

Maths exam

6 May 2021 at 10:49

1.

(i) $y = 7x^3 - 2x^2 + x + 2$

$$\frac{dy}{dx} = 21x^2 - 4x + 1$$

(ii) $y = (x+3)(x+2)$

$$y = x^2 + 3x + 2x + 6$$

$$y = x^2 + 5x + 6$$

$$\frac{dy}{dx} = 2x + 5$$

(iii) $y = \frac{1}{x^3} + \sin 3x$

$$\frac{dy}{dx} = x^{-3} + 3\cos 3x$$

(iv) $y = x^4 \tan 2x$

$$\frac{dy}{dx} = 4x^3 2\sec^2 2x$$

(v) $y = \frac{e^{2x}}{\cos 2x}$

$$\frac{dy}{dx} = \frac{2e^{2x}}{-2.5772x}$$

(vi) $y = (5x^2 + 2x + 1)^5$

$$y = (10x + 2)^5$$

$$\frac{dy}{dx} = 10x^4 + 32$$

(i) $\sin x = 0.65$

$$\sin^{-1}(0.65)$$

$$x = 40.54$$

(ii) $7\cos x = 4$

$$\cos x = \frac{4}{7}$$

$$x = \cos^{-1}\left(\frac{4}{7}\right)$$

(iii) $\tan x = -0.22$

$$\tan x = 0.22$$

$$x = \tan^{-1}(0.22)$$

2.

(i) $\sin x = 0.65$

$x = \sin^{-1}(0.65)$

$x = 40.54$

(ii) $7 \cos x = 4$

$\cos x = \frac{4}{7}$

$x = \cos^{-1}\left(\frac{4}{7}\right)$

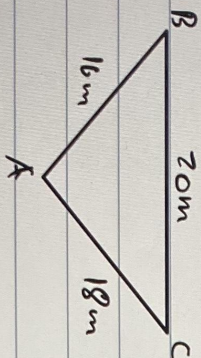
$x = 55.15$

(iii) $\tan x = -0.22$

$\tan x = -0.22$

$x = \tan^{-1}(-0.22)$

$x = 12.41$

3. find angle $\hat{A}$ 

$a^2 = b^2 + c^2 - 2bc \cos A$

$20^2 = 18^2 + 16^2 - 2(18)(16) \cos(A)$

$400 = 224 + 256 - 576 \cos(A)$

$400 - 324 - 256 = -576 \cos(A)$

$-180 = -576 \cos(A)$

$0.313 = \cos(A)$

$\cos^{-1}(0.313) = A$

$A = 71.6$

A

$$-180 = -576 \cos(A)$$

$$0.313 = \cos(A)$$

$$\cos^{-1}(0.313) = A$$

$$A = 71.6$$

4.

$$P = \sqrt{\frac{xy}{T}} + 3 \quad / = \quad P^2 = \frac{xy}{T} + 3 \quad / = \quad P^2 - 3 = \frac{xy}{T} \quad / = \quad T(P^2 - 3) = xy \quad /$$

$$= \frac{T(P^2 - 3)}{y} = x$$

$$5. (i) \quad 5^{x+2} = 2^x \quad 3 \text{ d.p.}$$

$$\log 5^{x+2} = \log 2^x$$

2nd law 3rd law

$$(x+2) \log 5 = (x) \log 2$$

$$(x+2) 0.699 = (x) 0.301$$

$$0.699x + 1.398 = 0.301x$$

$$(ii) \quad \log(x+2) + \log(x+3) = 2 \log(x+1)$$

$$\log(x+2) + \log(x+3) = 2 \log(x+1)$$

1st 3rd

$$\log(x+2)(x+3) = \log(x+1)^2$$

$$(x+2)(x+3) = (x+1)(x+1)$$

$$x^2 + 5x + 6 = x^2 + 2x + 1$$

$$\underline{5x} \quad \underline{2x}$$

y

$$5. (i) 5^{x+2} = 2^x \quad 3 \text{ d.p.}$$

$$\log 5^{x+2} = \log 2^x$$

2nd law 2nd law

$$\therefore (x+2) \log 5 = (x) \log 2$$

$$(x+2) 0.699 = (x) 0.301$$

$$= 0.699x + 1.398 = 0.301x$$

$$= 0.699x - 0.301x = 1.398$$

$$= 0.398x = \frac{1.398}{0.398}$$

$$x = \frac{1.398}{0.398}$$

$$x = 3.513$$

$$(ii) \log(x+2) + \log(x+3) = 2 \log(x+1)$$

$$\log(x+2) + \log(x+3) = 2 \log(x+1)$$

1st 2nd law

$$\cancel{\log}(x+2)(x+3) = \cancel{\log}(x+1)^2$$

$$(x+2)(x+3) = (x+1)(x+1)$$

$$x^2 + 5x + 6 = x^2 + 2x + 1$$

$$\cancel{x^2} + 5x + 6 = \cancel{x^2} + 2x + 1$$

$$5x + 6 = 2x + 1$$

$$5x - 2x = 1 - 6$$

$$3x = -5 \quad \therefore x = -\frac{5}{3}$$

$$\frac{-5}{3} \quad \text{or} \quad -1.67$$

$$= 0.398x = 1.398$$

$$0.398$$

$$0.398$$

$$x = \frac{1.398}{0.398}$$

$$x = 3.513$$

$$5x + 6 = 2x + 1$$

$$5x - 2x = 1 - 6$$

$$3x = -5 \quad \Rightarrow x = -\frac{5}{3}$$

$$x = -\frac{5}{3} \text{ or } -1.67$$

$$6. (i) \int x^4 + 3x^2 - 3x + 1 \, dx$$

$$\int \frac{x^5}{5} + \frac{3x^3}{3} - \frac{3x^2}{2} + x$$

$$\int \frac{x^5}{5} + x^3 - \frac{3x^2}{2} + x + c$$

$$(ii) \int (x-1)(2x-3) \, dx$$

$$\int x^2 - 3x - 2x + 3 \, dx$$

$$\int \frac{x^3}{3} - \frac{3x^2}{2} - 1 + c$$

$$(iii) \int 3 \sec^2 2x \, dx$$

$$3 \int \sec^2 2x \, dx$$

$$3 \int \frac{1}{2} \tan^2 2x \, dx$$

$$3 \times \frac{1}{2} \tan^2 2x$$

$$(iv) \int_0^4 (x+3)(x+2) \, dx$$

$$\int_0^4 x^2 + 2x + 3x + 6 \, dx$$

$$\int_0^4 x^2 + 5x + 6 \, dx$$

$$(iii) \int 3 \sec^2 x \, dx$$

$$3 \int \sec^2 x \, dx$$

$$3 \int \frac{1}{2} \tan^2 x \, dx$$

$$3 \times \frac{1}{2} \tan^2 x$$

$$\frac{3}{2} \tan^2 x + C$$

$$(iv) \int_2^3 x(x+1) + e^x \, dx$$

$$\text{let } u = x+1$$

$$\int_2^3 x u + e^x \, dx$$

$$\frac{du}{dx} = 1$$

$$du = dx$$

$$\int_2^3 x u + e^x \frac{du}{1}$$

$$(iv) \int_0^4 (x+3)(x+2) \, dx$$

$$\int_0^4 x^2 + 2x + 3x + 6 \, dx$$

$$\int_0^4 \frac{x^3}{3} + \cancel{2x^2} + 3\frac{x^2}{2} + 6 \, dx$$

$$\left[\frac{x^3}{3} + x^2 + \frac{3x^2}{2} + 6 \right]_0^4$$

$$= \left[\frac{(4)^3}{3} + (4)^2 + \frac{3(4)^2}{2} + 6 \right] - \left[\frac{(0)^3}{3} + (0)^2 + \frac{3(0)^2}{2} + 6 \right]$$

$$= 67.3 - 6 = 61.3$$

$$\frac{du}{dx} = 1$$

$$\frac{du}{1} = dx$$

$$= 67.3 - 6 = 61.3$$

$$\int_2^3 x u + e^x \frac{du}{1}$$

$$\frac{1}{1} \int_2^3 x u + e^x du$$

$$u = x + 1$$

$$u - 1 = x$$

$$\frac{1}{1} \int_2^3 (\cancel{u-1}) u + e^{\cancel{u-1}} du$$

$$\frac{1}{1} \int_2^3 u + e du$$

$$\frac{1}{1} \int_2^3 (x+1) e$$



$$u-1=x$$

$$\frac{1}{2} \int_2^3 (u-1)u + e^{(u-1)} du$$

$$\frac{1}{2} \int_2^3 u + e du$$

$$= \frac{1}{2} \int_2^3 (x+1)e$$

$$[(x+1)e]_2^3$$

$$[(3+1)e] - [(2+1)e]$$

$$= 4 + 3 = 7$$