

UWTSD

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Analytical Coursework

Analytical Methods

(ACHA4000)

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Question 1

(i). $(x + 6)^2 + 2x = x^2 - 5$

$$x^2 + 36 + 2x = x^2 - 5$$

$$x^2 - x^2 + 2x = -5 - 36$$

$$2x = -41$$

$$x = -41/2$$

$$x = -20.5$$

(ii). $2^{x+5} = 6^x$

$$\text{Log}(2^{x+5}) = \log(6^x)$$

(take the logs then place into brackets)

$$(x + 5) \times \log(2) = (x) \times \log(6)$$

$$(x + 5) \times 0.3010 = (x) \times 0.78$$

$$0.3010x + 1.505 = 0.78x$$

$$1.505 = 0.78x - 0.3010x$$

$$1.505 = 0.479x$$

$$1.505/0.479 = x$$

$$3.14 = x$$

(iii). $2\log(x + 3) = \log(x + 5) + \log(x + 8)$

$$\text{Log}(x + 3)^2 = \log(x + 5)(x + 8)$$

(logs cancel out)

$$(x + 3)^2 = (x + 5)(x + 8)$$

$$x^2 + 9 = x^2 + 5x + 8x + 40$$

$$9 = 13x + 40$$

(x^2 cancel out)

$$9 - 40 = 13x$$

$$-31 = 13x$$

$$-31/13 = x$$

$$-2.38 = x$$

$$(iv) x - 1^{-6x/5} / 3 = 4^{-x/3} / 12 + 5/4$$

$$x - 12(1^{-6x/5})/3 = 12(4^{-x/3})/12 + 12(5)/4$$

$$x - 4(1^{-6x/5}) = 4^{-x/3} + 3(5)$$

$$x - 4(5 - 6x)/5 = 4(3 - x)/3 + 15$$

$$x - 4(-6x) = 4(-x) + 15 \quad (\text{numbers cancel out})$$

$$x + 24x = -4x + 15$$

$$25x + 4x = 15$$

$$29x = 15$$

$$x = 15/29$$

$$x = 0.52$$

Question 2

1. $2x + 4y - 3z = 120$ $\rightarrow x5 = 10x + 20y - 15z = 600$
2. $5x + 2y + z = 140$ $\rightarrow x2 = 10x + 4y + 2z = 280$
3. $x - 3y - 5z = 100$ Eq1. $16y - 17z = 320$

$$2. \quad = 5x + 2y + z = 140$$

$$3. \quad x5 = 5x - 15y - 25z = 500$$

$$\text{Eq2. } 17y + 26z = -360$$

$$\text{Eq1. } x17 = 272y - 289z = 5440 \quad (\text{use both calculated equations to work out } z)$$

$$\text{Eq2. } x16 = 272y + 416z = -5760$$

$$-705z = 11200$$

$$Z = -14500/-670$$

$$z = -15.89$$

$$16y - 17(-15.89) = 320 \quad (\text{sub back into Eq1.})$$

$$16y + 270.13 = 320$$

$$16y = 320 - 270.13$$

$$y = 49.87/16$$

$$y = 3.12$$

$$2x + 4(3.12) - 3(-15.89) = 120 \quad (\text{sub numbers into original equation})$$

$$2x + 12.48 + 47.67 = 120$$

$$2x = 120 - 60.15$$

$$2x = 59.85$$

$$x = 59.85/2$$

$$x = 29.93$$

Check answer,

Sub into 1. $2(29.93) + 4(3.12) - 3(-15.89) = 120$

$$59.86 + 12.48 + 47.67 = 120$$

Sub into 2. $5(29.93) + 2(3.12) - 15.89 = 140$

$$149.65 + 6.24 - 15.89 = 140$$

Sub into 3. $29.93 - 3(3.12) - 5(-15.89) = 100$

$$29.93 - 9.36 + 79.45 = 100$$

Question 3

$$0.6x^2 + 3.4x = 7.4$$

$$a = 0.6x^2 \quad b = 3.4x \quad c = 7.4$$

$$x = 0.6x^2 + 3.4x - 7.4 = 0$$

$$x = \frac{-3.4 \pm \sqrt{3.4^2 - 4 \times 0.6 \times (-7.4)}}{2(0.6)}$$

$$x +ve = 1.68$$

$$x -ve = -7.35$$

Question 4

(i). $y = 5x^3 - 7x^2 + 3x - 1$

$$dy/dx = 15x^2 - 14x + 3$$

(ii). $y = -1/x^5$

$$dy/dx = -1/x^5$$

$$= -dy/dx [1/x^5]$$

$$= -(-5)x^{-6}$$

$$= 5/x^6$$

(iii) $y = e^{x/2} + \cos(x/6) + \sin 2x$

$$dy/dx = (1/2)e^{x/2} - (1/6)\sin(x/6) + 2\cos 2x$$

(iv). $y = (x^7 + 4)^5 (3x^2 - 1)^4$

$$du/dx = 5(x^7 + 4)^4 (7x^6)$$

$$= (35x^6)(x^7 + 4)^4$$

$$dv/dx = 4(3x^2 - 1)^3 (6x)$$

$$= (24x)(3x^2 - 1)^3$$

$$dy/dx = (3x^2 - 1)^4 [(35x^6)(x^7 + 4)^4] + (x^7 + 4)^5 [(24x)(3x^2 - 1)^3]$$

$$u = (x^7 + 4)^5$$

$$v = (3x^2 - 1)^4$$

(v). $y = (5x^2 + 3x + 1)^5$ $\leftarrow U$

$$e^x \quad \leftarrow V$$

$$du/dx = 5(5x^2 + 3x + 1)^4 (10x + 3)$$

$$= (50x + 15)(5x^2 + 3x + 1)^4$$

$$dv/dx = xe^x$$

$$\frac{dy}{dx} = (e^x)[(50x + 15)(5x^2 + 3x + 1)^4 - (5x^2 + 3x + 1)^5(xe^x)]$$

$$(e^x)^2$$

$$(vi). y = \sqrt{\frac{5x-1}{8x+5}}$$

$$y = \left(\frac{5x-1}{8x+5}\right)^{1/2}$$

$$u = 5x - 1$$

$$v = 8x + 5$$

$$du/dx = 5$$

$$dv/dx = 8$$

$$\frac{dy}{dx} = \frac{1}{2} (5x - 1 / 8x + 5)^{-1/2} [(8x+5)(5) - (5x-1)(8)]$$

$$(8x+5)^2$$

$$(vii). y = (10x + \sqrt[3]{x^5 + 1})^5$$

$$= 5(10 + [(1/3)(x^5 + 1)^{-1/3}]^4]$$

$$= 5(10x + \sqrt[3]{x^5 + 1})^4 (10 + (1/3)(5x^4)^{-1/3})$$

$$\frac{dy}{dx} = (50 + (1/3)(25x^4)^{-1/3})(10x + \sqrt[3]{x^5 + 1})^4$$

Question 5

$$1^{\text{st}} \text{ derivative (velocity)} = \frac{dx}{dt}$$

(Increase the ^ Power by one for each derivative)

$$2^{\text{nd}} \text{ derivative (acceleration)} = \frac{d^2x}{dt^2}$$

$$3^{\text{rd}} \text{ derivative (jerk)} = \frac{d^3x}{dt^3}$$

$$4^{\text{th}} \text{ derivative (snap)} = \frac{d^4x}{dt^4}$$

$$5^{\text{th}} \text{ derivative (crackle)} = \frac{d^5x}{dt^5}$$

$$6^{\text{th}} \text{ derivative (pop)} = \frac{d^6x}{dt^6}$$

Question 6

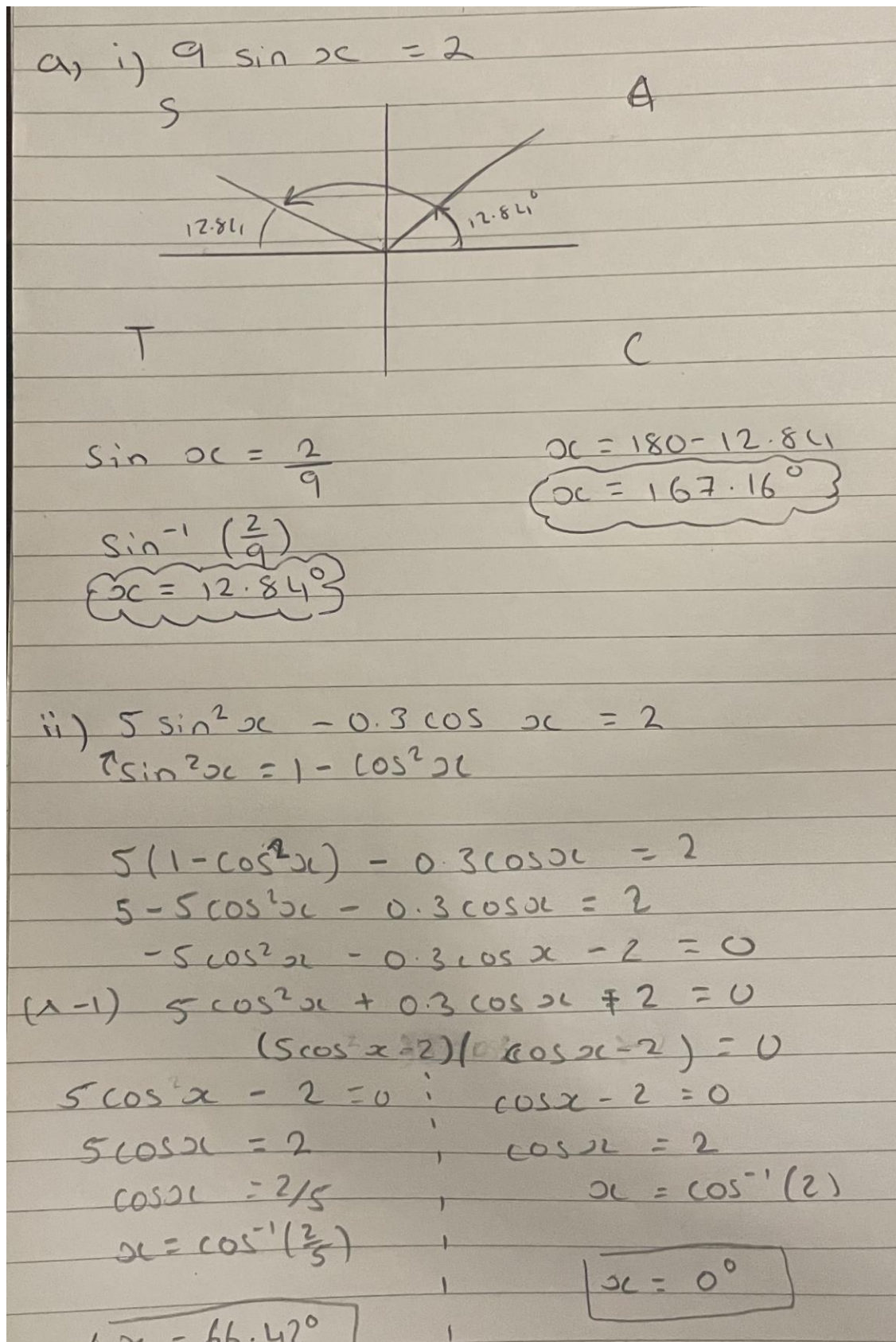


Figure 1-1 Question 6 Calculations (Finn Flye, 2025)

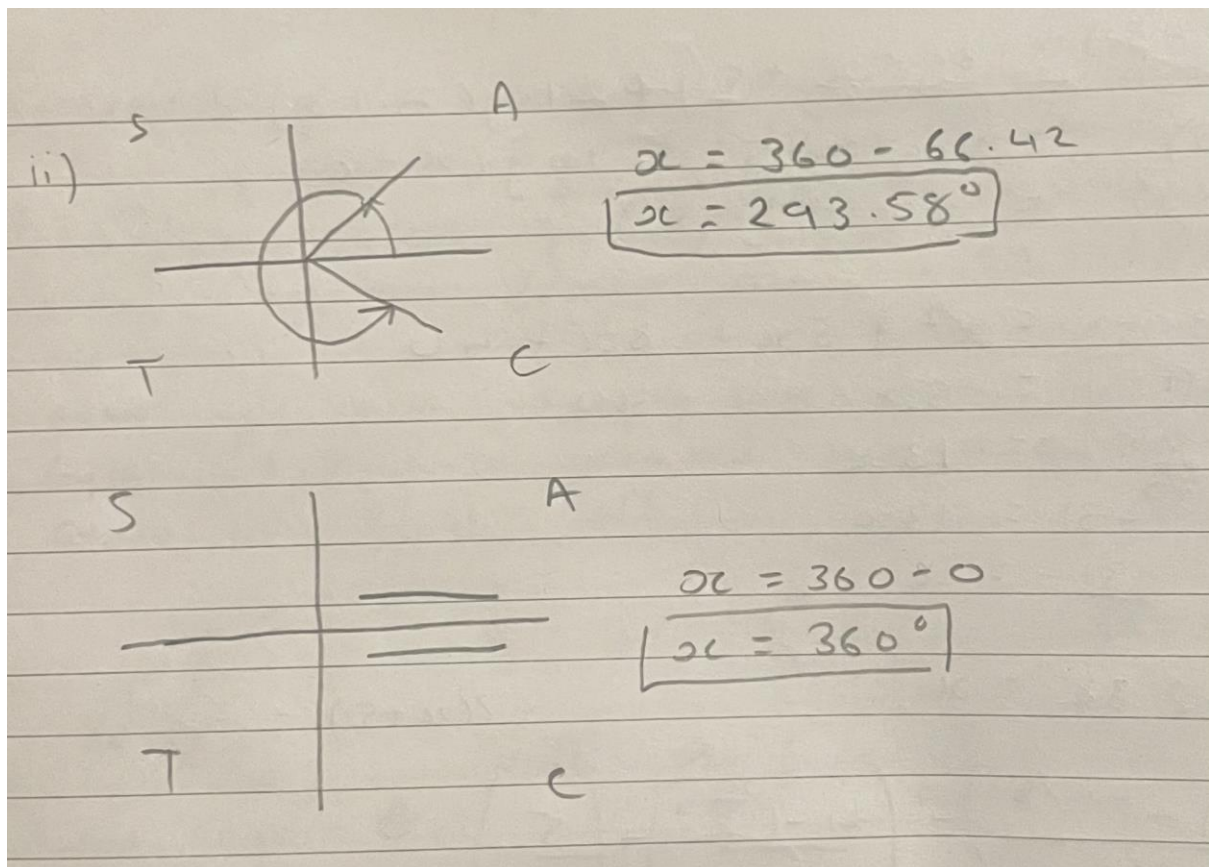
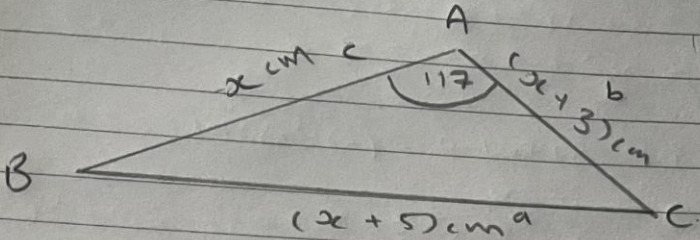


Figure 1-2 Question 6 Calculations, cast diagrams for part two (Finn Flye, 2025)

Question 7

Question 7



$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$(x+5)^2 = (x+3)^2 + (x^2) - 2(x+3)(x) \cos 117$$

$$(x+5)(x+5) = (x+3)(x+3) + (x^2) - 2(x^2+3x) \cos 117$$

$$x^2 + 10x + 25 = 2x^2 + 6x + 0.45394(2x^2 + 6x)$$

Work out cos value, $0.45394(2x^2+6x) = 0.90798x^2 + 2.72394x$

Sub back in $\rightarrow$

$$x^2 + 10x + 25 = 2x^2 + 6x + 4 + 0.90798x^2 + 2.72394x$$

$$x^2 + 10x + 25 = (2 + 0.90798)x^2 + (6 + 2.72394)x + 4$$

$$x^2 + 10x + 25 = 2.90798x^2 + 8.72394x + 4$$

All on one side $\rightarrow$

$$0 = 2.90798x^2 + x^2 + 8.72394x - 10x + 4 - 25$$

$$0 = 1.90798x^2 - 1.272394x - 16$$

$\uparrow$ $\uparrow$ $\uparrow$
 a b c

in quadratic

$$x = \frac{-(-1.272394) \pm \sqrt{(-1.272394)^2 - 4 \times (1.90798) \times (-16)}}{2(1.90798)}$$

$x + v = 3.24 \text{ cm}$

$x - v = -2.58$

$BA = 3.24 \text{ cm}$

$BC = (3.24 + 5)$

$BC = 8.24 \text{ cm}$

$AC = (3.24 + 3)$

$AC = 6.24 \text{ cm}$

Figure 1-3 Question 7 part 1 calculations (Finn Flye, 2025)

ii) $\text{Area} = \frac{1}{2} \times a \times b \times \sin C$

$\sin C = \frac{8.24}{\sin 117} = \frac{3.24}{\sin C}$

$8.24 \times \sin C = 3.24 \times \sin 117$

$8.24 \times \sin C = 2.886861138$

$\sin C = \frac{2.886861138}{8.24}$

$\sin C = 0.35$

$C^\circ = \sin^{-1}(0.35)$

$C^\circ = 20.49^\circ$

$\text{Area} = \frac{1}{2} \times 8.24 \times 6.24 \times \sin(20.49)$

$\text{Area} = 8.99 \text{ cm}^2$

Figure 1-4 Question 7 part 2 calculations (Finn Flye, 2025)