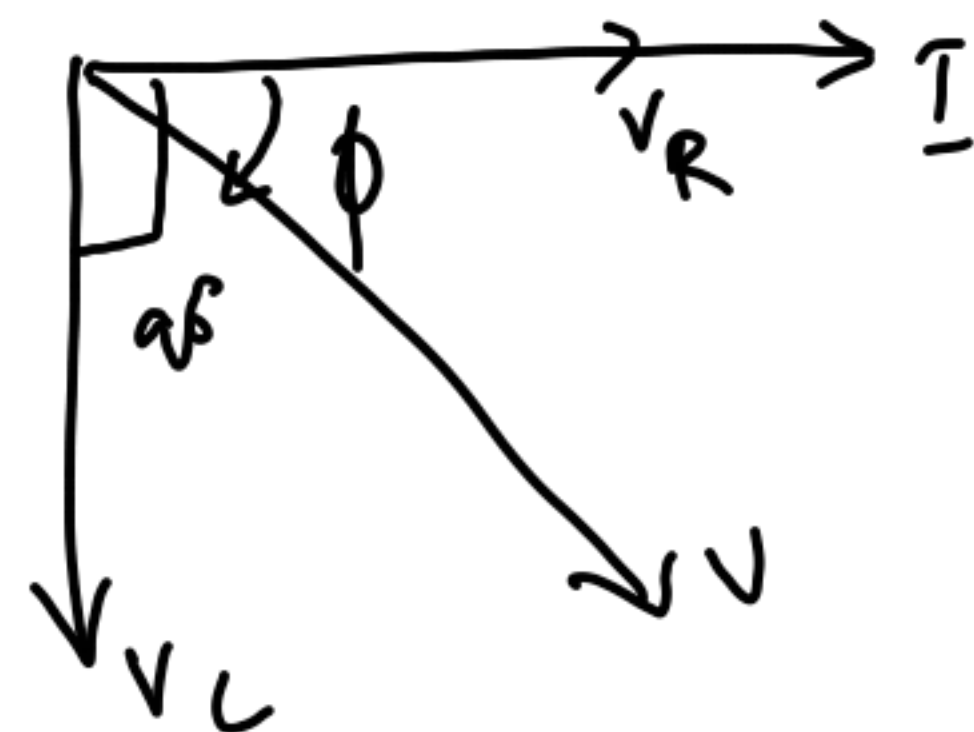
Voltage equation.

$$V = V_m \sin \omega t$$

Waveform.



Phase Diagram.



V_R in phase with I

V_C lags the current.

Phase:

$$\text{Phase } \phi = \tan^{-1} \left(\frac{X_C}{R} \right)$$

Power (or) Average power

$$P_{avg} = \frac{V_m \hat{I}_m}{2} \cos \phi$$

$$P_{avg} = V \hat{I} \cos \phi \text{ Watts}$$

Power factor!

$$P.F = \cos \phi$$

$$\cos \phi = \frac{R}{Z} \quad \checkmark$$

Voltage drop across capacitor

$$V_C = \hat{I} X_C$$

$$\sin \phi = \frac{X_C}{Z}$$

4) A capacitor is connected in Series with a 40Ω resistor across a supply of frequency 60Hz . A current of 3A flows and the circuit impedance is 50Ω . Calculate

- Value of capacitance
- Supply voltage
- Phase between the voltage and current
- P.D across the resistor.
- P.D across the capacitor
- Phasor diagram