

①

②

$$\sigma_1 - \sigma_3 = \sigma_y$$

FACTOR OF SAFETY, n SAYS 2D IN
QUESTION $\therefore \sigma_2 = 0$

$$\therefore \sigma_1 - \sigma_3 = \frac{\sigma_y}{n}$$

$$\therefore n = \frac{\sigma_y}{\sigma_1 - \sigma_3}$$

$$= \frac{240}{140 - (-45)}$$

$$= \frac{240}{185}$$

$$= \underline{\underline{1.3}}$$

$$\textcircled{b} \quad \sigma_y^2 = \frac{1}{2} [(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2]$$

FACTOR OF SAFETY, n

$$\left(\frac{\sigma_y}{n}\right)^2 = \frac{1}{2} [(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2]$$

$$\left(\frac{240}{n}\right)^2 = \frac{1}{2} [(140 - 0)^2 + (0 - (-45))^2 + (-45 - 140)^2]$$

$$\frac{57,600}{n^2} = \frac{1}{2} [(140)^2 + 45^2 + (-185)^2]$$

$$\frac{57,600}{n^2} = 27,925$$

$$\therefore n = 1.4$$

①

(2) FROM COMPLEX STRESS FOR AN ELEMENT SUBJECTED TO SHEAR STRESS ONLY: (2)

$$\sigma_\theta = \tau_{xy} \sin 2\theta$$

$$\sigma_1 = \tau$$

$$\sigma_3 = -\tau$$



ASSUMING 2D STRESS PLANE $\therefore \sigma_2 = 0$

FROM THE SIMPLE THEORY OF ELASTIC TORSION

$$\frac{\tau}{r} = \frac{T}{J} \left(= \frac{6\theta}{L} \right)$$

$$\therefore \tau = \frac{Tr}{J}$$

• MEAN $\phi = 100 \text{ mm}$

THICKNESS = 3 mm

• OUTSIDE $\phi = 103 \text{ mm} = D$

INSIDE $\phi = 97 \text{ mm} = d$

• $J_o = \frac{\pi D^4}{32} = \frac{\pi}{32} (D^4 - d^4)$

$$= \frac{\pi}{32} \left((103 \times 10^{-3})^4 - (97 \times 10^{-3})^4 \right)$$

$$= \underline{\underline{2.36 \times 10^{-6} \text{ m}^4}}$$

• $\tau = \frac{Tr}{J}$

$$= \frac{51.5 \times 10^{-3}}{2.36 \times 10^{-6}} \times T$$

$$\tau = 21,822 T \quad *$$

(3)

$$(a) \quad \sigma_1 - \sigma_3 = \frac{\sigma_y}{n}$$

$$\tau + \tau = \frac{225 \times 10^6}{2.25}$$

$$2\tau = 100 \times 10^6$$

$$\tau = \underline{\underline{50 \times 10^6 \text{ N/m}^2}}$$

FROM *

$$T = \frac{\tau}{21.822}$$

$$= \frac{50 \times 10^6}{21.822}$$

$$= \underline{\underline{2.3 \text{ kNm}}}$$

$$(b) \quad \sigma_y^2 = \sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\nu(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1)$$

$$\left(\frac{225 \times 10^6}{2.25}\right)^2 = \tau^2 + 0 + (-\tau)^2 - 2 \times 0.3(\tau + -\tau)$$

$$= 2.6\tau^2$$

$$1 \times 10^{16} = 2.6(21.822T)^2 \quad \text{FROM *}$$

$$\therefore \sqrt{\frac{1 \times 10^{16}}{2.6}} \div 21.822 = T$$

$$\underline{\underline{2.8 \text{ kNm} = T}}$$

$$c) \sigma_y^2 = \frac{1}{2} [(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2]$$

(4)

$$\left(\frac{225 \times 10^6}{2.25} \right)^2 = \frac{1}{2} [(\tau - 0)^2 + (0 - (-\tau))^2 + (-\tau - \tau)^2]$$

$$1 \times 10^{16} = 3\tau^2$$

$$= 3 \times (21,822 \text{ T})^2$$

$$\therefore \sqrt{\frac{1 \times 10^{16}}{3}} = 21,822 = \tau$$

$$\underline{\underline{2.6 \text{ kNm}}} = \tau$$

③ = SIMPLE THEORY OF ELASTIC BENDING

⑤

$$\frac{\sigma}{y} = \frac{M}{I} \left(= \frac{E}{R} \right)$$

$$\sigma = \frac{My}{I}$$

$$I_0 = \frac{\pi d^4}{64}$$

$$\therefore \sigma = \frac{64 My}{\pi d^4}$$

$$= \frac{64 \times 1.9 \times 10^3 \times 25 \times 10^{-3}}{\pi \times (50 \times 10^{-3})^4}$$
$$= \underline{\underline{155 \text{ MN/m}^2}}$$

• SIMPLE THEORY OF ELASTIC TORSION

$$\frac{T}{J} = \frac{\tau}{R} = \left(\frac{G \theta}{L} \right)$$

$$\therefore \tau = \frac{TR}{J}$$

$$J_0 = \frac{\pi d^4}{32}$$

$$\therefore \tau = \frac{32 TR}{\pi d^4}$$

$$= \frac{32 \times T \times 25 \times 10^{-3}}{\pi \times (50 \times 10^{-3})^4}$$

$$= \underline{\underline{40.7 \times 10^3 \text{ T}}}$$

(6)

MAX SHEAR STRESS THEORY

$$\sigma_1 - \sigma_3 = \sigma_y$$

FROM COMPLEX STRESS THEORY

$$\text{LET } a = (\sigma_x + \sigma_y) \\ b = \sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}$$

$$\sigma_1 = \frac{1}{2} [(\sigma_x + \sigma_y) + \sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}] = \frac{1}{2}(a + b) = \frac{1}{2}a + \frac{1}{2}b$$

$$\sigma_3 = \frac{1}{2} [(\sigma_x + \sigma_y) - \sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}] = \frac{1}{2}(a - b) = \frac{1}{2}a - \frac{1}{2}b$$

$$\sigma_1 - \sigma_3 = \frac{1}{2}a + \frac{1}{2}b - (\frac{1}{2}a - \frac{1}{2}b)$$

$$= \frac{1}{2}a + \frac{1}{2}b - \frac{1}{2}a + \frac{1}{2}b$$

$$= b$$

$$= \sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}$$

~~$$\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}$$~~

~~$$\sigma_x = 185 \text{ MPa}$$~~

~~$$\sigma_y = 0 \text{ MPa}$$~~

~~$$\tau_{xy} = 40.7 \times 10^{-3} \text{ MPa} \quad (= 40.7 \times 10^{-3} \text{ MPa})$$~~

$$\therefore \sqrt{\sigma_x^2 + 4\tau_{xy}^2} = \sigma_{y(\max)}$$

$$\sqrt{\frac{\sigma_y^2 - \sigma_x^2}{4}} = \tau_{xy}$$

$$\sqrt{\frac{(200 \times 10^6)^2 - (155 \times 10^6)^2}{4}} = \tau_{xy}$$

$$63.2 \times 10^6 = \tau_{xy}$$

$$63.2 \times 10^6 = 40.7 \times 10^3 T$$

$$\therefore \underline{T = 1.55 \text{ kNm}}$$

$$(b) (\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 = 2\sigma_y^2$$

(8)

$$\sigma_1 = \frac{1}{2}a + \frac{1}{2}b$$

$$\sigma_2 = 0$$

$$\sigma_3 = \frac{1}{2}a - \frac{1}{2}b$$

$$\left(\frac{1}{2}a + \frac{1}{2}b\right)^2 + \left(-\frac{1}{2}a + \frac{1}{2}b\right)^2 + \left(\frac{1}{2}a - \frac{1}{2}b - \frac{1}{2}a - \frac{1}{2}b\right)^2 = 2\sigma_y^2$$

$$\left(\frac{1}{2}a + \frac{1}{2}b\right)\left(\frac{1}{2}a + \frac{1}{2}b\right) + \left(-\frac{1}{2}a + \frac{1}{2}b\right)\left(-\frac{1}{2}a + \frac{1}{2}b\right) + b^2 = 2\sigma_y^2$$

$$\frac{1}{4}a^2 + \frac{1}{4}ab + \frac{1}{4}ab + \frac{1}{4}b^2 + \frac{1}{4}a^2 - \frac{1}{4}ab - \frac{1}{4}ab + \frac{1}{4}b^2 + b^2 = 2\sigma_y^2$$

$$\frac{1}{2}a^2 + \frac{3}{2}b^2 = 2\sigma_y^2 \quad * *$$

$$a = \sigma_x + \sigma_y$$

$$= \sigma_x \quad (\text{since } \sigma_y = 0)$$

$$b = \sqrt{\sigma_x^2 + 4\tau_{xy}}$$

From * *

$$\frac{1}{2}\sigma_x^2 + \frac{3}{2}(\sigma_x^2 + 4\tau_{xy}) = 2\sigma_y^2$$

$$\tau_{xy} = \frac{2(2\sigma_y^2 - \frac{1}{2}\sigma_x^2)}{4} = \sigma_x^2$$

(9)

FROM * *

$$\frac{1}{2} \sigma_x^2 + \frac{3}{2} (\sigma_x^2 + 4\tau_{xy}^2) = 2\sigma_y^2$$

$$\frac{1}{2} \sigma_x^2 + \frac{3}{2} \sigma_x^2 + 6\tau_{xy}^2 = 2\sigma_y^2$$

$$2\sigma_x^2 + 6\tau_{xy}^2 = 2\sigma_y^2$$

$$\sigma_x^2 + 3\tau_{xy}^2 = \sigma_y^2$$

$$\sigma_x = 155 \text{ MPa}$$

$$\tau_{xy} = 40.7 \times 10^{-3} \text{ TPa} \quad (= 40.7 \times 10^{-3} \text{ TPa})$$

$$155^2 + 3((40.7 \times 10^{-3})^2 \text{ T}^2) = 200^2$$

$$(40.7 \times 10^{-3})^2 \text{ T}^2 = \frac{200^2 - 155^2}{3}$$

$$\text{T}^2 = \frac{5325}{(40.7 \times 10^{-3})^2}$$

$$\therefore \text{T} = 1.79 \times 10^3 \text{ Nm}$$

$$= \underline{\underline{1.79 \text{ kNm}}}$$

(4) THIN CYLINDER THEORY

(10)

3 MUTUALLY $\perp^r$ STRESSES

$$\text{HOOP} = \frac{Pd}{2t} = \sigma_1 \quad (\text{MAX})$$

$$\text{LONG} = \frac{Pd}{4t} = \sigma_2 \quad (\text{MIDDLE})$$

$$\text{RADIAL} = 0 = \sigma_3 \quad (\text{MIN})$$

(5) TRFKA

$$\sigma_1 - \sigma_3 = \sigma_y$$

$$\frac{Pd}{2t} = \frac{\sigma_y}{2}$$

$$\begin{aligned} \therefore t &= \frac{Pd}{\sigma_y} \\ &= \frac{2.5 \times 10^6 \times 2}{400 \times 10^6} \\ &= \underline{\underline{12.5 \text{ mm}}} \end{aligned}$$

(b) von Mises

(11)

$$(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 = 2 \sigma_y^2$$

IF $\sigma_3 = 0$ & FACTOR OF SAFETY

$$(\sigma_1 - \sigma_2)^2 + \sigma_2^2 + \sigma_1^2 = 2 \left(\frac{\sigma_y}{2} \right)^2$$

$$\left(\frac{Pd}{2t} - \frac{Pd}{4t} \right)^2 + \left(\frac{Pd}{4t} \right)^2 + \left(\frac{Pd}{2t} \right)^2 = 2 \left(\frac{\sigma_y}{2} \right)^2$$

$$\left(\frac{Pd}{2t} - \frac{Pd}{4t} \right) \left(\frac{Pd}{2t} - \frac{Pd}{4t} \right) + \frac{P^2 d^2}{16t^2} + \frac{P^2 d^2}{4t^2} = \frac{\sigma_y^2}{2}$$

$$\frac{P^2 d^2}{4t^2} - \frac{P^2 d^2}{8t^2} - \frac{P^2 d^2}{8t^2} + \frac{P^2 d^2}{16t^2} + \frac{P^2 d^2}{16t^2} + \frac{P^2 d^2}{4t^2} = \frac{\sigma_y^2}{2}$$

$$\frac{P^2 d^2}{t^2} \left(\frac{1}{4} - \frac{1}{8} - \frac{1}{8} + \frac{1}{16} + \frac{1}{16} + \frac{1}{4} \right) = \frac{\sigma_y^2}{2}$$

$$\frac{3P^2 d^2}{8t^2} = \frac{\sigma_y^2}{2}$$

$$\frac{3P^2 d^2}{4t^2} = \sigma_y^2$$

$$t = \sqrt{\frac{3P^2 d^2}{4 \sigma_y^2}} = \sqrt{\frac{3 \times (2.5 \times 10^6)^2 \times 2}{4 \times (400 \times 10^6)^2}}$$

$$t = \underline{\underline{10.8 \text{ mm}}}$$