

module code: ACH4400

10:00

Question One:

(a)

$$V_{\min} = \sqrt{\frac{W}{S} \times \frac{2}{P} \times \frac{1}{C_L}}$$

$$\downarrow$$

$$(V_{\min})^2 = \frac{W}{S} \times \frac{2}{P} \times \frac{1}{C_L} \quad \Rightarrow \quad V_{\min}^2 = \frac{2W}{S \times P \times C_L}$$

$$\boxed{S = \frac{2W}{V_{\min}^2 \times P \times C_L}}$$

$$\boxed{S = \frac{2 \cdot 480332}{200^2 \times 0,41 \times 0,52} = 115,00 \text{ m}^2}$$

(b)

$$\textcircled{I} \quad \frac{8x-3}{5} = \frac{-5x+6}{-3}$$

$$\Rightarrow (8x-3) \cdot (-3) = 5 \cdot (-5x+6)$$

$$\Rightarrow -24x + 9 = -25x + 30$$

$$x = 30 - 9$$

$$\boxed{x = 21}$$

$$\textcircled{II} \quad \left(\frac{(4x-1)}{3} - \frac{x}{4} = \frac{(4x+3)}{2} \right) \times 4$$

$$\left(\frac{4(4x-1)}{3} - x = 2(4x+3) \right) \times 3$$

$$4(4x-1) - 3x = 6(4x+3)$$

$$16x - 4 - 3x = 24x + 18$$

$$16x - 3x - 24x = 18 + 4$$

$$-11x = 22$$

$$\boxed{x = \frac{-22}{11} = -2}$$

III

$$2 \log(5x+3) = \log(x+1) + \log(x+2)$$

$$\log(5x+3)^2 = \log(x+1)(x+2)$$

$$(5x+3)^2 = (x+1)(x+2)$$

$$25x^2 + 30x + 9 = x^2 + 2x + x + 2$$

$$25x^2 + 30x + 9 = x^2 + 3x + 2$$

$$24x^2 + 27x + 7 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-27 \pm \sqrt{27^2 - 4 \cdot 24 \cdot 7}}{48}$$

$$x = \frac{-27 \pm \sqrt{57}}{48} \quad \begin{cases} x = \frac{-27 + \sqrt{57}}{48} = -0,41 \\ x = \frac{-27 - \sqrt{57}}{48} = -0,72 \end{cases}$$

$$\boxed{x = -0,41}$$

$$\boxed{x = -0,72}$$

Question one b:

(iv)

$$7^x \times 3^{x+2} = 5^{x-1}$$

$$\log(7^x \times 3^{x+2}) = \log 5^{x-1}$$

$$\log(7^x) + \log 3^{x+2} = (x-1) \cdot \log 5$$

$$x \cdot \log 7 + (x+2) \log 3 = (x-1) \log 5$$

$$0,35x + 0,48x + 0,95 = 0,70x - 0,70$$

$$\Delta \quad 0,63x = -1,65$$

$$x = \frac{-1,65}{0,63}$$

$$\boxed{x = -2,62}$$

Question one c:

(i) $27R^2 + 24R + 5$

(ii) $810^2 - 1440^2$

$$\boxed{(90 - 120)(90 + 120)}$$

(iii) $x - 6y + 0x - 0y = 0$

$$\boxed{(x + y)(x - y)}$$

$$x^2 - y^2 + 0x - 0y$$

Question one D:

(I) $x^2 + 9x + 8 = 0$

$(x+1)(x+8) = 0$

$x = -1$

$x = -8$

(II) $12x^2 - 9x + 3 = 0$

(III) $2x^2 + 17x - 30 = 0$

$(2x-3)(x+10) = 0$

$2x-3=0$

$x+10=0$

$2x=3$

$x = \frac{3}{2}$

$x = -10$

(IV) $x^2 - 0x - 256 = 0$

$(x+16)(x-16) = 0$

$x+16=0$

$x-16=0$

$x = -16$

$x = 16$

Question one e:

$$(x+5)^2 - x^2 - 32 = (x+1)(x+2) - x^2$$

$$\cancel{x^2} + 10x + 25 - \cancel{x^2} - 32 = \cancel{x^2} + 2x + x + 2 - \cancel{x^2}$$

$$10x + 25 - 32 = 3x + 2$$

$$7x = 2 - 25 + 32$$

$$7x = 9$$

$$x = \frac{9}{7} = 1,29$$

Question two A

I $y = 8x^3 - 2x^2 + x + 10$

$$\frac{dy}{dx} = 24x^2 - 4x + 1$$

II $y = (5x-3)(6x-1)$

$$y = 30x^2 - 5x - 18x + 3$$

$$y = 30x^2 - 23x + 3$$

$$\frac{dy}{dx} = 60x - 23$$

III $y = \frac{3}{\sqrt{x}} + \ln x$

$$y = 3x^{-\frac{1}{2}} + \ln x$$

$$\frac{dy}{dx} = -\frac{3}{2} x^{-\frac{3}{2}} + \frac{1}{x}$$

IV $y = 5 \sin \frac{x}{2} - 6 \cos 3x - \cos \frac{x}{3}$

$$\frac{dy}{dx} = \frac{5}{2} \cos \frac{x}{2} + 18 \sin 3x + 3 \sin \frac{x}{3}$$

V $y = e^{7x} \cos 3x$

$$\frac{dy}{dx} = 7e^{7x} \cos 3x + e^{7x} \cdot (-3) \sin 3x$$

VI $y = \frac{\sin 2x}{e^{3x}}$

$$\frac{dy}{dx} = \frac{2 \cos 2x \cdot e^{3x} - \sin 2x \cdot 3e^{3x}}{e^{6x}}$$

VII $y = (4x^3 - 2x + 1)^{\frac{1}{3}}$

$$\frac{dy}{dx} = \frac{1}{3} (4x^3 - 2x + 1)^{-\frac{2}{3}} \cdot (12x^2 - 2)$$

Question Two B

$$y = \sin 2x$$

$$\frac{dy}{dx} = 2 \cos 2x$$

$$\frac{d^2y}{dx^2} = -4 \sin 2x$$

$$\frac{d^3y}{dx^3} = -8 \cos 2x$$

$$\frac{d^4y}{dx^4} = 16 \sin 2x$$

Question Two C

$$Z = x^5 + 3x^3y^2 + 4y^5$$

↓_x

$$\frac{dZ}{dx} = 5x^4 + 9x^2y^2$$

$$\frac{d^2Z}{dx^2} = 20x^3 + 18xy^2$$

$$\frac{d^2Z}{dydx} = 18xy^2$$

↘

$$\frac{dZ}{dy} = 6x^3 + 20y^4$$

$$\frac{d^2Z}{dy^2} = 6x^3 + 80y^3$$

$$\frac{d^2Z}{dx dy} = 18x^2y$$

Question Two D

$$\begin{cases} y = 3x^2 + 16x + 30 \\ y = 2x^2 + 4x + 3 \end{cases}$$

$$y = 3(-3)^2 + 16(-3) + 30$$

$$y = 9$$

$$y = 2(-9)^2 + 4(-9) + 3$$

$$y = 3$$

$$3x^2 + 16x + 30 = 2x^2 + 4x + 3$$

$$x^2 + 12x + 27 = 0$$

$$x = \frac{-12 \pm \sqrt{12^2 - 4 \cdot 1 \cdot 27}}{2}$$

$$x = -3$$

$$x = -9$$

Question 1

Question two e:

$$y = 5x^2 + 4x - 1$$

$$\frac{dy}{dx} = 10x + 4$$

$$\frac{d^2y}{dx^2} = 10$$

$$6 \cdot 10 - 5 \cdot (10x + 4) + 7x - 1$$

$$60 - 50x - 20 + 7x - 1$$

$$39 - 43x = 39 - 43x$$