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ADVANCED ENGINE DESIGN:
CALCULATIONS ON THE PISTON,
CONNECTING ROD AND CRANKSHAFT
ASSEMBLIES

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1 Introduction

This report shall investigate the stresses on key engine components (the gudgeon pin, connecting rod (con-rod) assembly, and crankshaft) throughout both the standard and modified rpm range of a Ford 1.6 litre GDTI EcoBoost engine. A datasheet of cylinder pressures were provided, and from this the author measured and researched further data values, compiling all information into a MATLAB script to carry out various calculations. A critical analysis of fatigue safety factors within the engine was conducted, supported by investigation into how certain components can be redesigned to reduce mass, whilst maintaining strength. The equations used can be found within each section, the MATLAB script can be found in appendices one to four.

This report consists of four main sections, each analysing a different component. Throughout the report, tensile stress shall be considered negative due to its direction in the cylinder.

2 Global Constants and Initial Calculations

A new script was created for each section to reduce computing time, each loading in global constants shared across the questions, found below, from an external script.

Table 1 - Global Variables

Piston Data		
Data	Value	Source
Bore [m]	0.0790	(Ford, 2009)
Stroke [m]	0.0814	
Piston area [m]	0.0049	
R - Crank throw [m]	0.0407	
L - Rod length [m]	0.1340	
Materials Data		
Gudgeon pin UTS (alloy steel) [MPa]	1000	(MatWeb, 2014)
Gudgeon pin endurance limit (alloy steel) [Mpa]	450	(Hart, 2020)
Gudgeon pin density (steel) [Kg/m³]	7800	(MatWeb, 2014)
Con-rod UTS (powder forged steel) [MPa]	750	(AZO Materials, 2016)
Con-rod endurance limit (powder forged steel) [MPa]	337.5	(Hart, 2020)
Standard bolt UTS (alloy steel) [MPa]	1100	(MatWeb, 2014)
Standard bolt endurance limit (alloy steel) [MPa]	495	(Hart, 2020)
Standard bolt Young’s Modulus (alloy steel) [N/m³]	210	(AZO Materials, 2016)
Modified bolt UTS (Custom Age 625 PLUS alloy steel) [MPa]	1861	(Voort <i>et al.</i> , 2012)
Modified bolt endurance limit (Custom Age 625 PLUS alloy steel) [MPa]	837.45	(Hart, 2020)
Modified bolt Young’s Modulus (Custom Age 625 PLUS alloy steel) [N/m³]	210	(AZO Materials, 2016)
Con-rod cap Young’s Modulus (powder forged steel) [N/m³]	170	(AZO Materials, 2016)

Table 2 - Component Masses

Component Masses		
Con-rod mass [kg]	0.2674	(Hart, 2020)
Con-rod cap mass [kg]	0.1038	
Con-rod bolt mass [kg]	0.0352	
Total con-rod assembly mass [kg]	0.4416	
Gudgeon pin mass [kg]	0.0802	
Piston mass [kg]	0.2577	
Piston ring mass [kg]	0.0055	
Gudgeon pin circlip mass [kg]	0.0008	
Total piston assembly mass [kg]	0.3559	

The author used the con-rod and piston assemblies in the workshop to measure the dimension and mass values seen above in tables one and two.

Endurance limit values have been calculated as half of the ultimate tensile strength (UTS). This is an estimate of the magnitude of alternating stress that a material can infinitely withstand without failure (Budynas–Nisbett, 2014).

$$\text{Endurance limit} = (0.5) \cdot \text{UTS}$$

Equation 1 - Endurance Limit

This equation assumes the part is on correct working order. If it is rusty or otherwise degraded, or the surface is damaged or poorly machined, the endurance limit may only be around 20-25% UTS (Budynas–Nisbett, 2014).

2.1 Cylinder Pressures

Cylinder pressures were provided, the author used the 'xlsread' function to import these into MATLAB.

```
% 1000rpm
Average_Cylinder_Pressure_1000rpm = xlsread('DATA', '1000rpm', 'G4:G723'); % Pa
```

2.2 Gas Forces

These pressures could be used to find the force exerted on the piston by the gas.

$$\text{Force} = \text{Pressure} \times \text{Area}$$

$$\therefore \text{Gas Force} = \text{Cylinder Pressure} \times \frac{\pi \times \text{bore}^2}{4}$$

Equation 2 - Gas Force

2.3 Piston Acceleration

From the rpm, the angular velocity could be found. Using this with known constants, the piston acceleration at each crank angle could be calculated.

First Order:

Second Order:

$$R \times \omega^2 \times \cos \theta + \frac{R}{L} \times L \times \omega^2 \times 2 \cos \theta$$

Equation 3 - Piston Acceleration

3 Gudgeon Pin Stresses (Question 1)

A gudgeon pin forms the connection between the con-rod and the piston, providing a bearing to accommodate the pivot point caused by crankshaft rotation and piston reciprocation. As an integral engine component, it is crucial that a gudgeon pin can withstand the stresses acting upon it. This section shall analyse how the stresses effect the component, and its potential for being modified whilst retaining strength (Heywood, 2018).

3.1 Constant Values

For the following calculations, various specific constant values were measured by the author.

Table 3 - Question 1 Constants

<u>Gudgeon Pin Dimensions</u>		
<u>Data</u>	<u>Value</u>	<u>Source</u>
d1 - Outside diameter [mm]	21.00	(Hart, 2020)
d2 - Inside diameter [mm]	15.44	
L1 - Length of one side pin protrusion from conrod [mm]	16.29	
L2 - Distance to Centre of Gravity [mm]	5.46	

From these, the neutral axis of the pin was taken to be half the outside diameter:

$$Neutral\ Axis\ (Y) = \frac{Outside\ diameter}{2}$$

Equation 4 - Gudgeon Pin Neutral Axis

Specific masses could then be calculated to be used in later calculations:

$$Mass\ of\ pin\ protrusion\ from\ conrod = Length\ of\ protrusion \cdot \left(\frac{Outside\ radius}{2} \right)^2 - \left(\frac{Inside\ radius}{2} \right)^2 \cdot Alloy\ steel\ density$$

Equation 5 - Mass of Pin Protrusion

The mass of the reciprocating part of the assembly could then be calculated. For the following calculations, one side of the pin's protrusion from the small end of the con-rod is being considered, thus the mass of one protrusion is considered as the pin's reciprocating mass. The piston's reciprocating mass is taken as the piston total, plus the section of con-rod reciprocating with it, considered to be 1/3.

$$Reciprocating\ mass\ of\ pin = Mass\ of\ pin\ protrusion$$

$$Reciprocating\ mass\ piston = Mass\ of\ piston\ assembly + \left(\frac{1}{3} \cdot conrod\ mass \right)$$

Equation 6 - Gudgeon Pin and Piston Reciprocating Masses

The author calculated the second moment of area for the gudgeon pin. This is a numerical measure of how a cross sectional area resists bending when loaded, thus can be used to calculate bending stresses c

$$\text{Second moment of area} = \frac{\pi}{64} \cdot (\text{Outside diameter})^4 - (\text{Inside diameter})^4$$

Equation 7 - Second Moment of Area

3.2 Inertia Forces

For any given engine component, the two main loadings will be compressive stress, a result of gas force, and tensile stress, a result of inertia force. To calculate both, the inertia force of both the piston and pin are found.

$$\text{Piston inertia force} = \text{Piston acceleration} \cdot \text{Piston reciprocating mass}$$

$$\text{Pin inertia force} = \text{Piston acceleration} \cdot \text{Pin reciprocating mas}$$

Equation 8 - Piston and Pin Inertia Forces

3.3 Net Forces

By calculating the net force within the cylinder, the maximum compressive and tensile forces can be found. Compressive force will be positive and tensile negative.

$$\text{Net force} = \text{Gas force} - \text{Inertia Force}$$

Equation 9 - Net Force on Piston

As stated, the maximum values here will give maximum loadings, thus the MATLAB script was written to output the maximum and minimum values at each rpm, and the corresponding crank angle location, to allow the author to find corresponding values at later calculations.

```
% 1000rpm
Net_Force_Piston_1000rpm = (Gas_Force_1000rpm)-(Inertia_Force_Piston_1000rpm); % N

[Max_Net_Force_Piston_1000rpm,I_Max_Net_1000rpm] = max(Net_Force_Piston_1000rpm); % N
[Min_Net_Force_Piston_1000rpm,I_Min_Net_1000rpm] = min(Net_Force_Piston_1000rpm); % N
```

This, for example, outputs the maximum value to variable “Max_Net_Force_Piston_1000rpm” and the corresponding crank angle to variable “I_Max_Net_1000rpm”.

3.4 Bending Moment

The bending moment at maximum compressive and tensile load can be calculated to start to analyse stress. This is effectively the net torque applied to the gudgeon pin, causing bending stress.

Bending moment

$$= \frac{\text{Net force}}{2} \cdot \text{Pin protrusion length} - \text{Pin inertia force} \cdot \text{Distance to CoG}$$

Equation 10 - Bending Moment on Gudgeon Pin

3.5 Bending Stress

As the bending moment is applied, material above the neutral axis extends, and below shortens (or visa vera). The resistance of the material to this is quantified as bending stress (Young and Budynas, 2002).

$$\text{Bending Stress} = \frac{\text{Bending moment} \cdot \text{Neutral axis}}{\text{Second moment of area}}$$

Equation 11 - Bending Stress on Gudgeon Pin

3.6 Shear Stress

Shear stress is the resistance of the material to deform due to loading on a perpendicular plane.

$$\text{Shear stress} = \frac{\frac{\text{Net Force}}{2}}{\text{Surface area}}$$

Equation 12 - Shear Stress on Gudgeon Pin

3.7 Combined Stress

To analyse how these stresses interact, the combined (or resultant) stress is analysed.

$$\text{Combined stress} = \sqrt{\text{Bending stress}^2 + 2 \cdot \text{Shear stress}^2}$$

Equation 13 - Combined Stress on Gudgeon Pin

3.8 Nominal Stresses

With all stresses calculated and combined, the maximum overall compressive and tensile stresses could be found by plotting the maximum values for each rpm and comparing.

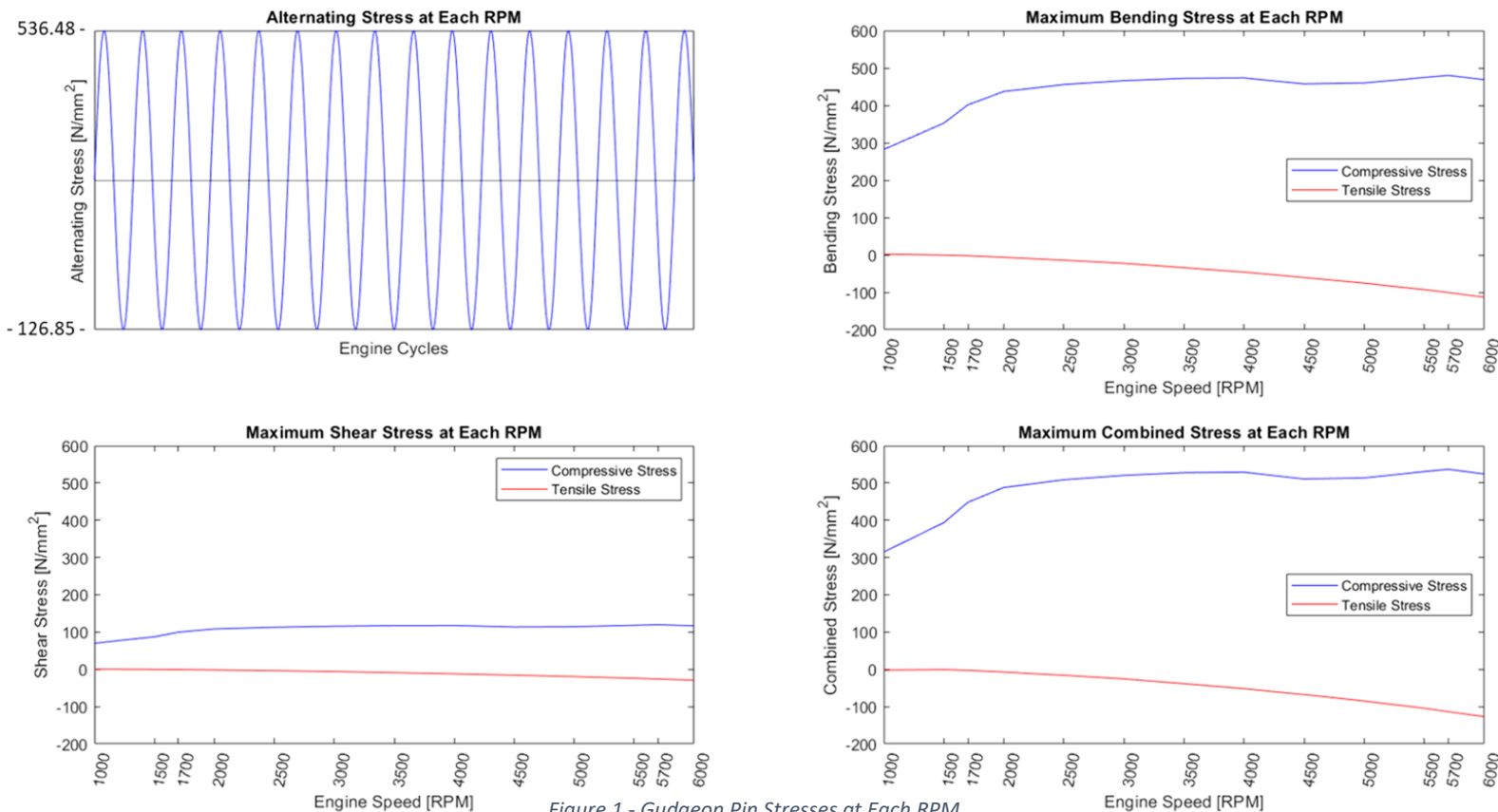


Figure 1 - Gudgeon Pin Stresses at Each RPM

Figure one shows that compressive stresses are considerably higher than tensile stresses. The compressive stresses noticeably increase around 1700rpm, this is presumed to be where the turbo has spooled. As compressive forces are related to gas forces, the forced induction is expected to cause an increase here; moreover, the compressive stresses reduce again around 4500rpm, this is roughly the point that the standard turbo can no longer efficiently operate, thus reducing the rate of increase of compressive stress.

Tensile stress steadily increases throughout the rpm range as it is calculated from inertia force, a product of rpm.

It is clear that the main component of the combined stress is the bending stress, with shear stress accounting for only 24% of the overall value. The nature of the gudgeon pin's position within the small end of the con-rod means each end of the pin is effectively cantilevered, with a large force producing a large bending moment. There are much fewer sources of load parallel to the pin's plane, explaining the lower shear stress and therefore difference in magnitude.

Figure one also shows how the gudgeon pin is subject to particularly demanding stresses, as the sinusoidal reciprocation of an engine will cycle stresses constantly between compressive and tensile, creating a constantly varying load.

3.9 Fatigue Safety Factor

A fatigue safety factor numerically assesses how likely a part is to have an infinite service life. By taking the worst case scenario in terms of loading and stresses, calculations can be made to compare the maximum permissible range of stresses before fatigue, which can be compared to the range of stresses seen. It is accepted that if this figure is above one, the part will be able to survive an infinite number of cycles without failing (Smith, 2010). This can be tested experimentally also; it is generally accepted that if an engine component can withstand 10^7 cycles, it will withstand infinite cycles (Taylor, 1985).

To calculate the fatigue safety factor, the maximum compressive and tensile combined stresses were taken throughout the entire engine speed range, compressive at 5700rpm and tensile at 6000rpm as seen on figure one. Calculating a stress mean and range from these two values gives an overall approximation of the component's safety factor, although in reality the engine would not see these extreme values in one cycle. However this figure truly acts as a worst case scenario and ensures the component can withstand any normal operation. This may also go some way to simulating cyclic variation.

The allowable range calculates the maximum alternating stress a material can handle without fatigue, the equation is derived in section 3.10.

$$Allowable\ range = 2 \cdot End\ limit \cdot 1 - \frac{Mean\ stress}{UTS}$$

Equation 14 - Allowable Range on Gudgeon Pin

From this,

$$Fatigue\ safety\ factor = \frac{Allowable\ range}{Actual\ range}$$

Equation 15 - Fatigue Safety Factor

For the gudgeon pin a safety factor of 1.19 was calculated, showing the component should have infinite life, even at very worst case scenarios.

3.10 Goodman Diagram

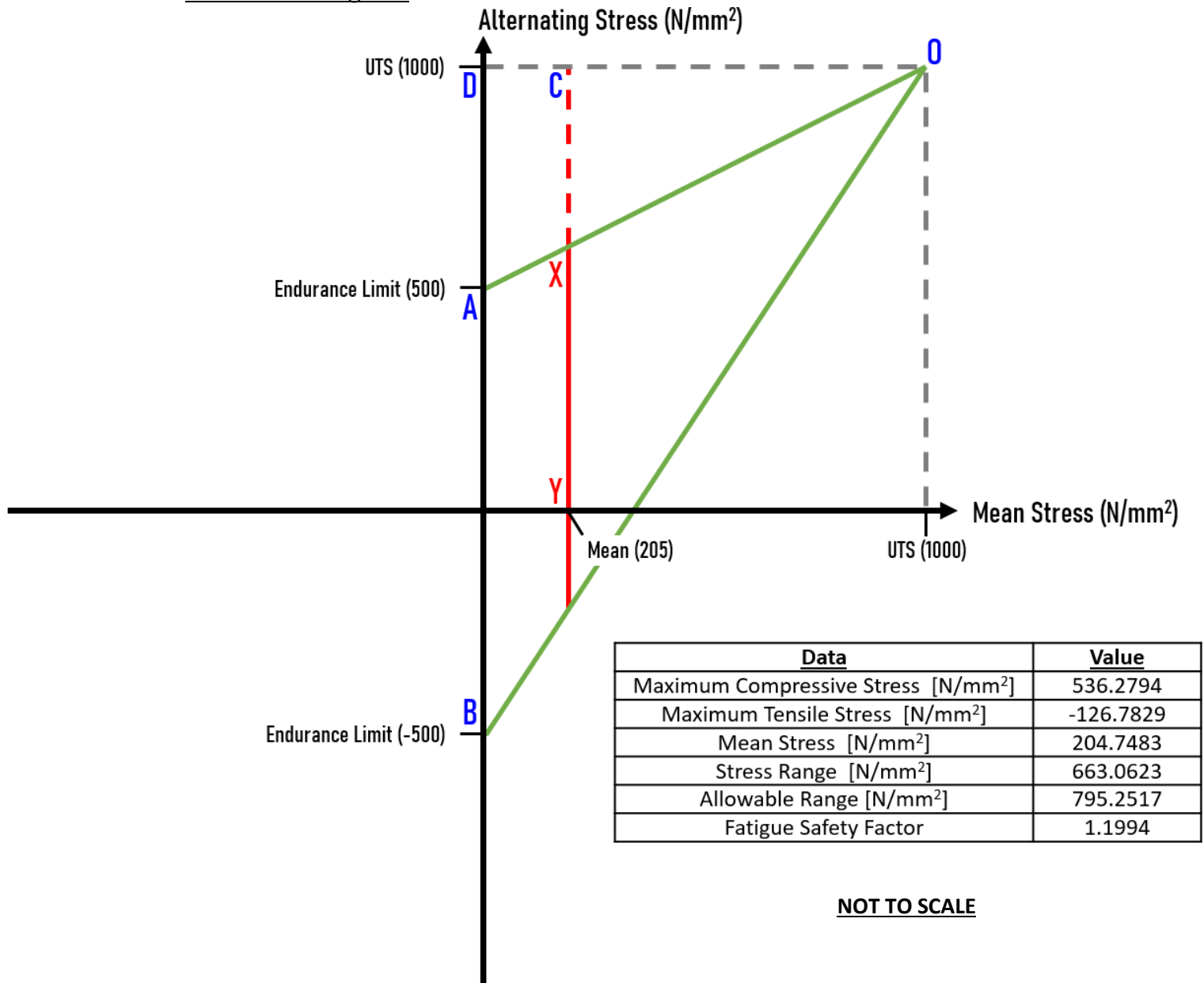


Figure 2 - Gudgeon Pin Stresses Goodman Diagram

A Goodman diagram visually displays how the fatigue safety factor can be calculated, through finding the allowable range.

Figure one shows two pairs of similar triangles: OXY & OAB, OXC & OAD.

$$\therefore \frac{XY}{AB} = \frac{OX}{OA} = \frac{OC}{OD} = \frac{UTS - Mean}{UTS}$$

$$\therefore \text{Allowable range at mean stress} = XY = AB \cdot \frac{UTS - Mean}{UTS}$$

Equation 16 - Allowable Range Derivation

$$\therefore XY = (2 \cdot \text{End limit}) \cdot \left(1 - \frac{Mean}{UTS}\right)$$

Thus, the derivation of equation 15.

3.11 Gudgeon Pin Analysis

Overall there is no sign of the gudgeon pin breaking within the standard engine speed range. With a worst case scenario safety factor of 1.19, the alternating stresses seen will not fatigue the component and thus it can be said it has an infinite service life. This leads the author to believe there is room for design modification, the component could be made lighter at the cost of a small reduction in strength and still be sufficient. Moreover, this component could handle light engine modification before needing to be re-designed.

To improve the design further, in the case of engine modification, nitriding could be applied. This is the process of blasting the surface of the component with nitrogen molecules, the impact of which leaves a residual 100N/mm² compressive force on the surface (Budynas–Nisbett, 2014) .

Following this process:

$$\begin{aligned}\sigma_{MAX} &= 536 - 100 = 436 \\ \sigma_{MIN} &= -127 - 100 = -137 \\ \therefore \sigma_{RANGE} &= 436 - (-137) = 573 \\ \therefore \sigma_{MEAN} &= \frac{436 + (-137)}{2} = 146.5\end{aligned}$$

Therefore the mean has moved left on the Goodman diagram, thus creating a larger endurance limit, creating a larger allowable range and therefore a higher safety factor.

4 Con-rod Stresses (Question 2)

A con-rod within an internal combustion engine connects the piston to the crankshaft. It is the main link between rotating and reciprocating motion, therefore is required to withstand all compressive and tensile stresses experienced within the engine. A connecting rod will experience bending stresses due to centrifugal effects, compressive shear stress due to gas force, and tensile shear stress due to inertial load (Desai, 2014). Section four shall analyse how the con-rod reacts to these stresses at three crucial points:



Figure 3 - Three Con-rod Points of Interest (Hart, 2020)

4.1 Constant Values

For the following calculations, various specific constant values were measured by the author.

Table 5 - Question 1 Constants

<u>Gudgeon Pin Dimensions</u>		
<u>Data</u>	<u>Value</u>	<u>Source</u>
Length at A [mm]	18.18	(Hart, 2020)
Width at A [mm]	4.261	
CSA at A [mm ²]	16.295	
Length 1 at B [mm]	13.00	
Width 1 at B [mm]	7.13	
Length 2 at B [mm]	4.57	
Width 2 at B [mm]	3.24	
CSA at B [mm ²]	122.303	
Length at C [mm]	20.51	
Width at C [mm]	8.96	
CSA at C [mm ²]	183.7696	

The reciprocating mass of the piston was calculated as in equation 6, and the individual con-rod reciprocating masses were calculated as follows:

$$\text{Reciprocating mass at A} = \text{Mass of piston assembly}$$

Equation 17 - Reciprocating mass at A

At point A, the con-rod reciprocating mass was taken to be negligible. This section of the con-rod is a very small proportion of the overall mass and would not increase accuracy of the calculation.

$$\text{Reciprocating mass at B} = \text{Mass of piston assembly} + \frac{1}{3} \cdot \text{Conrod mass}$$

Equation 18 - Reciprocating mass at B

At point B, a third of the con-rod mass is assumed to be reciprocating. This point does not include the cap or bolts .

$$\begin{aligned} \text{Reciprocating mass at C} = \\ \text{Mass of piston assembly} + \frac{2}{3} \cdot \text{Total conrod Assembly mass} \end{aligned}$$

Equation 19 - Reciprocating mass at C

At point C, two thirds of the total con-rod assembly is assumed to be reciprocating. This includes a proportion of the cap and two bolt masses also.

4.2 Net Forces

For the con-rod investigation, the piston inertia forces for each RPM are calculated as per equation eight, similarly the net compressive and tensile forces are calculated as per equation nine.

4.3 Gas Force

For this section, the individual net forces at each point need to be calculated to accurately understand the stresses acting upon them. To start this, the MATLAB script assigned to a variable the gas force at the location of the highest compressive and tensile forces for each RPM using the assigned location variable "I_Net".

```
% 1000rpm
Max_Gas_Force_1000rpm = Gas_Force_1000rpm(I_Max_Net_1000rpm); % N
Min_Gas_Force_1000rpm = Gas_Force_1000rpm(I_Min_Net_1000rpm); % N
```

4.4 Individual Inertia Forces

Similarly, the inertia force acting on each point at every RPM was calculated using the corresponding piston acceleration multiplied by each reciprocating mass, as per equation eight.

```
% 1000rpm
Max_Inertia_Force_A_1000rpm = Piston_Accel_1000rpm(I_Max_Net_1000rpm)*(Recip_Mass_A); % N
Max_Inertia_Force_B_1000rpm = Piston_Accel_1000rpm(I_Max_Net_1000rpm)*(Recip_Mass_B); % N
Max_Inertia_Force_C_1000rpm = Piston_Accel_1000rpm(I_Max_Net_1000rpm)*(Recip_Mass_C); % N
```

4.5 Individual Net Forces

As per equation nine, the maximum compressive and tensile forces specific to each point on the con-rod could then be calculated for each RPM using the individual values.

```
% 1000rpm
Max_Net_Force_A_1000rpm = (Max_Gas_Force_1000rpm) - (Max_Inertia_Force_A_1000rpm); % N
Max_Net_Force_B_1000rpm = (Max_Gas_Force_1000rpm) - (Max_Inertia_Force_B_1000rpm); % N
Max_Net_Force_C_1000rpm = (Max_Gas_Force_1000rpm) - (Max_Inertia_Force_C_1000rpm); % N

Min_Net_Force_A_1000rpm = (Min_Gas_Force_1000rpm) - (Min_Inertia_Force_A_1000rpm); % N
Min_Net_Force_B_1000rpm = (Min_Gas_Force_1000rpm) - (Min_Inertia_Force_B_1000rpm); % N
Min_Net_Force_C_1000rpm = (Min_Gas_Force_1000rpm) - (Min_Inertia_Force_C_1000rpm); % N
```

4.6 Shear Stress

When analysing the con-rod, the loading is perpendicular to the component. There will be minimal bending stresses caused by the rotation of the big end, but these are considered negligible; therefore, only shear stresses are considered (Desai, 2014; Heywood, 2018).

$$\text{Shear stress at A} = \frac{\text{Net Force}}{2} / \text{Surface area at A}$$

Equation 20 - Shear Stress at A

As point A analyses the small end, the net force is split around the gudgeon pin hole, thus half net force is used to analyse half of the component.

$$\text{Shear stress at B} = \frac{\text{Net force}}{\text{Surface area at B}}$$

Equation 21 - Shear Stress at B

At point B, the net force is distributed evenly along the body of the rod, thus full net force is used.

$$\text{Shear stress at C} = \frac{\frac{\text{Net Force}}{2}}{\text{Surface area at C}}$$

Equation 22 - Shear Stress at C

As point C analyses the big end, the net force is split around the crankshaft journal hole, thus half net force is used to analyse half of the component.

4.7 Nominal Stresses

For the maximum compressive and tensile stresses, the maximum and minimum shear stresses were found respectively for every rpm, to once again calculate an overall worst case scenario fatigue safety factor for the entire rpm range.

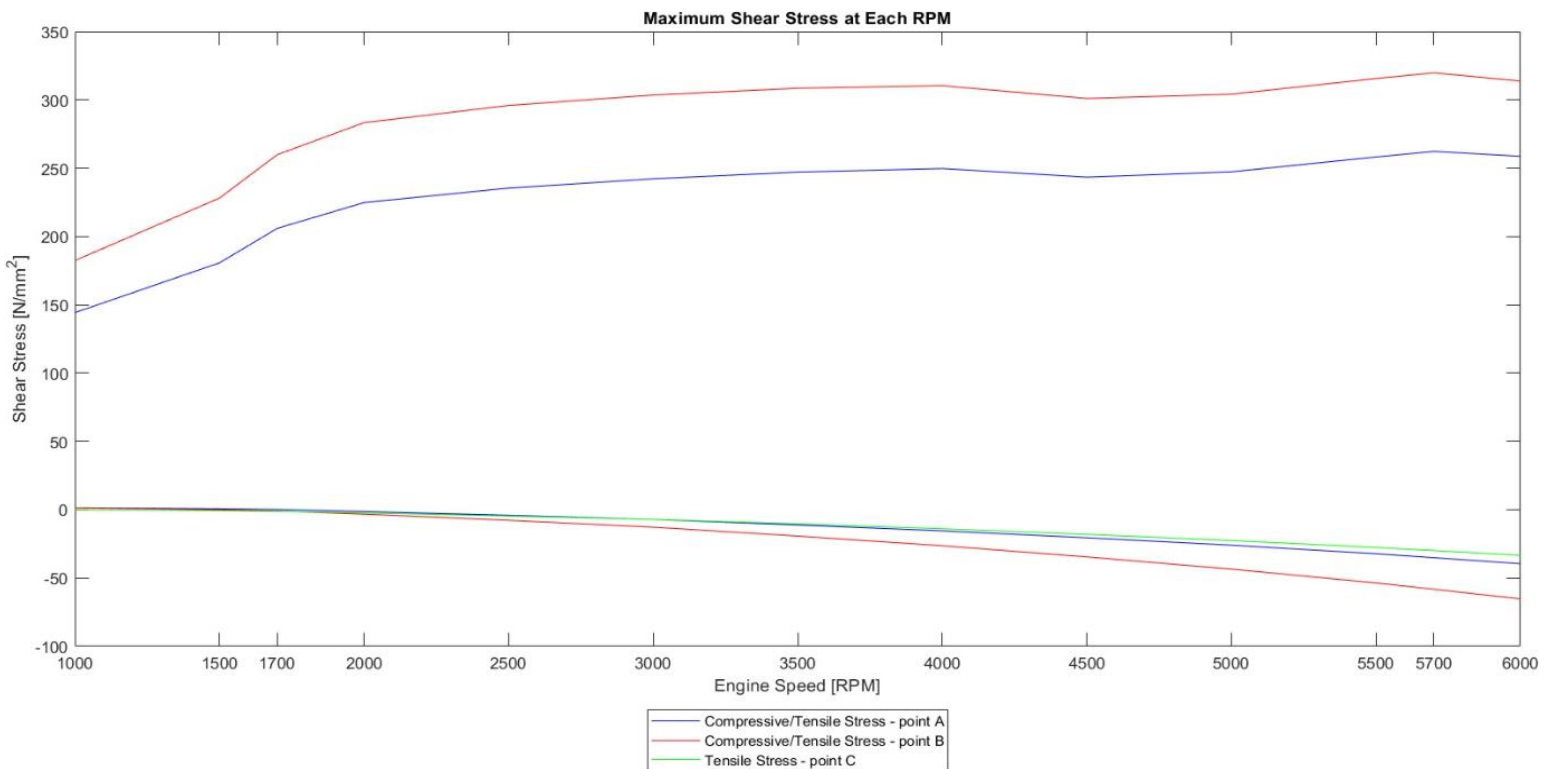


Figure 4 - Con-rod Shear Stresses at Each RPM

Figure four follows the same trends seen as figure 1, with a considerably higher compressive stress, increasing significantly when the turbo spools and tailing off again at higher RPM. Point B sees the highest stresses, the maximum compressive stress being over three times larger than at C.

Point C see does not experience any compressive stress as the compressive force acts on the top of the crankshaft journal hole; point C is below this. Therefore this point is only acted upon by tensile stress.

Again as per figure 1, the tensile stresses increase with engine speed and follow the same magnitude trends as compressive stresses.

4.8.1 Point A Goodman Diagram



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4.8.2 Point B Goodman Diagram

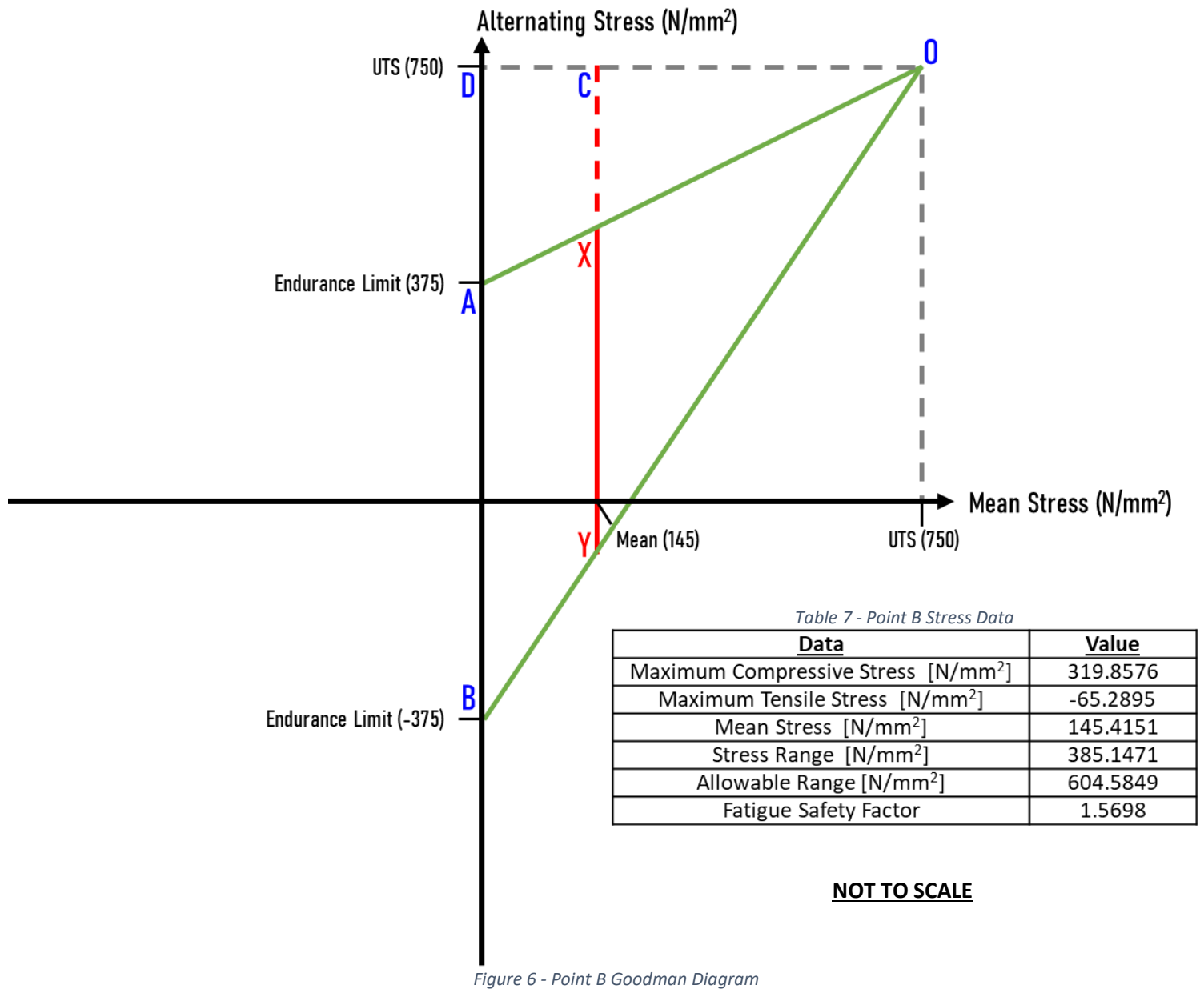


Figure six shows that point B also does not pose significant risk to the component when under stress. Although a lower FSF than point A, it is still over 1 and thus this point is expected to have an infinite service life.

4.8.3 Point C Goodman Diagram

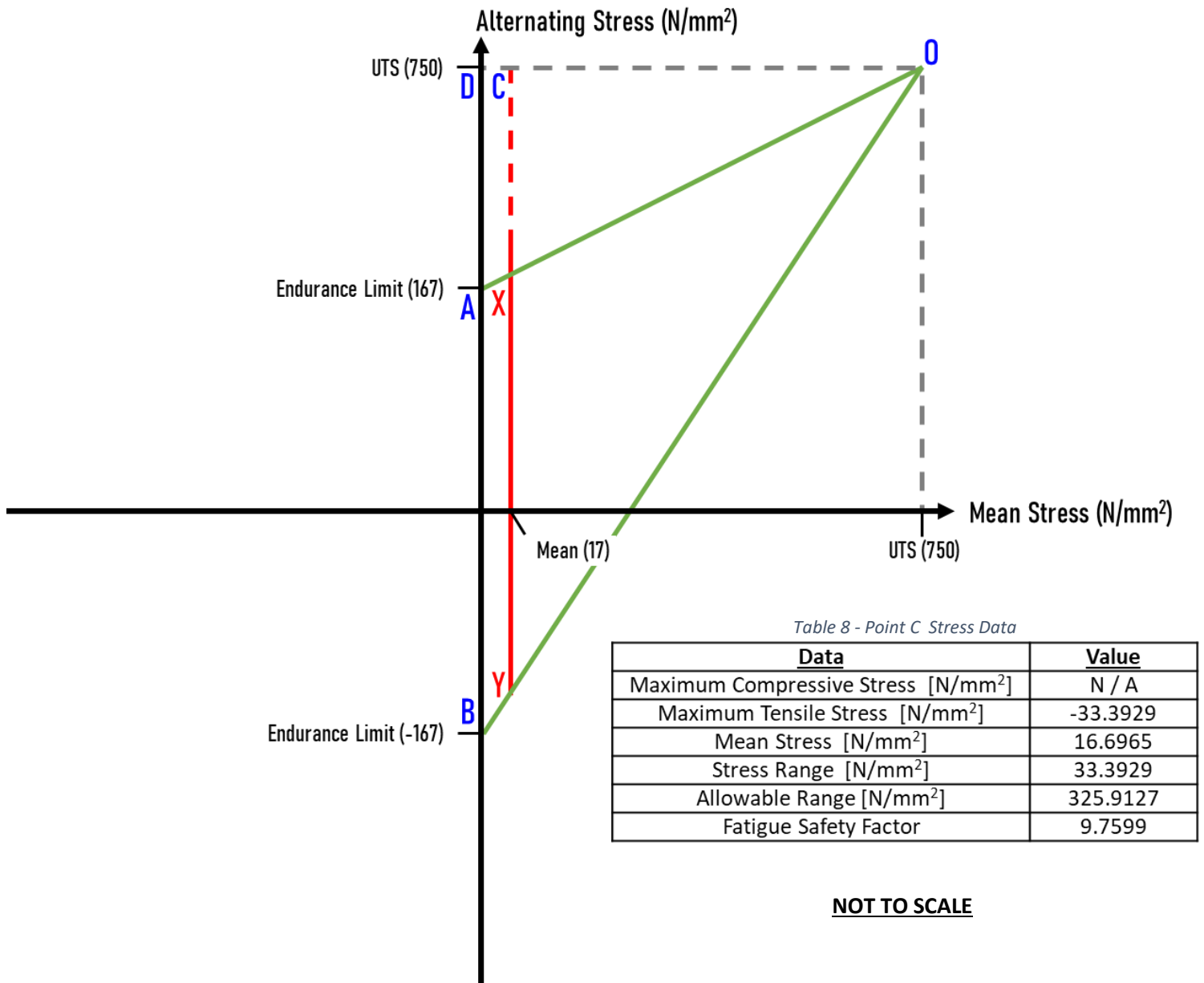


Figure 7 - Point C Goodman Diagram

Figure seven shows that the stresses acting on point C have no chance of causing failure. As shown on figure four, the main stresses are from compressive force, which do not act at this point; thus, the relatively low tensile forces from inertial loading do not pose significant risk to the component here.

4.9 Stress Concentration Factors

When the geometry of a component abruptly changes, a stress concentration is created. The stresses running through that area cannot be evenly distributed as per usual, resulting in the theoretical lines of stress running through the part converging, and increasing the levels of stress experienced. When mathematically modelling the stresses throughout a part, this is not considered; therefore stress concentration factors are included to reduce the material's endurance limit, as the high stresses of the original limit would likely cause the part to fail (Sengupta, 2009).

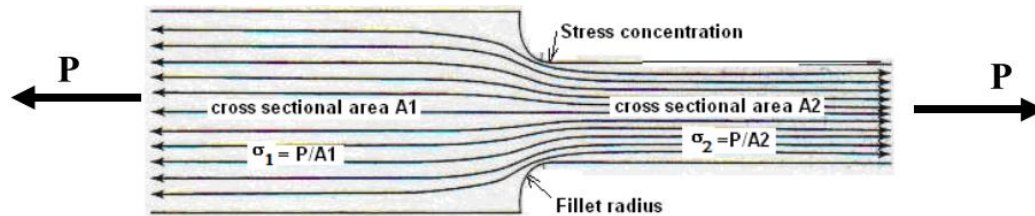


Figure 8 - Stress Concentration Example (Sengupta, 2009)

4.9.1 Peterson Stress Concentration Factors

To understand what stress concentration factor to use, the author consulted the Peterson Stress Concentration Factors book, containing multiple scenarios to choose from. The radii at points A and B were too large to apply stress concentration, however, point C matched the criteria of figure 10, so the following measurements were taken:

Table 9 - Con-rod Point C Stress Concentration Factor Data

Stress Concentration Factor Data		
Data	Value	Source
H - Height [mm]	11.21	(Hart, 2020)
D - thickness [mm]	9.00	
r - radius [mm]	0.47	
r/d	0.0522	
H/d	1.2456	

Knowing these values, the correct graph was chosen and thus the correct stress concentration factors were found.

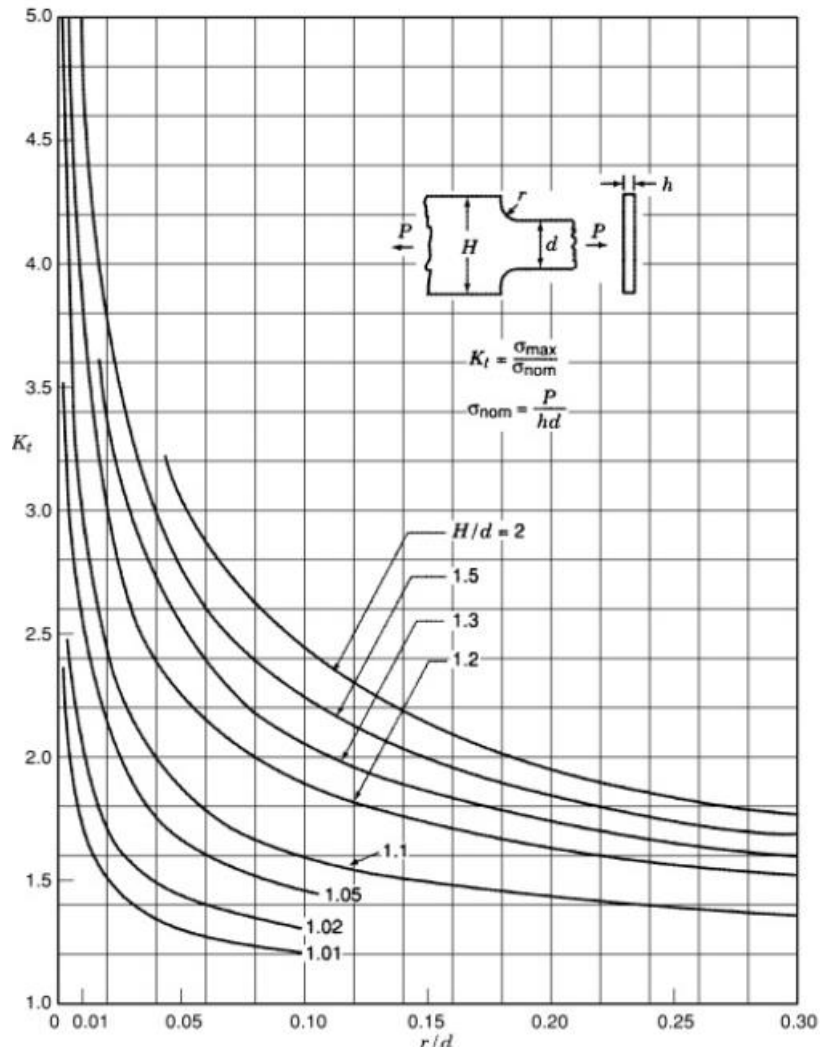


Figure 9 - Peterson Shoulder Fillet Stress Concentration Graph (Pilkey, 2008)

Using figure 10, a stress concentration factor of 2.25 was chosen for point C.

4.9.2 Final Endurance Limit

As mentioned, the stress concentration factor can be included by reducing the endurance limit.

$$\text{Final endurance limit} = \frac{\text{Endurance limit}}{\text{Stress concentration factor}}$$

Figure 10 - Final Endurance Limit

4.10 Fatigue Safety Factors

From here, the allowable range was calculated for each point as per equation 14, then the fatigue safety factor was calculated as per equation 15.

Table 10 - Question 2 Fatigue Safety Factors

Point A Fatigue Safety Factor	2.0937
Point B Fatigue Safety Factor	1.5698
Point C Fatigue Safety Factor	9.7599

5 Con-rod Bolt Stresses (Question 3)

Con-rod bolts fasten together the con-rod cap and the con-rod itself. It is necessary these parts separate for assembly; however they need to be securely attached as the bottom end experiences high magnitudes of various stresses. A con-rod bolt will experience tensile stress from tightening, and from inertial loads from the engine. The nature of two fastened components under load will also result in shear stress as the two components try to pull apart. The bolt will also experience compressive stress as the gas combusts. Moreover, a bending stress will be seen as the bottom end distorts. On firing, the bolts will be inclined to bend outwards, conversely, on exhaust stroke the bolts will bend inwards (Griza *et al.*, 2009). For this section, the alternating tensile shear stresses will be analysed, as the bending and compressive stresses are considered negligible.

5.1 Constant Values

For the following calculations, various specific constant values were measured by the author.

Table 11 - Question 3 Constants

Con-rod Bolt Dimensions		
<u>Data</u>	<u>Value</u>	<u>Source</u>
Bolt head diameter [mm]	12.90	(Hart, 2020)
Bolt shank diameter [mm]	7.35	
Average bolt diameter [mm]	10.125	
Bolt length to second engaged thread [mm]	25.75	
Total surface area [mm ²]	175.07	
Cap surface area [mm ²]	135.87	
Bolt surface area [mm ²]	39.20	(Bosch, 2017)
Bolt thread μ	0.12	
Bolt head μ	0.18	

The reciprocating mass for this question was calculated as in equation 6. The rotational mass was also required for the following calculations, which was taken to be two thirds of the con-rod mass.

5.2 Surface Area Ratio (Question 3A)

The bolt stiffness to cap stiffness ratio is important to know. This will give an indication as to what component is likely to deform or fail more. If the bolt stiffness is comparatively small, it will stretch more than the cap, and visa versa. A well designed con-rod bolt and cap combination will both be as stiff as possible to reduce deformation at high RPM, resulting in less bolt stretch (Desai, 2014).

To estimate the stiffness ratio, the surface areas can be compared.

$$\text{Surface area ratio} = \frac{\text{Bolt surface area}}{\text{Cap surface area}}$$

Equation 23 - Surface Area Ratio

This equates to 0.2885, suggesting that the bolt is only around 28% as stiff as the cap, this starts to explain why con-rod bolts are prone to stretching.

5.3 Bolt Stiffness (3B)

The bolt stiffness can be calculated by the following:

$$\text{Bolt stiffness } (K_B) = \frac{\text{Young's modulus} \cdot \text{Surface area}}{\text{length}}$$

Equation 24 - Stiffness Equation

The standard Ford bolt stiffness was calculated to be 319.689 N/mm.

The cap stiffness was calculated to be 897.0058 N/mm.

This gives a stiffness ratio of 0.3564, 20% bigger than the estimated value, showing the bolt is slightly stiffer than previously thought.

5.4 Maximum Alternating Load (3C)

To quantify the stresses acting on the con-rod bolts, the alternating stress shall be calculated through the following equations.

5.4.1 Angular Velocity

To calculate the angular velocity, the engine speed must be converted to radians per second using the following identity:

$$\text{Angular velocity} \equiv \left(\frac{2 \cdot \pi \cdot \text{RPM}}{60} \right) \text{rads} \cdot \text{sec}^{-1}$$

Equation 25 - Angular Velocity Identity

5.4.2 Inertia Force

Inertia force is calculated using the above angular velocity and masses:

$$\begin{aligned} \text{Inertia force} = & (\text{Recip. mass}) \cdot (\text{Crank throw}) \cdot (\omega^2) \cdot \left(1 - \left(\frac{\text{Crank throw}}{\text{Rod length}} \right) \right) + \\ & (\text{Rot. mass}) \cdot (\text{Crank throw}) \cdot (\omega^2) \end{aligned}$$

Equation 26 - Con-rod Bolt Inertia Force

5.4.3 Alternating Load

The alternating load acting on the measured surface area creates the alternating stress:

$$\text{Alternating load} = \frac{K_{CAP}}{K_{BOLT}} \cdot \text{Inertia force}$$

Equation 27 - Con-rod Bolt Alternating Load

5.4.4 Alternating Stress

From this, the alternating stress, the range of stresses acting on the bolt, can be calculated:

$$\text{Alternating stress} = \frac{\text{Alternating load}}{\text{Bolt surface area}}$$

Equation 28 - Con-rod Bolt Alternating Stress

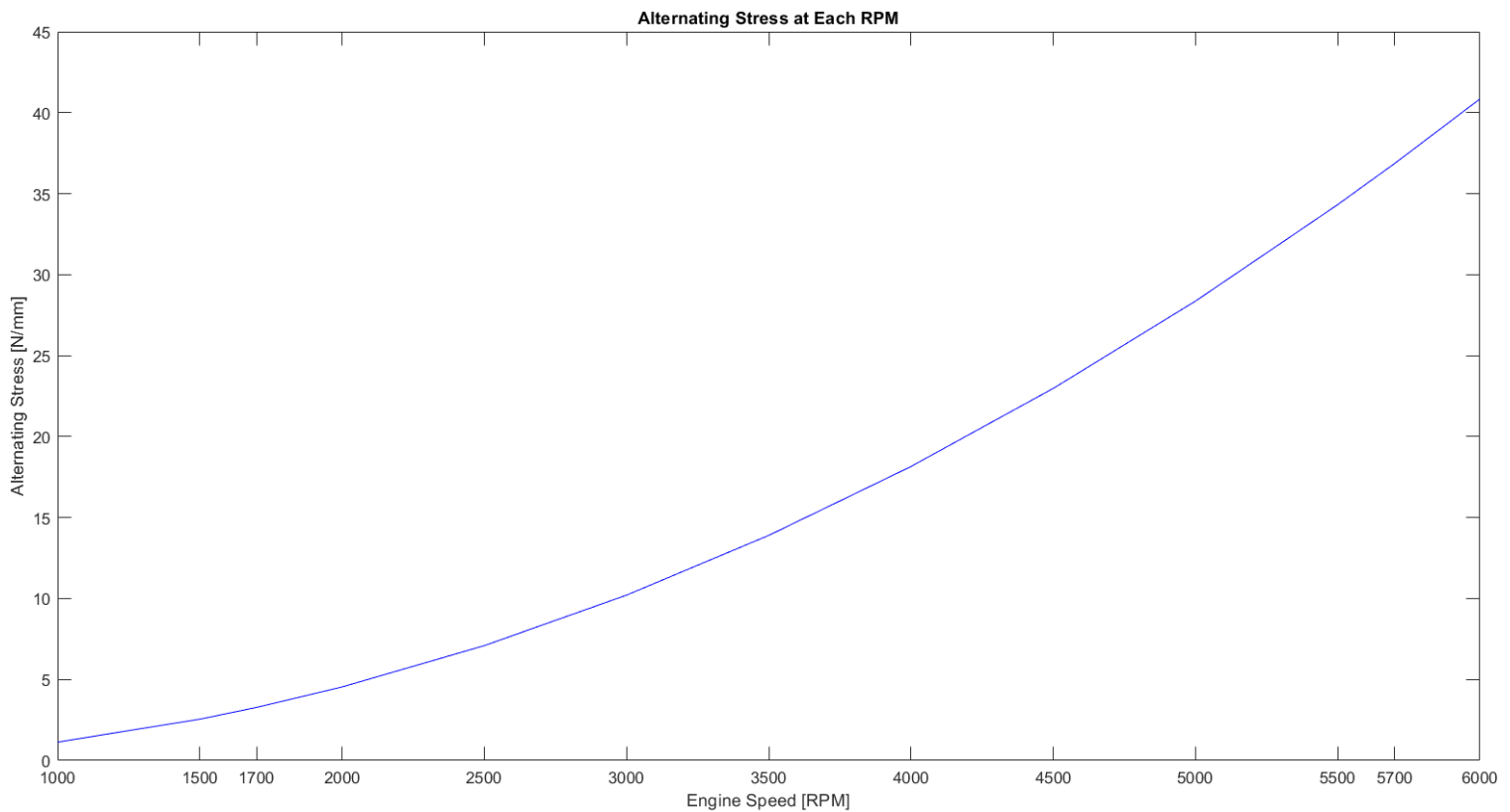


Figure 11 - Alternating Stress at Each RPM

As expected, due to being calculated from inertia forces, the tensile load increases with engine speed. The maximum value is 40.846 N/mm² at 6000rpm. This shows that the bolt will be severely impacted by increased rpm. This is something to consider closely later when modifying the engine.

5.5 Bolt Tightening Torque (3D)

Every bolt within a mass-produced component will have a specified tightening torque. This torque is calculated by taking factors such as yield point and overcoming frictional resistive torque into account.

5.5.1 Yield and Axial Stress

This section is specifically looking at the bolt tightening into 90% Yield. Yield strength can be taken as roughly 90% of the UTS, and shall be for the following calculations (AZO Materials, 2016).

From this, axial strength can be calculated, quantified as 60% of the yield strength.

$$\text{Yield strength} = (0.9) \cdot (\text{UTS})$$

Equation 29 - Yield Strength

$$\text{Axial Stress} = (0.6) \cdot (\text{Yield strength})$$

Equation 30 - Yield Strength

5.5.2 Frictional Resistive Torque

To find the torque required to overcome the friction of the head against the con-rod cap and the bolt threads, resistive torques were calculated.

$$\text{Clamping force} = \text{Surface area} \cdot \text{Axial stress}$$

Equation 31 - Clamping Force

$$\text{Circumferential friction} = \text{Bolt thread } \mu \cdot \text{Clamping force}$$

Equation 32 - Circumferential Friction

$$\text{Thread resistive torque} = \frac{\text{Bolt diameter}}{2} \cdot \text{Circumferential friction}$$

Equation 33 - Thread Resistive Torque

$$\text{Bolt head friction force} = \text{Bolt head } \mu \cdot \text{Clamping force}$$

Equation 34 - Bolt Head Friction Force

$$\text{Head resistive torque} = \frac{\text{Bolt diameter}}{2} \cdot \text{Bolt head friction force}$$

Equation 35 - Head Resistive Torque

$$\therefore \text{Total torque required} = \text{Thread resistive torque} + \text{Head resistive torque}$$

Equation 36 - Total Torque Required

The calculated values, using constants in table 11, for the standard bolt torque were:

Table 12 - Standard Bolt Torque Data

Yield Strength [MPa]	990
Axial Strength [N/mm ²]	594
Clamping Force [N]	23285
Circumferential Friction [N]	2794
Thread Resistive Torque [Nmm]	10269
Bolt Head Friction Force [N]	4191
Head Resistive Torque [Nmm]	27034
Total Torque Required [Nm]	37.3022

5.5.3 Required Torque

From the above calculations, it was calculated that the bolt would require 37.3022Nm of torque to achieve 90% yield. This will be considered when re-designing the bolt to ensure the torque required does not increase.

5.6 Con-rod Bolt Stresses (3E)

Later in this section, the author shall analyse con-rod bolt stresses within an engine modified to redline at 9000rpm. For comparison, the standard bolt stresses were analysed at both 6000 and 9000rpm using standard stress analysis equations, as per previous sections. Pre-existing cylinder pressure data at 9000rpm was available to the author from linear interpolation of the data from the rest of the rpm range.

The minimum stress was taken to be 90% of yield as this is where bolt is tightened to, so will be the starting tensile stress applied. The following script was written for both 6000 and 9000rpm:

```
%% STANDARD RPM
Bolt_Stress_Min = (0.9)*(Bolt_Yield_Standard); % N/mm^2
Bolt_Stress_Range_Standard = (Alternating_Stress_Maximum); % N/mm^2
Bolt_Stress_Max_Standard = (Bolt_Stress_Range_Standard + Bolt_Stress_Min); % N/mm^2
Bolt_Stress_Mean_Standard = (Bolt_Stress_Max_Standard + Bolt_Stress_Min)/2; % N/mm^2
```

Table 13 - Standard and Modified Bolt Stresses

<u>Data</u>	<u>Standard</u>	<u>Modified</u>
Minimum Stress [N/mm ²]	891.000	891.000
Maximum Stress [N/mm ²]	931.846	982.903
Stress Range [N/mm ²]	40.846	91.903
Mean Stress [N/mm ²]	911.423	936.950

5.7 Standard Bolt Fatigue Safety Factors (3F)

5.7.1 Stress Concentration Factors

From the above data, the safety factors could be calculated for the bolt.

To analyse the stresses within the bolt, a stress concentration factor was applied as per section 4.9. The author measured the following data from the bolt:

Table 14 - Standard Bolt Stress Concentration Factor Data

<u>Stress Concentration Factor Data</u>		
<u>Data</u>	<u>Value</u>	<u>Source</u>
H - Height [mm]	8.00	(Hart, 2020)
D - thickness [mm]	7.35	
r - radius [mm]	0.14	
t - D to edge [mm]	0.28	
r/d	0.0197	
H/d	1.0884	
t/r	1.9379	

The chosen graph for this application was as follows:

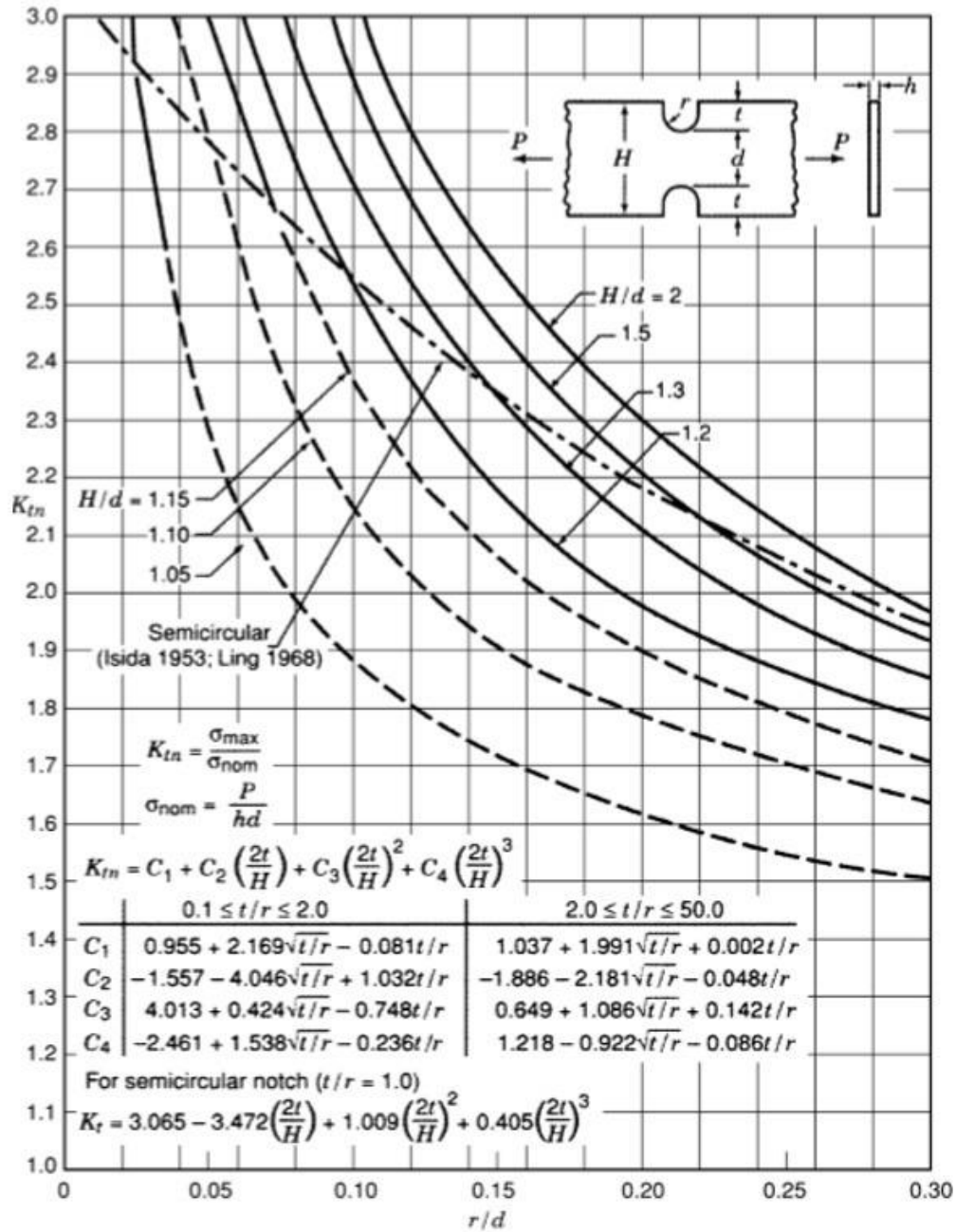


Figure 12 - Peterson Elliptical Fillet Stress Concentration Graph (Pilkey, 2008)

Table 14 shows that the r/d ratio does not have a corresponding y-axis coordinate on any lines of the graph. Because of this, the author used the provided equations for a t/r ratio of less than 2.0, as seen in figure 12.

This produced a final stress concentration factor of:

$$K_t = (3.8175) - (5.1895) \cdot \left(\frac{(2)(0.28)}{8.00} \right) + (3.1537) \cdot \left(\frac{(2)(0.28)}{8.00} \right)^2 - 0.7773 \cdot \left(\frac{(2)(0.28)}{8.00} \right)^3$$

Equation 37 - Standard Con-rod Bolt Stress Concentration Factor

This gave a bolt stress concentration factor of **3.4682**.

5.7.2 Standard Bolt FSF - 6000rpm

Using the previously calculated data, the author produced a Goodman diagram for the standard bolt at both 6000rpm and 6000rpm to find fatigue safety factors using equations as per section 4.9 and 4.10.

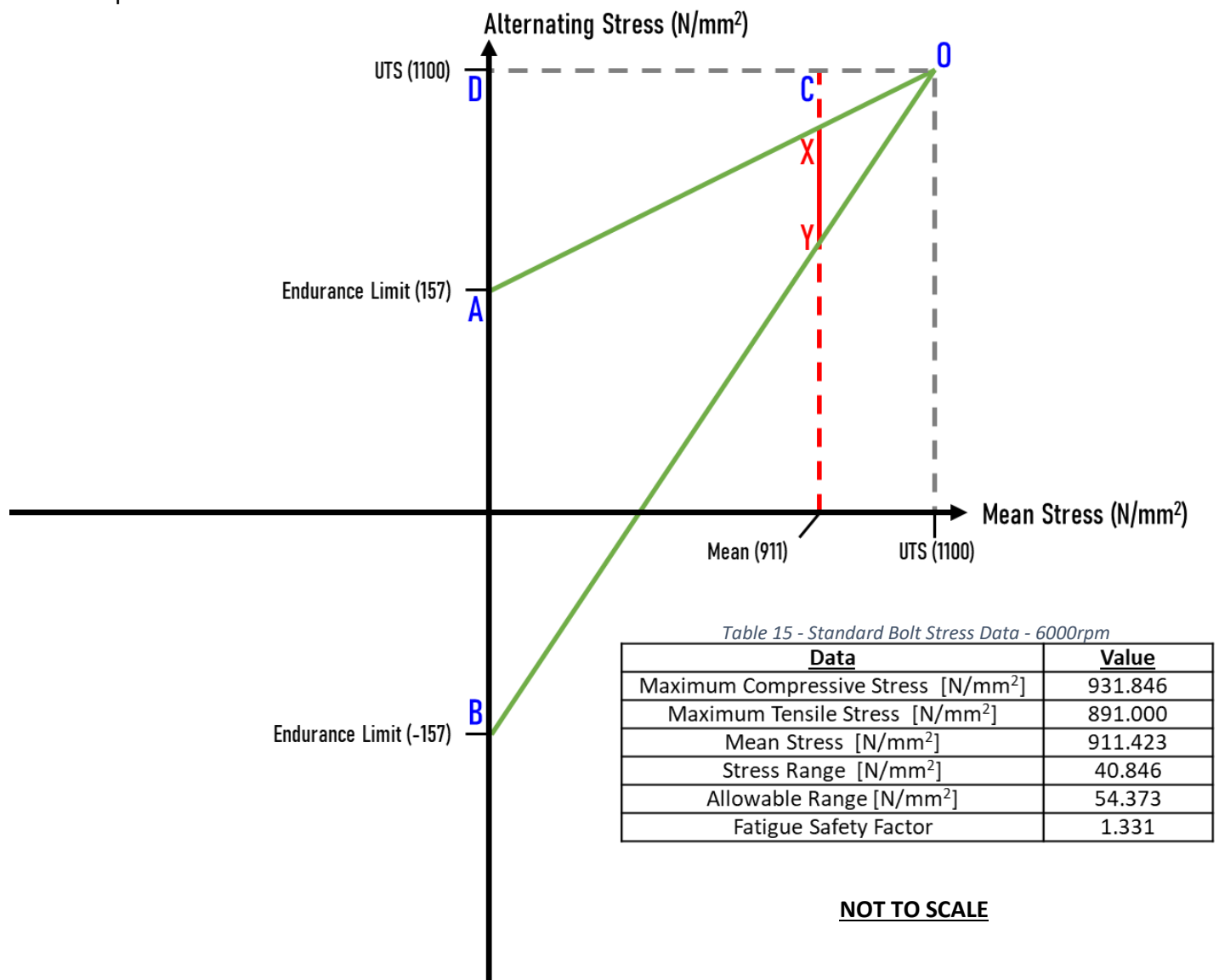


Figure 13 - Standard Bolt Goodman Diagram - 6000rpm

Figure 13 shows an acceptable fatigue safety factor. The standard bolt is engineered to never fail within standard operating range of the engine.

5.7.3 Standard Bolt FSF - 9000rpm

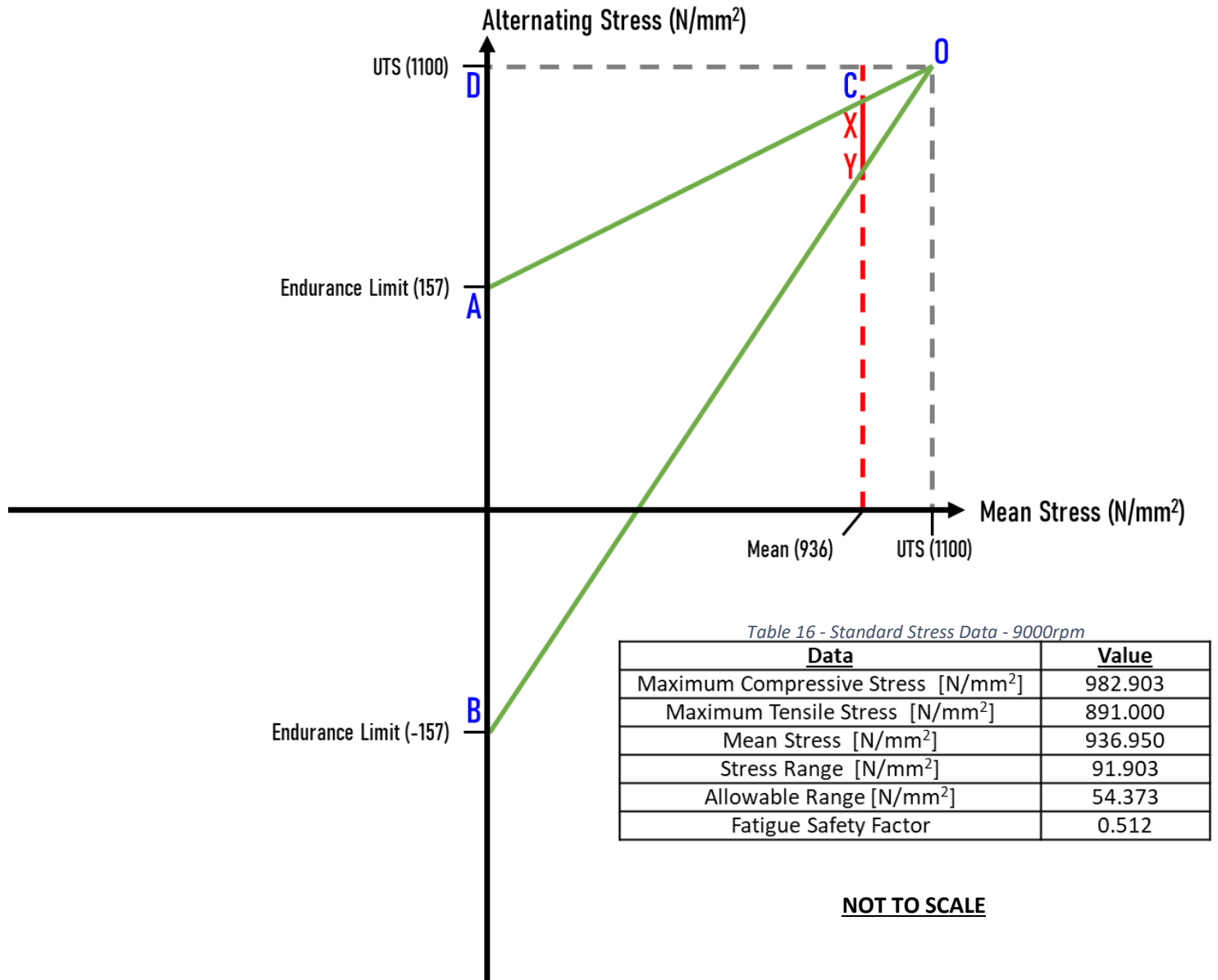


Figure 14 - Standard Bolt Goodman Diagram - 9000rpm

Figure 14 shows that the standard con-rod bolt would fail when the engine speed increases to 9000rpm, as the fatigue safety factor is less than one. For a modified engine, the con-rod bolt will need to be modified also.

5.8 Bolt Redesign (3G, 3H)

The analysis from section 5.7 clearly shows that the standard bolt has many areas of potential improvement. The bolt is very much over-engineered at standard operating range, and very close to failure at 9000rpm. This shows potential for the bolt to be lightened, but also strengthened.

The author researched alternative materials to make the bolt from, mainly based around the ARP website. ARP are an American bolt manufacturer known for producing high quality con-rod bolts from a variety of materials. A material used by them is Carpenter Custom Age 625 PLUS® nickel-base alloy. It is very resistant to corrosion and can achieve a high yield strength without warm or cold working whilst withstanding 'superior fatigue cycle life' (Voort *et al.*, 2012; ARP, 2018). This material will add strength to the bolt and is a similar mass to the original, this will also allow for the mass to be reduced through a new design.

5.8.1 Fatigue Safety Factor Modified Engine, Modified Material

As a baseline, the author made another Goodman diagram analysing the stress on original bolt made of Custom Age 625 PLUS® alloy at 9000rpm to understand how much stronger the new material is.

To start this, the goal seek function on EXCEL was used to find how far into yield the bolt needed to be tightened to keep a constant tightening torque of 37.3022Nm. This ensured no deformation of the big end. The result was 53% yield. This could also have been calculated through transposition of formulae; however this method was quicker and easier. From this, yield and axial stress were re-calculated as per equations 29 and 30.

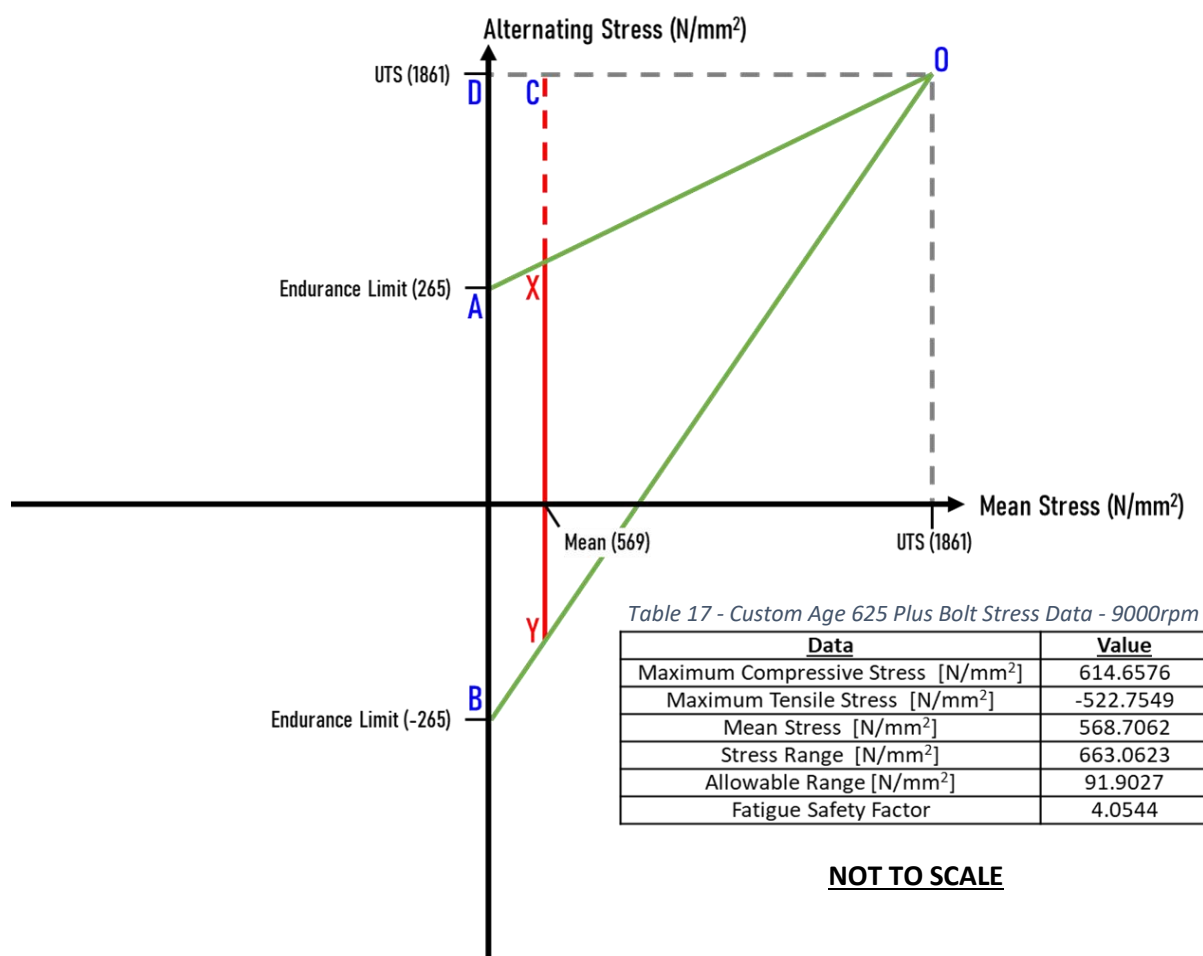


Figure 15 - Custom Age 625 Plus Bolt Goodman Diagram - 9000rpm

Figure 15 shows an FSF 7.9 times larger than the standard material, proving this material is considerably stronger and therefore a good modification choice.

5.8.2 Minimum Safe Diameter

Once the material was proven to be strong enough, the author explored the possibility of reducing the size of areas of the bolt to reduce mass. This will in turn reduce the rotating mass of the cap, potentially resulting in a gain in power (Heywood, 2018).

To keep the same axial stress as a percentage of UTS:

$$\frac{\text{Force} / \text{Standard surface area}}{\text{Standard UTS}} = \frac{\text{Force} / \text{Modified surface area}}{\text{Modified UTS}}$$

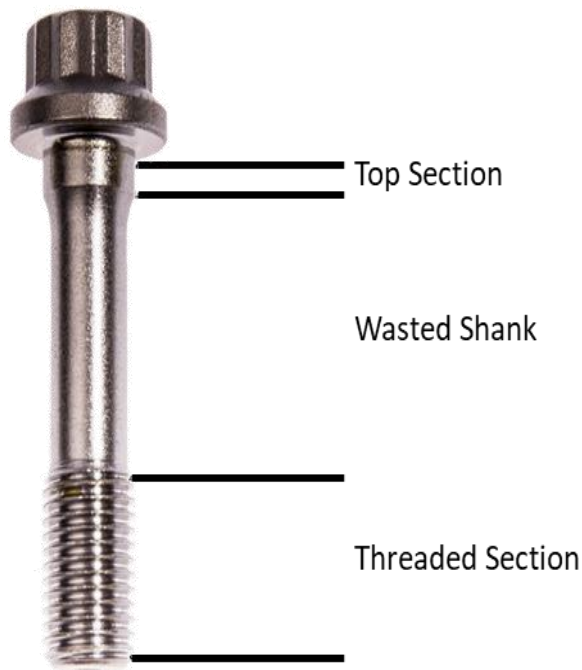
$$\therefore (\text{Standard diameter})^2 \cdot (\text{Standard UTS}) = (\text{Modified diameter})^2 \cdot (\text{Modified UTS})$$

$$\therefore \text{Minimum modified diameter} = \sqrt{\frac{\text{Standard UTS}}{\text{Modified UTS}}} \cdot (\text{Standard diameter})^2$$

Equation 38 - Minimum Modified Bolt Diameter

From this, the minimum safe diameter was calculated to be **5.6508mm**. The author shall use this, plus 10% for added safety as the diameter of the wasted shank section of the bolt: **6.2159mm**.

For the improved design, the bolt shall be taken as three main areas:



The top section and head shall maintain the same dimensions as the standard bolt to ensure correct fitment into the con-rod cap holes.

The 'wasted shank' is calculated by measuring how much thread is actually used within the con-rod. By measuring the overall bolt hole length, and the non-threaded section of the rod, it was found that 24mm of thread was unused. This section's diameter can be reduced.

The threaded section must stay the same diameter to ensure the con-rod cap can stay the same, and the modified bolt can be a direct replacement.

Figure 16 - Con-rod Bolt Sections (MED Engineering, 2020)

5.8.3 Top Section

Using equations as per section 5.3, the following dimensions for the top section were calculated:

Table 18 - Top Section Dimensions

<u>Data</u>	<u>Standard</u>
Length [mm]	5.00
Diameter [mm]	7.35
Area [mm]	42.43
Stiffness [N/mm]	1782.00

5.8.4 Wasted Shank

For the wasted shank section, the following dimensions were calculated in the same way:

Table 19 - Wasted Shank Dimensions

<u>Data</u>	<u>Standard</u>
Length [mm]	24.00
Diameter [mm]	6.22
Area [mm]	30.35
Stiffness [N/mm]	265.52

5.8.5 Threaded Section

For the threaded section, the following dimensions were again calculated in the same way:

Table 20 - Threaded Section Dimensions

<u>Data</u>	<u>Standard</u>
Length [mm]	8.00
Diameter [mm]	7.35
Area [mm]	42.43
Stiffness [N/mm]	1113.80

5.8.6 New Bolt Properties

The three sections, with individual stiffnesses, act in series. Because of this the section with the smallest stiffness (wasted shank) will always dictate overall stiffness, which cannot be lower than this value. There is room to increase the diameter here if the bolt is required to be stiffer, however for this application it is sufficient (Heywood, 2018).

$$K_T = \left(\frac{1}{K_{TOP}} + \frac{1}{K_{WASTED}} + \frac{1}{K_{THREAD}} \right)^{-1}$$

Equation 39 - Total Bolt Stiffness

- This gave a total bolt stiffness of **191.38N/mm**.
- From here each section's surface area was added for a total surface area of **38.40mm²**.
- An average bolt diameter was taken across all sections, equalling **6.97mm**.

A new Alternating load was calculated, considering the new bolt stiffness, and thus a new alternating stress was calculated. Knowing this value, as per section 5.6, stress calculations were carried out so a Goodman diagram could be made to find the new fatigue safety factor.



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5.9 Standard and Modified Bolt Comparison

5.9.1 Standard Bolt



Figure 18 - Standard Con-rod Bolt Design (Hart, 2020)

5.9.2 Modified Bolt Design

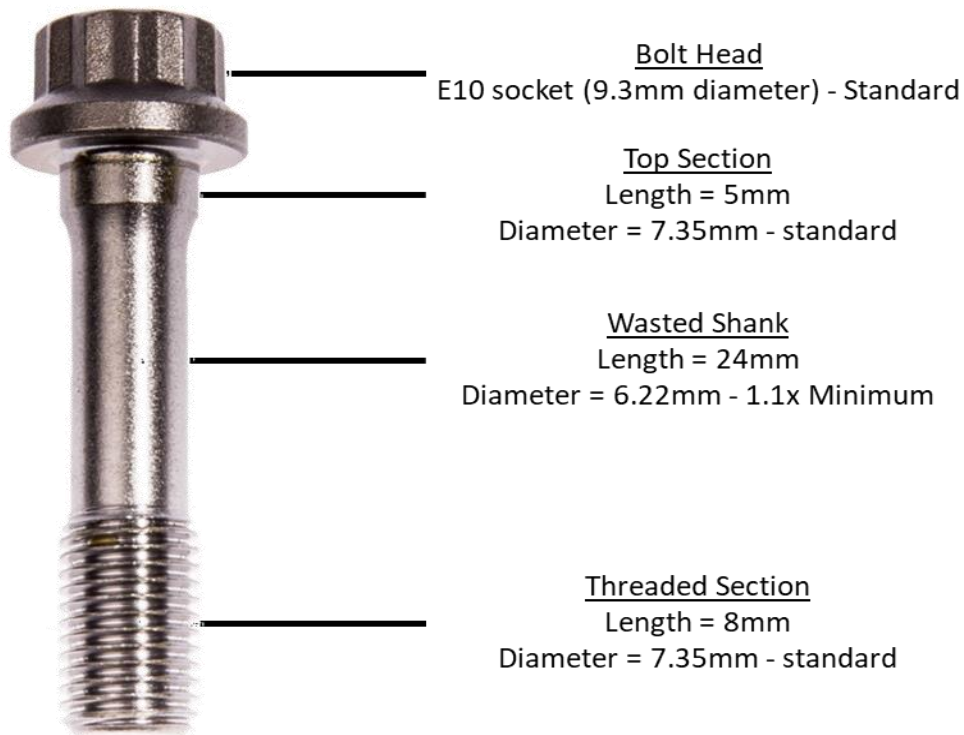


Figure 19 - Modified Con-rod Bolt Design (MED Engineering, 2020)

6 Crankshaft Stresses (Question 4).

In an internal combustion engine a crankshaft, along with the con-rods, transform piston reciprocation into rotational motion. All loading from the engine is sent directly to this component to transmit drive to the flywheel, therefore making it an integral component that needs to withstand stresses from inertial loading of the piston and crankshaft, compressive load from combustion, and torsional load and vibration stresses (Heywood, 2018).

For this section, the stresses at the worst effected parts will be investigated. These are the areas where the journal and crankpin join. These areas include small fillets, resulting in a noticeable stress concentration.

6.1 Constant Values

For the following calculations, various specific constant values were measured by the author.

Table 22 - Question 4 Constants

<u>Gudgeon Pin Dimensions</u>		
<u>Data</u>	<u>Value</u>	<u>Source</u>
L2 - Length of point B to centre of journal [mm]	33.41	(Hart, 2020)
D1 - Top journal diameter [mm]	43.97	
D2 - Bottom journal diameter [mm]	48.00	
b - Effective width of critical beam [mm]	45.985	
AB - Distance between two points [mm]	20.75	
D - Crankpin thickness [mm]	20.50	
I - Second moment of inertia [mm ⁴]	33014	
Y - Neutral axis	10.375	
Surface Area [mm ²]	1518.5	

The reciprocating mass of the piston was calculated as in equation 6, and the crankshaft reciprocating and rotating masses were calculated as follows:

$$\text{Reciprocating mass} = (\text{Mass of piston assembly}) + \left(\frac{1}{3} \cdot \text{Conrod mass}\right)$$

$$\text{Rotating mass} = (\text{Mass of conrod assembly}) - \left(\frac{1}{3} \cdot \text{Conrod mass}\right)$$

Equation 40 - Crankshaft Masses

6.2 Crank Net Force

To calculate inertial loads on the crankshaft, angular velocity was calculated as per equation 25, and inertia force calculated as per equation 26. These values were subtracted from the gas forces at each rpm as per equation 9.

6.3 Bending Stresses

The bending moment on the crank was calculated as per equation 10, using half net force and distance L2. This is considered maximum at point A because of its location, and how it transfers load from the con-rod to the bearings.

From here, bending stress is calculated as per equation 11.

6.4 Nominal Combined Stresses

Shear stresses on the crankshaft are calculated as per equation 12. Similarly to the gudgeon pin analysis, the shear stresses act with the bending stresses, thus are combined as per equation 13.

6.5 Stress Concentration Factors

As mentioned, there are small fillets at points A and B. These were taken to be the worst sections for stress concentrations and the following measurements were taken:

Equation 41 - Crankshaft Stress Concentration Factor Data

<u>Stress Concentration Factor Data</u>		
<u>Data</u>	<u>Value</u>	<u>Source</u>
H - Height [mm]	7.40	(Hart, 2020)
D - thickness [mm]	44.01	
r - radius [mm]	1.50	
t - D to edge [mm]	0.52	
r/d	0.0341	
t/r	0.3467	

The chosen graph for this application was as follows:

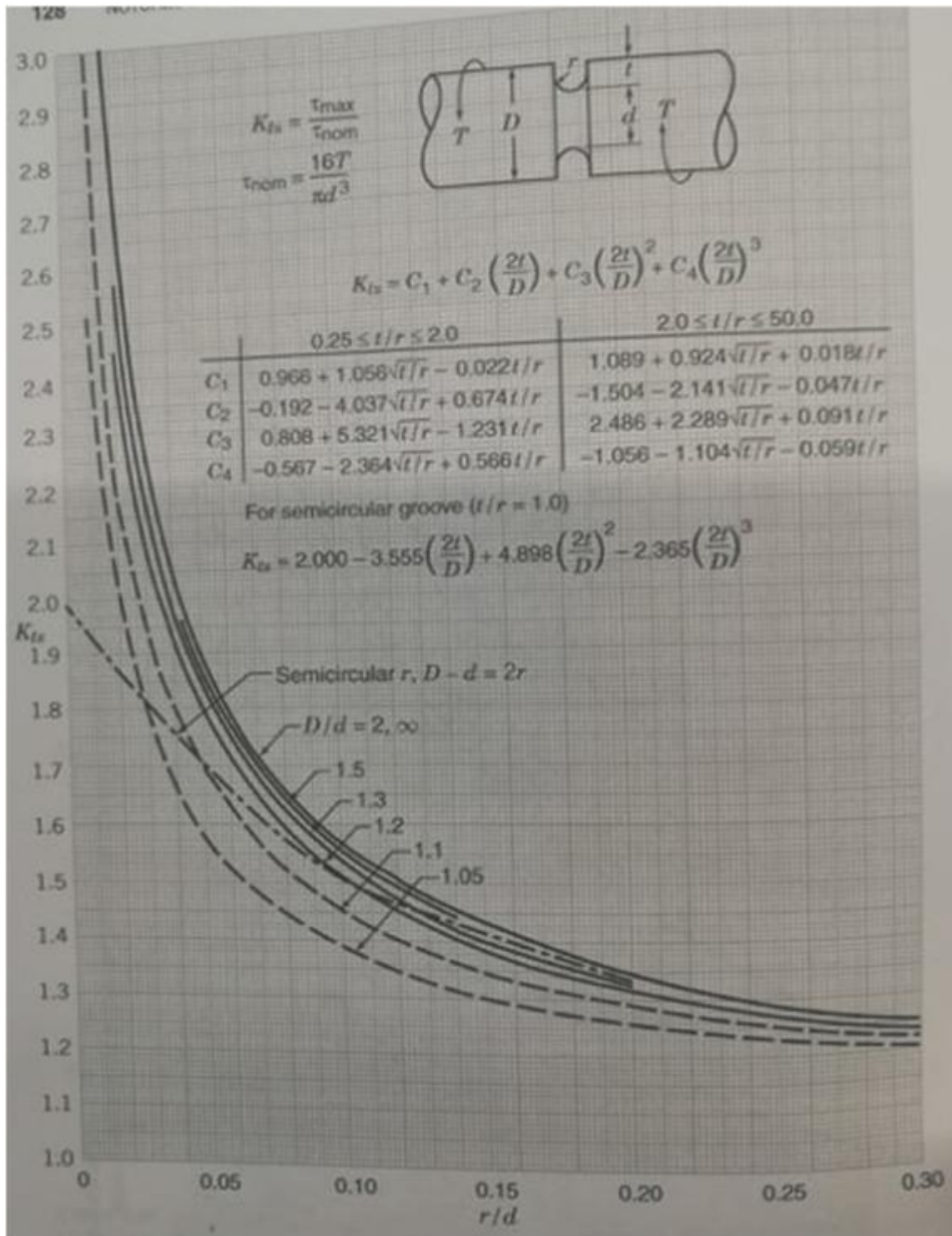


Figure 20 - Crankshaft Stress Concentration Graph (Pilkey, 2008)

Table 14 shows that the r/d ratio does not have a corresponding y-axis coordinate on any lines of the graph. Because of this, the author used the provided equations for a t/r ratio of less than 2.0, as seen in figure 19.

This produced a final stress concentration factor of:

$$K_t = (1.5801) - (2.3353) \cdot \left(\frac{(2)(0.52)}{44.01} \right) + (3.5142) \cdot \left(\frac{(2)(0.52)}{44.01} \right)^2 - 2.1551 \cdot \left(\frac{(2)(0.52)}{44.01} \right)^3$$

Equation 42 - Crankshaft Stress Concentration Factor

6.6 Fatigue Safety Factor

From the above data, a Goodman diagram could be made to find the worst case scenario fatigue safety factor.

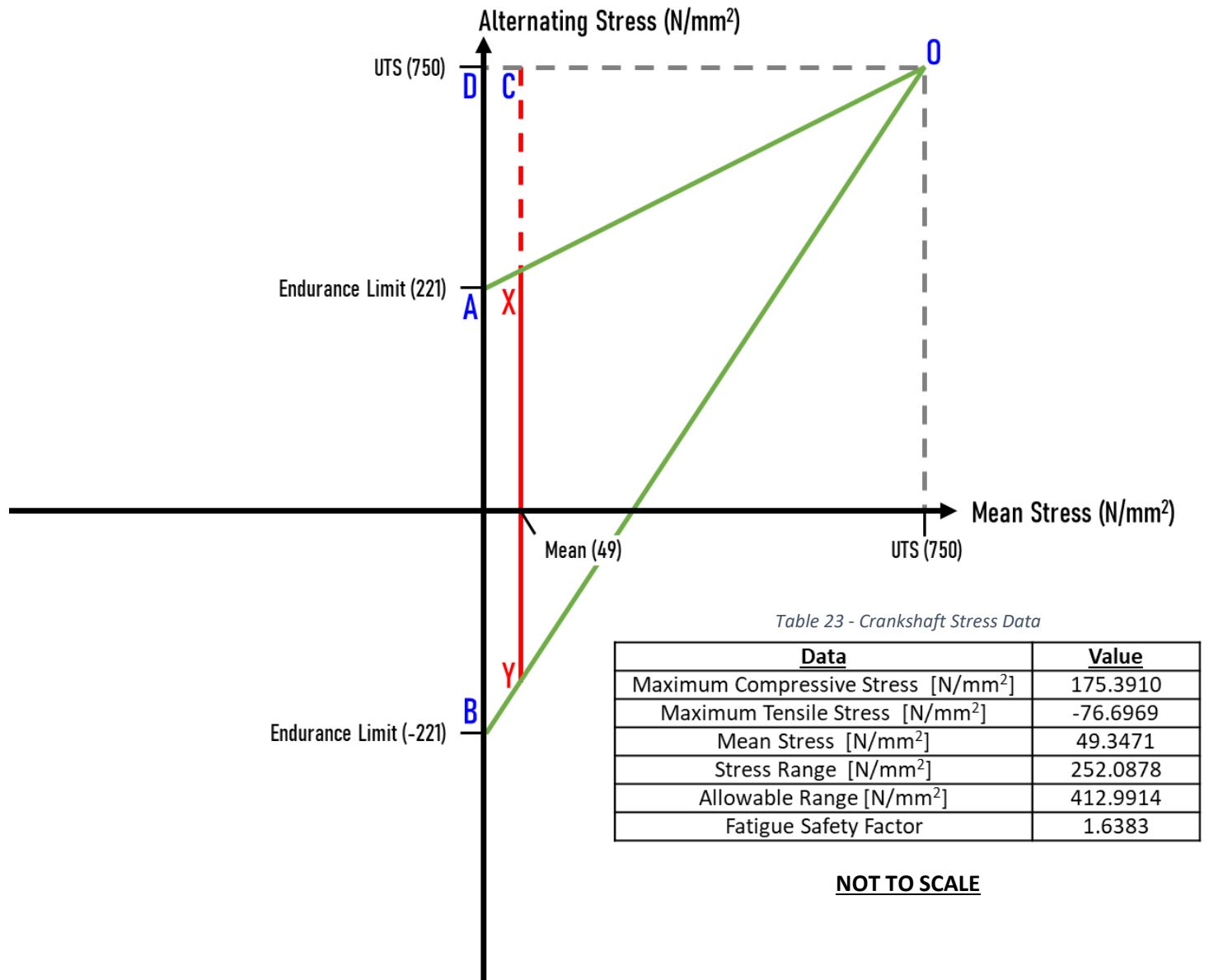


Figure 21 - Crankshaft Goodman Diagram

Figure 20 clearly shows that the crankshaft will have infinite service life. With a fatigue safety factor above 1, even at very worst case scenarios, the part is engineered to be strong enough to withstand any loading from a standard engine. The stress concentration areas, although weakening the part, are a necessity because of the nature of the design and do not have enough of an effect to seriously threaten the lifecycle of the part.

7 Conclusion

This report has allowed the author to conduct critical analysis into different stresses on key components within an internal combustion engine.

The gudgeon pin overall has an ample fatigue safety factor and will easily fulfil an infinite service life. The part could be modified to reduce mass, thus reducing piston assembly reciprocating mass and possibly achieving an increase in power, this could be developed further to fully understand the components reactions to the stresses placed upon it.

The con-rod similarly sees ample fatigue safety factor figures within the standard engine speed range. The figure at point B is the lowest, suggesting that if the engine was modified, this would be the first section to fail. To develop this analysis further, the author could research developing this design at this region, or potentially changing the con-rod material to increase FSF here.

The con-rod bolts are equally engineered to have infinite service life. Upon engine modification, the bolts do shear, and the author learnt lots about how to modify a component for added strength using different materials. From the added strength this brought, the author learnt how to redesign component geometry to reduce mass. These techniques could be applied to other components in further research.

The crankshaft similarly is engineered to easily achieve infinite service life. The necessary stress concentration points around the analysed fillets do not impact the durability of the component. To develop this research further, the author would assess the possibility of reducing these stress concentrations or changing to a stronger material.

This report details a successful study with valid, trustworthy results. For more accurate findings, more materials research should be carried out, and mathematical analysis should be complimented with CAD and FEA analysis.

8 Referenced Work

ARP (2018) *The Official ARP Web Site | Technical Information, ARP Material Specifications*. Available at: <https://arp-bolts.com/p/technical.php>.

AZO Materials (2016) 'Material Properties and Applications'. Available at: <https://www.azom.com/properties.aspx?ArticleID=712>.

Bosch (2017) 'Bosch Automotive Handbook', (9), p. 1534.

Budynas–Nisbett, J. (2014) 'Shigley's Mechanical Engineering Design', *Mechanical Engineering*. doi: 10.4324/9780080496498.

Desai, F. (2014) 'Stress Analysis of Connecting Rod for Weight Reduction- A Review', *International Journal of Engineering Research & Technology (IJERT)*, 3(2). Available at: www.ijert.org.

Ford (2009) '1.6 GTDI EcoBoost'. Available at: <https://www.ford.co.uk/content/dam/guxeu/uk/documents/home/experience-ford/about-ford/ford-component-sales/specification-details-data-sheets/16-ecoboost-np.pdf>.

Griza, S. et al. (2009) *Fatigue in engine connecting rod bolt due to forming laps*, *Engineering Failure Analysis*. doi: 10.1016/j.engfailanal.2008.10.002.

Hart, M. (2020) 'Component mass / dimension measurement carried out in workshop'.

Heywood, J. (2018) 'Internal Combustion Engine Fundamentals 2E', p. 1056. Available at: <https://www.accessengineeringlibrary.com/browse/internal-combustion-engine-fundamentals-second-edition>.

MatWeb (2014) *General Properties of Steels*. Available at: <http://www.matweb.com/search/datasheet.aspx?matguid=10e1c14130cd4ed6ae64b85723be53af&n=1&ckck=1>.

MED Engineering (2020) *REPLACEMENT ROD BOLT - STEEL CONRODS*.

Pilkey, W. D. (2008) *Peterson's Stress Concentration Factors*. Second. New York: Wiley-Interscience. doi: 10.1002/9780470211106.ch3.

Sengupta, A. K. (2009) *Stress Concentration Caused by Sudden Change in Form*, *Jurnal Pesona PAUD*.

Smith, H. (2010) *Factors of Safety*. Available at: https://www.engineeringtoolbox.com/factors-safety-fos-d_1624.html.

Taylor, C. (1985) *The Internal Combustion Engine in Theory and Practice*. Massachusetts: MIT Press.

Voort, G. F. V., Bowman, J. W. and Frank, R. B. (2012) *Microstructural Characterization of Custom Age 625 Plus Alloy*. Reading, Pennsylvania: The Minerals, Metals and Materials Society. doi: 10.7449/1994/superalloys_1994_489_498.

Young, W. and Budynas, R. (2002) *Roark's Formulas for Stress and Strain*. New York: McGraw Hill.

9 Appendices

For each MATLAB script appendix, if a calculation is repeated for every RPM, it is only shown for 1000rpm. A for loop was written into the MATLAB script to automatically repeat these calculations for every RPM.

9.1 Appendix One - Question One MATLAB Script

LOADING OF CONSTANTS

```
Load_Constants = ☐ ; % TICK TO LOAD IN CONSTANTS

if Load_Constants == 1
    Constants_And_Cylinder_Data
end
```

CONSTANT VALUES

```
%% GUDGEON PIN DIMENSIONS
Pin_Inside_d = 15.44; % mm - d2
Pin_Outside_d = 21.00; % mm - d1
Pin_Extrusion = 16.29; % mm - L1
Pin_CoG_Length = 5.46; % mm - L2

Neutral_Axis = (Pin_Outside_d / 2) % mm

Neutral_Axis = 10.5000
```

```
%% MASSES
Mass_Pin_Extrusion = (Pin_Extrusion/1000)*(pi)*(((Pin_Outside_d/2000)^2) - ((Pin_Inside_d/2000)^2))*(Gudgeon_Pin_Density) % Kg

Mass_Pin_Extrusion = 0.0202
```

```
Recip_Mass_Pin = (Mass_Pin_Extrusion) % Kg

Recip_Mass_Pin = 0.0202
```

```
Recip_Mass_Piston = (Mass_Piston_Total) + (1/3*Mass_Conrod_Total) % Kg

Recip_Mass_Piston = 0.5031
```

```
Second_Moment_Of_Area_Pin = ((pi/64)*(Pin_Outside_d^4 - Pin_Inside_d^4)) % Nmm

Second_Moment_Of_Area_Pin = 6.7569e+03
```

```
Surface_Area = (pi/4)*(Pin_Outside_d^2 - Pin_Inside_d^2) % mm^2

Surface_Area = 159.1267
```

INERTIA FORCES

```
% 1000rpm
Inertia_Force_Piston_1000rpm = Piston_Accel_1000rpm.*(Recip_Mass_Piston); % N
Inertia_Force_Pin_1000rpm = Piston_Accel_1000rpm.*(Recip_Mass_Pin); % N
```

NET FORCES

```
% 1000rpm
Net_Force_Piston_1000rpm = (Gas_Force_1000rpm)-(Inertia_Force_Piston_1000rpm); % N

[Max_Net_Force_Piston_1000rpm,I_Max_Net_1000rpm] = max(Net_Force_Piston_1000rpm); % N
[Min_Net_Force_Piston_1000rpm,I_Min_Net_1000rpm] = min(Net_Force_Piston_1000rpm); % N
```

BENDING MOMENT

```
% 1000rpm
Max_Bending_Moment_1000rpm = ((Max_Net_Force_Piston_1000rpm/2)*(Pin_Extrusion))-((Inertia_Force_Pin_1000rpm(I_Max_Net_1000rpm))*(Pin_CoG_Length)); % N/mm
Min_Bending_Moment_1000rpm = ((Min_Net_Force_Piston_1000rpm/2)*(Pin_Extrusion))+((Inertia_Force_Pin_1000rpm(I_Min_Net_1000rpm))*(Pin_CoG_Length)); % N/mm
```

BENDING STRESS

```
% 1000rpm
Max_Bending_Stress_1000rpm = ((Max_Bending_Moment_1000rpm)*(Pin_Outside_d/2))/(Second_Moment_Of_Area_Pin); % N/mm^2
Min_Bending_Stress_1000rpm = ((Min_Bending_Moment_1000rpm)*(Pin_Outside_d/2))/(Second_Moment_Of_Area_Pin); % N/mm^2
```

SHEAR STRESS

```
% 1000rpm
Max_Shear_Stress_1000rpm = (Max_Net_Force_Piston_1000rpm/2)/(Surface_Area); % N/mm^2
Min_Shear_Stress_1000rpm = (Min_Net_Force_Piston_1000rpm/2)/(Surface_Area); % N/mm^2
```

COMBINED STRESS

```
% 1000rpm
Max_Combined_Stress_1000rpm = sqrt(Max_Bending_Stress_1000rpm^2+(2*Max_Shear_Stress_1000rpm)^2); % N/mm^2
Min_Combined_Stress_1000rpm = sqrt(Min_Bending_Stress_1000rpm^2+(2*Min_Shear_Stress_1000rpm)^2); % N/mm^2
```

OVERALL MAXIMUM STRESSES

Maximum_Compressive_Stress = Max_Combined_Stress_5700rpm % N/mm²

Maximum_Compressive_Stress = 536.2794

Maximum_Tensile_Stress = (Min_Combined_Stress_6000rpm)*(-1) % N/mm²

Maximum_Tensile_Stress = -126.7829

Mean_Stress = (Maximum_Compressive_Stress + Maximum_Tensile_Stress)/2 % N/mm²

Mean_Stress = 204.7483

Range_Stress = (Maximum_Compressive_Stress - Maximum_Tensile_Stress) % N/mm²

Range_Stress = 663.0623

ALLOWABLE RANGE

Allowable_Range = (2)*(Gudgeon_Pin_End_Lim)*((1-(Mean_Stress/Gudgeon_Pin_UTS))) % N/mm²

Allowable_Range = 795.2517

FATIGUE SAFETY FACTOR

Fatigue_Safety_Factor = (Allowable_Range)/(Range_Stress) % Ratio

Fatigue_Safety_Factor = 1.1994

9.2 Appendix Two - Question Two MATLAB Script

LOADING OF CONSTANTS

```
Load_Constants = ☐; % TICK TO LOAD IN CONSTANTS
```

```
if Load_Constants == 1  
    Constants_And_Cylinder_Data  
end
```

CONSTANT VALUES

```
Recip_Mass_A = (Mass_Piston_Total) % Kg
```

```
Recip_Mass_A = 0.3559
```

```
Recip_Mass_B = (Mass_Piston_Total)+(1/3*Mass_Conrod) % Kg
```

```
Recip_Mass_B = 0.4451
```

```
Recip_Mass_C = (Mass_Piston_Total)+(2/3*Mass_Conrod_Total) % Kg
```

```
Recip_Mass_C = 0.6503
```

```
Recip_Mass_Piston = (Mass_Piston_Total) + (1/3*Mass_Conrod_Total) % Kg
```

```
Recip_Mass_Piston = 0.5031
```

```
Length_A = 18.18; % mm
```

```
Width_A = 4.26; % mm
```

```
CSA_A = (Length_A)*(Width_A) % mm^2
```

```
CSA_A = 77.4468
```

```
Length_B1 = 13.00; % mm
```

```
Width_B1 = 7.13; % mm
```

```
Length_B2 = 4.57; % mm
```

```
Width_B2 = 3.24; % mm
```

```
CSA_B = (Length_B1*Width_B1)+((Length_B2*Width_B2)*2) % mm^2
```

```
CSA_B = 122.3036
```

```
Length_C = 20.51; % mm
```

```
Width_C = 8.96; % mm
```

```
CSA_C = (Length_C*Width_C) % mm^2
```

```
CSA_C = 183.7696
```

PISTON INERTIA FORCE

```
% 1000rpm
Inertia_Force_Piston_1000rpm = Piston_Accel_1000rpm.*(Recip_Mass_Piston); % N
```

NET FORCES

```
% 1000rpm
Net_Force_Piston_1000rpm = (Gas_Force_1000rpm)-(Inertia_Force_Piston_1000rpm); % N

[Max_Net_Force_Piston_1000rpm,I_Max_Net_1000rpm] = max(Net_Force_Piston_1000rpm); % N
[Min_Net_Force_Piston_1000rpm,I_Min_Net_1000rpm] = min(Net_Force_Piston_1000rpm); % N
```

GAS FORCES

```
% 1000rpm
Max_Gas_Force_1000rpm = Gas_Force_1000rpm(I_Max_Net_1000rpm); % N
Min_Gas_Force_1000rpm = Gas_Force_1000rpm(I_Min_Net_1000rpm); % N
```

INDIVIDUAL INERTIA FORCES

```
% 1000rpm
Max_Inertia_Force_A_1000rpm = Piston_Accel_1000rpm(I_Max_Net_1000rpm)*(Recip_Mass_A); % N
Max_Inertia_Force_B_1000rpm = Piston_Accel_1000rpm(I_Max_Net_1000rpm)*(Recip_Mass_B); % N
Max_Inertia_Force_C_1000rpm = Piston_Accel_1000rpm(I_Max_Net_1000rpm)*(Recip_Mass_C); % N

Min_Inertia_Force_A_1000rpm = Piston_Accel_1000rpm(I_Min_Net_1000rpm)*(Recip_Mass_A); % N
Min_Inertia_Force_B_1000rpm = Piston_Accel_1000rpm(I_Min_Net_1000rpm)*(Recip_Mass_B); % N
Min_Inertia_Force_C_1000rpm = Piston_Accel_1000rpm(I_Min_Net_1000rpm)*(Recip_Mass_C); % N
```

INDIVIDUAL NET FORCES

```
% 1000rpm
Max_Net_Force_A_1000rpm = (Max_Gas_Force_1000rpm) - (Max_Inertia_Force_A_1000rpm); % N
Max_Net_Force_B_1000rpm = (Max_Gas_Force_1000rpm) - (Max_Inertia_Force_B_1000rpm); % N
Max_Net_Force_C_1000rpm = (Max_Gas_Force_1000rpm) - (Max_Inertia_Force_C_1000rpm); % N

Min_Net_Force_A_1000rpm = (Min_Gas_Force_1000rpm) - (Min_Inertia_Force_A_1000rpm); % N
Min_Net_Force_B_1000rpm = (Min_Gas_Force_1000rpm) - (Min_Inertia_Force_B_1000rpm); % N
Min_Net_Force_C_1000rpm = (Min_Gas_Force_1000rpm) - (Min_Inertia_Force_C_1000rpm); % N
```

SHEAR STRESS

%% 1000rpm

% A

Max_Shear_Stress_A_1000rpm = (Max_Net_Force_A_1000rpm/2)/(CSA_A) % N/mm^2

Max_Shear_Stress_A_1000rpm = 144.3048

Min_Shear_Stress_A_1000rpm = (Min_Net_Force_A_1000rpm/2)/(CSA_A) % N/mm^2

Min_Shear_Stress_A_1000rpm = 1.4622

Mean_Shear_Stress_A_1000rpm = (Max_Shear_Stress_A_1000rpm + Min_Shear_Stress_A_1000rpm)/2 % N/mm^2

Mean_Shear_Stress_A_1000rpm = 72.8835

% B

Max_Shear_Stress_B_1000rpm = (Max_Net_Force_B_1000rpm)/(CSA_B) % N/mm^2

Max_Shear_Stress_B_1000rpm = 182.4153

Min_Shear_Stress_B_1000rpm = (Min_Net_Force_B_1000rpm)/(CSA_B) % N/mm^2

Min_Shear_Stress_B_1000rpm = 1.4560

Mean_Shear_Stress_B_1000rpm = (Max_Shear_Stress_B_1000rpm + Min_Shear_Stress_B_1000rpm)/2 % N/mm^2

Mean_Shear_Stress_B_1000rpm = 91.9357

% C

Max_Shear_Stress_C_1000rpm = (Max_Net_Force_C_1000rpm/2)/(CSA_C) % N/mm^2

Max_Shear_Stress_C_1000rpm = 60.4390

Min_Shear_Stress_C_1000rpm = (Min_Net_Force_C_1000rpm/2)/(CSA_C) % N/mm^2

Min_Shear_Stress_C_1000rpm = 0.1812

Mean_Shear_Stress_C_1000rpm = (Max_Shear_Stress_C_1000rpm + Min_Shear_Stress_C_1000rpm)/2 % N/mm^2

Mean_Shear_Stress_C_1000rpm = 30.3101

STRESS CONCENTRATION FACTORS

```
% STRESS CONCENTRATION DATA FOR POINT C:
```

```
H = 11.21; % mm - Cap thickness
```

```
d = 9; % mm - Rod Thickness
```

```
R = 0.47; % mm - Radius
```

```
R_over_d = (R)/(d)
```

```
R_over_d = 0.0522
```

```
H_over_d = (H)/(d)
```

```
H_over_d = 1.2456
```

```
KT_A = 1;
```

```
KT_B = 1;
```

```
KT_C = 2.25;
```

FINAL ENDURANCE LIMIT

```
Con_Rod_End_Lim_A = (Con_Rod_End_Lim)/(KT_A) % N/mm^2
```

```
Con_Rod_End_Lim_A = 375
```

```
Con_Rod_End_Lim_B = (Con_Rod_End_Lim)/(KT_B) % N/mm^2
```

```
Con_Rod_End_Lim_B = 375
```

```
Con_Rod_End_Lim_C = (Con_Rod_End_Lim)/(KT_C) % N/mm^2
```

```
Con_Rod_End_Lim_C = 166.6667
```

ALLOWABLE RANGE

```
Allowable_Range_A = (2)*(Con_Rod_End_Lim_A)*((1-(max(Mean_Shear_Stresses_A)/Con_Rod_UTS))) % N/mm^2
```

```
Allowable_Range_A = 632.0092
```

```
Allowable_Range_B = (2)*(Con_Rod_End_Lim_B)*((1-(max(Mean_Shear_Stresses_B)/Con_Rod_UTS))) % N/mm^2
```

```
Allowable_Range_B = 604.5849
```

```
Allowable_Range_C = (2)*(Con_Rod_End_Lim_C)*((1-(Mean_Shear_Stress_C)/Con_Rod_UTS)) % N/mm^2
```

```
Allowable_Range_C = 325.9127
```

FATIGUE SAFETY FACTOR

Fatigue_Safety_Factor_A = (Allowable_Range_A)/(Range_Shear_Stresses_A) % Ratio

Fatigue_Safety_Factor_A = 2.0937

Fatigue_Safety_Factor_B = (Allowable_Range_B)/(Range_Shear_Stresses_B) % Ratio

Fatigue_Safety_Factor_B = 1.5698

Fatigue_Safety_Factor_C = (Allowable_Range_C)/(Range_Shear_Stress_C) % Ratio

Fatigue_Safety_Factor_C = 9.5411

9.3 Appendix Three - Question Three MATLAB Script

LOADING OF CONSTANTS

```
Load_Constants = ☐; % TICK TO LOAD IN CONSTANTS

if Load_Constants == 1
    Constants_And_Cylinder_Data
end
```

CONSTANT VALUES

```
Recip_Mass = (Mass_Piston_Total)+(1/3*Mass_Conrod) % Kg
```

```
Recip_Mass = 0.4451
```

```
Rot_Mass = (2/3*Mass_Conrod) % Kg
```

```
Rot_Mass = 0.1783
```

```
Bolt_Head_Diameter = 12.9; % mm
```

```
Bolt_Diameter = 7.35; % mm
```

```
Avg_Bolt_Diameter = (Bolt_Head_Diameter + Bolt_Diameter)/2 % mm
```

```
Avg_Bolt_Diameter = 10.1250
```

```
Bolt_Length = 25.75; % mm - to second engaged thread
```

```
Total_Surface_Area = 175.07; % mm^2
```

```
Bolt_Surface_Area = 39.20; % mm^2
```

```
Cap_Surface_Area = (Total_Surface_Area - Bolt_Surface_Area) % mm^2
```

```
Cap_Surface_Area = 135.8700
```

```
Bolt_Thread_Mu = 0.12;
```

```
Bolt_Head_Mu = 0.18;
```

PART A

```
Surface_Area_Ratio = (Bolt_Surface_Area)/(Cap_Surface_Area) % Ratio
```

```
Surface_Area_Ratio = 0.2885
```

PART B

```
Bolt_Stiffness = ((Bolt_E_Standard)*(Bolt_Surface_Area))/(Bolt_Length) % N/mm
```

```
Bolt_Stiffness = 319.6893
```

```
Cap_Stiffness = ((Cap_E)*(Cap_Surface_Area))/(Bolt_Length) % N/mm
```

```
Cap_Stiffness = 897.0058
```

```
Stiffness_Ratio = (Bolt_Stiffness)/(Cap_Stiffness) % Ratio
```

```
Stiffness_Ratio = 0.3564
```

PART C

ANGULAR VELOCITY

```
% 1000rpm
```

```
W_1000rpm = (2*pi*1000)/60; % rads/sec
```

INERTIA FORCE

```
% 1000rpm
```

```
Inertia_Force_1000rpm = ((Recip_Mass)*(Crank_Throw_R)*(W_1000rpm)^2*(1+(Crank_Throw_R/Rod_Length_L))) + ((Rot_Mass)*(Crank_Throw_R)*(W_1000rpm)^2); % Kg
```

ALTERNATING LOAD

```
% 1000
```

```
Alternating_Load_1000rpm = (1/(1 + (1/Stiffness_Ratio)))*((Inertia_Force_1000rpm)/2); % N
```

ALTERNATING STRESS

```
% 1000  
Alternating_Stress_1000rpm = (Alternating_Load_1000rpm)/(Bolt_Surface_Area); % N/mm^2
```

PART D

```
Bolt_Yield_Standard = (0.9)*(Bolt_UTS_Standard) % MPa
```

```
Bolt_Yield_Standard = 990
```

```
Axial_Stress = (0.6)*(Bolt_Yield_Standard) % N/mm^2
```

```
Axial_Stress = 594
```

```
Clamping_Force_Standard = (Bolt_Surface_Area)*(Axial_Stress) % N
```

```
Clamping_Force_Standard = 2.3285e+04
```

```
Circumfencial_Friction_Standard = (Bolt_Thread_Mu)*(Clamping_Force_Standard) % N
```

```
Circumfencial_Friction_Standard = 2.7942e+03
```

```
Resisting_Torque_Threads_Standard = (Bolt_Diameter/2)*(Circumfencial_Friction_Standard) % Nmm
```

```
Resisting_Torque_Threads_Standard = 1.0269e+04
```

```
Bolt_Head_Friction_Force_Standard = (Bolt_Head_Mu)*(Clamping_Force_Standard) % N
```

```
Bolt_Head_Friction_Force_Standard = 4.1913e+03
```

```
Resisting_Torque_Head_Standard = (Bolt_Head_Friction_Force_Standard)*(Bolt_Head_Diameter/2) % Nmm
```

```
Resisting_Torque_Head_Standard = 2.7034e+04
```

```
Total_Torque_Standard = (Resisting_Torque_Head_Standard/1000) + (Resisting_Torque_Threads_Standard/1000) % Nm
```

```
Total_Torque_Standard = 37.3022
```

PART E

%% STANDARD RPM

Bolt_Stress_Min = (0.9)*(Bolt_Yield_Standard) % N/mm²

Bolt_Stress_Min = 891

Bolt_Stress_Range_Standard = (Alternating_Stress_Maximum) % N/mm²

Bolt_Stress_Range_Standard = 40.8456

Bolt_Stress_Max_Standard = (Bolt_Stress_Range_Standard + Bolt_Stress_Min) % N/mm²

Bolt_Stress_Max_Standard = 931.8456

Bolt_Stress_Mean_Standard = (Bolt_Stress_Max_Standard + Bolt_Stress_Min)/2 % N/mm²

Bolt_Stress_Mean_Standard = 911.4228

%% MODIFIED RPM

Bolt_Stress_Range_Modified = (Alternating_Stress_9000rpm) % N/mm²

Bolt_Stress_Range_Modified = 91.9027

Bolt_Stress_Max_Modified = (Bolt_Stress_Range_Modified + Bolt_Stress_Min) % N/mm²

Bolt_Stress_Max_Modified = 982.9027

Bolt_Stress_Mean_Modified = (Bolt_Stress_Max_Modified + Bolt_Stress_Min)/2 % N/mm²

Bolt_Stress_Mean_Modified = 936.9513

PART F

%% STRESS CONCENTRATION DATA

H_Standard = 8.00; % mm - Height

d_Standard = 7.35; % mm - Distance between fillets

R_Standard = 0.14; % mm - Radius

T_Standard = 0.28; % mm - Thickness of edge and fillet

R_over_d_Standard = (R_Standard)/(d_Standard)

R_over_d_Standard = 0.0190

```
H_over_d_Standard = (H_Standard)/(d_Standard)
```

```
H_over_d_Standard = 1.0884
```

```
T_over_R_Standard = (T_Standard)/(R_Standard)
```

```
T_over_R_Standard = 2
```

```
C1_Standard = (0.955) + (2.169)*(sqrt(T_over_R_Standard)) - (0.081)*(T_over_R_Standard)
```

```
C1_Standard = 3.8604
```

```
C2_Standard = (-1.557) - (4.046)*(sqrt(T_over_R_Standard)) + (1.032)*(T_over_R_Standard)
```

```
C2_Standard = -5.2149
```

```
C3_Standard = (4.013) + (0.424)*(sqrt(T_over_R_Standard)) - (0.748)*(T_over_R_Standard)
```

```
C3_Standard = 3.1166
```

```
C4_Standard = (-2.461) + (1.538)*(sqrt(T_over_R_Standard)) - (0.236)*(T_over_R_Standard)
```

```
C4_Standard = -0.7579
```

```
KT_Standard = (C1_Standard) + (C2_Standard)*(2*T_Standard/H_Standard) + (C3_Standard)*(2*T_Standard/H_Standard)^2 + (C4_Standard)*(2*T_Standard/H_Standard)^3
```

```
KT_Standard = 3.5104
```

```
%% STANDARD RPM
```

```
Final_Bolt_End_Lim = (Bolt_End_Lim_Standard)/(KT_Standard) % N/mm^2
```

```
Final_Bolt_End_Lim = 156.6774
```

```
Allowable_Range_Standard = (2)*(Final_Bolt_End_Lim)*((1-(Bolt_Stress_Mean_Standard)/Bolt_UTS_Standard)) % N/mm^2
```

```
Allowable_Range_Standard = 53.7196
```

```
Fatigue_Safety_Factor_Standard = (Allowable_Range_Standard)/(Bolt_Stress_Range_Standard) % Ratio
```

```
Fatigue_Safety_Factor_Standard = 1.3152
```

PART G & H

BOLT RE-DESIGN

```
%% PART G - MINIMUM SAFE DIAMETER
```

```
Min_Safe_Diameter = sqrt((Bolt_UTS_Standard/Bolt_UTS_Modified_625)*(Bolt_Diameter^2)) % mm
```

```
Min_Safe_Diameter = 5.6508
```

```
% SECTION 1
```

```
L1_Redesign = 5; % mm
```

```
Diam_1_Redesign = Bolt_Diameter % mm
```

```
Diam_1_Redesign = 7.3500
```

```
A1_Redesign = ((pi)*(Diam_1_Redesign^2))/4 % mm
```

```
A1_Redesign = 42.4292
```

```
K1_Redesign = ((Bolt_E_Modified_625)*(A1_Redesign))/(L1_Redesign) % N/mm
```

```
K1_Redesign = 1.7820e+03
```

```
% SECTION 2
```

```
L2_Redesign = 24; % mm
```

```
Diam_2_Redesign = (Min_Safe_Diameter)*1.1 % mm
```

```
Diam_2_Redesign = 6.2159
```

```
A2_Redesign = ((pi)*(Diam_2_Redesign^2))/4 % mm
```

```
A2_Redesign = 30.3456
```

```
K2_Redesign = ((Bolt_E_Modified_625)*(A2_Redesign))/(L2_Redesign) % N/mm
```

```
K2_Redesign = 265.5243
```

```
% SECTION 3
```

```
L3_Redesign = 8; % mm
```

```
Diam_3_Redesign = Bolt_Diameter % mm
```

```
Diam_3_Redesign = 7.3500
```

```
A3_Redesign = ((pi)*(Diam_3_Redesign^2))/4 % mm
```

```
A3_Redesign = 42.4292
```

```
K3_Redesign = ((Bolt_E_Modified_625)*(A3_Redesign))/(L3_Redesign) % N/mm
```

```
K3_Redesign = 1.1138e+03
```

```
% TOTAL STIFFNESS
```

```
KTotal_Redesign = ((1/K1_Redesign)+(1/K2_Redesign)+(1/K3_Redesign))^-1 % N/mm
```

```
KTotal_Redesign = 191.3822
```

```
% SURFACE AREA
```

```
Bolt_Surface_Area_Redesign = ((A1_Redesign)+(A2_Redesign)+(A3_Redesign))/3 % mm^2
```

```
Bolt_Surface_Area_Redesign = 38.4013
```

```
% DIAMETER
```

```
Bolt_Diameter_Redesign = ((Diam_1_Redesign)+(Diam_2_Redesign)+(Diam_3_Redesign))/3 % mm
```

```
Bolt_Diameter_Redesign = 6.9720
```

```
% STRESS CONCENTRATION DATA - MODIFIED
```

```
H_Modified = 8.00; % mm - Cap thickness
```

```
d_Modified = Bolt_Diameter_Redesign; % mm - Rod Thickness
```

```
R_Modified = 0.14; % mm - Radius
```

```
T_Modified = 0.28; % mm - D to edge
```

```
R_over_d_Modified = (R_Modified)/(d_Modified)
```

```
R_over_d_Modified = 0.0201
```

```
H_over_d_Modified = (H_Modified)/(d_Modified)
```

```
H_over_d_Modified = 1.1475
```

```
T_over_R_Modified = (T_Modified)/(R_Modified)
```

```
T_over_R_Modified = 2
```

```
C1_Modified = (0.955) + (2.169)*(sqrt(T_over_R_Modified)) - (0.081)*(T_over_R_Modified)
```

```
C1_Modified = 3.8604
```

```
C2_Modified = (-1.557) - (4.046)*(sqrt(T_over_R_Modified)) + (1.032)*(T_over_R_Modified)
```

```
C2_Modified = -5.2149
```

```
C3_Modified = (4.013) + (0.424)*(sqrt(T_over_R_Modified)) - (0.748)*(T_over_R_Modified)
```

```
C3_Modified = 3.1166
```

```
C4_Modified = (-2.461) + (1.538)*(sqrt(T_over_R_Modified)) - (0.236)*(T_over_R_Modified)
```

```
C4_Modified = -0.7579
```

```
KT_Modified = (C1_Modified) + (C2_Modified)*(2*T_Modified/H_Modified) + (C3_Modified)*(2*T_Modified/H_Modified)^2 + (C4_Modified)*(2*T_Modified/H_Modified)^3
```

```
KT_Modified = 3.5104
```

MODIFICATION SAFETY FACTORS

```
%% MODIFIED RPM - STANDARD MATERIALS
```

```
Final_Bolt_End_Lim = (Bolt_End_Lim_Standard)/(KT_Standard) % N/mm^2
```

```
Final_Bolt_End_Lim = 156.6774
```

```
Allowable_Range_Modified = (2)*(Final_Bolt_End_Lim)*((1-(Bolt_Stress_Mean_Modified)/Bolt_UTS_Standard)) % N/mm^2
```

```
Allowable_Range_Modified = 46.4474
```

```
Fatigue_Safety_Factor_Modified = (Allowable_Range_Modified)/(Bolt_Stress_Range_Modified) % Ratio
```

```
Fatigue_Safety_Factor_Modified = 0.5054
```

```
%% PART D - MODIFIED RPM - MODIFIED MATERIALS
```

```
Bolt_Yield_Modified_625 = (0.53)*(Bolt_UTS_Modified_625) % MPa
```

```
Bolt_Yield_Modified_625 = 986.3300
```

```
Axial_Stress = (0.6)*(Bolt_Yield_Modified_625) % N/mm^2
```

```
Axial_Stress = 591.7980
```

```
Clamping_Force_Modified_625 = (Bolt_Surface_Area)*(Axial_Stress) % N
```

```
Clamping_Force_Modified_625 = 2.3198e+04
```

```
Circumfrenclal_Friction_Modified_625 = (Bolt_Thread_Mu)*(Clamping_Force_Modified_625) % N
```

```
Circumfrenclal_Friction_Modified_625 = 2.7838e+03
```

```
Resisting_Torque_Threads_Modified_625 = (Bolt_Diameter/2)*(Circumfrenclal_Friction_Modified_625) % Nmm
```

```
Resisting_Torque_Threads_Modified_625 = 1.0231e+04
```

```
Bolt_Head_Friction_Force_Modified_625 = (Bolt_Head_Mu)*(Clamping_Force_Modified_625) % N
```

```
Bolt_Head_Friction_Force_Modified_625 = 4.1757e+03
```

```
Resisting_Torque_Head_Modified_625 = (Bolt_Head_Friction_Force_Modified_625)*(Bolt_Head_Diameter/2) % Nmm
```

```
Resisting_Torque_Head_Modified_625 = 2.6933e+04
```

```
Total_Torque_Modified_625 = (Resisting_Torque_Head_Modified_625/1000) + (Resisting_Torque_Threads_Modified_625/1000) % Nm
```

```
Total_Torque_Modified_625 = 37.1640
```

```
%% STIFFNESS RATIO - MODIFIED RPM - MODIFIED MATERIALS
```

```
Bolt_Stiffness_Modified = ((Bolt_E_Modified_625)*(Bolt_Surface_Area))/(Bolt_Length) % N/m
```

```
Bolt_Stiffness_Modified = 319.6893
```

```
Stiffness_Ratio_Modified = (Bolt_Stiffness_Modified)/(Cap_Stiffness) % Ratio
```

```
Stiffness_Ratio_Modified = 0.3564
```

```
% ALTERNATING LOAD - MODIFIED RPM - MODIFIED MATERIALS
```

```
Alternating_Load_9000rpm_Modified = (1/(1 + (1/Stiffness_Ratio_Modified)))*((Inertia_Force_9000rpm)/2) % N
```

```
Alternating_Load_9000rpm_Modified = 3.6026e+03
```

```
%% ALTERNATING STRESS - MODIFIED RPM - MODIFIED MATERIALS
```

```
Alternating_Stress_9000rpm_Modified = (Alternating_Load_9000rpm_Modified)/(Bolt_Surface_Area) % N/mm^2
```

```
Alternating_Stress_9000rpm_Modified = 91.9027
```

```
%% PART E - MODIFIED RPM - MODIFIED MATERIALS
```

```
Bolt_Stress_Min_Modified_625 = (0.53)*(Bolt_Yield_Modified_625) % N/mm^2
```

```
Bolt_Stress_Min_Modified_625 = 522.7549
```

```
Bolt_Stress_Range_Modified_625 = (Alternating_Stress_9000rpm_Modified) % N/mm^2
```

```
Bolt_Stress_Range_Modified_625 = 91.9027
```

```
Bolt_Stress_Max_Modified_625 = (Bolt_Stress_Range_Modified_625 + Bolt_Stress_Min_Modified_625) % N/mm^2
```

```
Bolt_Stress_Max_Modified_625 = 614.6576
```

```
Bolt_Stress_Mean_Modified_625 = (Bolt_Stress_Max_Modified_625 + Bolt_Stress_Min_Modified_625)/2 % N/mm^2
```

```
Bolt_Stress_Mean_Modified_625 = 568.7062
```

```
%% PART F - MODIFIED RPM - MODIFIED MATERIALS
```

```
Final_Bolt_End_Lim_625 = (Bolt_End_Lim_Modified_625)/(KT_Modified) % N/mm^2
```

```
Final_Bolt_End_Lim_625 = 265.0697
```

```
Allowable_Range_Modified_625 = (2)*(Final_Bolt_End_Lim_625)*((1-(Bolt_Stress_Mean_Modified_625)/Bolt_UTS_Modified_625)) % N/mm^2
```

```
Allowable_Range_Modified_625 = 368.1332
```

```
Fatigue_Safety_Factor_Modified_625 = (Allowable_Range_Modified_625)/(Bolt_Stress_Range_Modified_625) % Ratio
```

```
Fatigue_Safety_Factor_Modified_625 = 4.0057
```

```
%% STIFFNESS RATIO - MODIFIED RPM - MODIFIED MATERIALS - REDESIGN
```

```
Bolt_Stiffness_Redesign = ((Bolt_E_Modified_625)*(Bolt_Surface_Area_Redesign))/(Bolt_Length) % N/m
```

```
Bolt_Stiffness_Redesign = 313.1759
```

```
Stiffness_Ratio_Redesign = (Bolt_Stiffness_Redesign)/(Cap_Stiffness) % Ratio
```

```
Stiffness_Ratio_Redesign = 0.3491
```

```
% ALTERNATING LOAD - MODIFIED RPM - MODIFIED MATERIALS - REDESIGN
```

```
Alternating_Load_9000rpm_Redesign = (1/(1 + (1/Stiffness_Ratio_Redesign)))*((Inertia_Force_9000rpm)/2) % N
```

```
Alternating_Load_9000rpm_Redesign = 3.5482e+03
```

```
%% ALTERNATING STRESS - MODIFIED RPM - MODIFIED MATERIALS - REDESIGN
```

```
Alternating_Stress_9000rpm_Redesign = (Alternating_Load_9000rpm_Redesign)/(Bolt_Surface_Area_Redesign) % N/mm^2
```

```
Alternating_Stress_9000rpm_Redesign = 92.3973
```

```
%% PART D - MODIFIED RPM - MODIFIED MATERIALS - REDESIGN
```

```
Bolt_Yield_Modified_625_Redesign = (0.9)*(Bolt_UTS_Modified_625) % MPa
```

```
Bolt_Yield_Modified_625_Redesign = 1.6749e+03
```

```
Axial_Stress = (0.6)*(Bolt_Yield_Modified_625_Redesign) % N/mm^2
```

```
Axial_Stress = 1.0049e+03
```

```
Clamping_Force_Modified_625_Redesign = (Bolt_Surface_Area_Redesign)*(Axial_Stress) % N
```

```
Clamping_Force_Modified_625_Redesign = 3.8591e+04
```

```
Circumfrenclal_Friction_Modified_625_Redesign = (Bolt_Thread_Mu)*(Clamping_Force_Modified_625_Redesign) % N
```

```
Circumfrenclal_Friction_Modified_625_Redesign = 4.6309e+03
```

```
Resisting_Torque_Threads_Modified_625_Redesign = (Bolt_Diameter_Redesign/2)*(Circumfrenclal_Friction_Modified_625_Redesign) % Nmm
```

```
Resisting_Torque_Threads_Modified_625_Redesign = 1.6143e+04
```

```
Bolt_Head_Friction_Force_Modified_625_Redesign = (Bolt_Head_Mu)*(Clamping_Force_Modified_625_Redesign) % N
```

```
Bolt_Head_Friction_Force_Modified_625_Redesign = 6.9464e+03
```

```
Resisting_Torque_Head_Modified_625_Redesign = (Bolt_Head_Friction_Force_Modified_625_Redesign)*(Bolt_Head_Diameter/2) % Nmm
```

```
Resisting_Torque_Head_Modified_625_Redesign = 4.4804e+04
```

```
Total_Torque_Modified_625_Redesign = (Resisting_Torque_Head_Modified_625_Redesign/1000) + (Resisting_Torque_Threads_Modified_625_Redesign/1000) % Nm
```

```
Total_Torque_Modified_625_Redesign = 60.9475
```

```
%% PART E - MODIFIED RPM - MODIFIED MATERIALS - REDESIGN
```

```
Bolt_Stress_Min_Modified_625_Redesign = (0.6432)*(Bolt_Yield_Modified_625) % N/mm^2
```

```
Bolt_Stress_Min_Modified_625_Redesign = 634.4075
```

```
Bolt_Stress_Range_Modified_625_Redesign = (Alternating_Stress_9000rpm_Redesign) % N/mm^2
```

```
Bolt_Stress_Range_Modified_625_Redesign = 92.3973
```

```
Bolt_Stress_Max_Modified_625_Redesign = (Bolt_Stress_Range_Modified_625_Redesign + Bolt_Stress_Min_Modified_625_Redesign) % N/mm^2
```

```
Bolt_Stress_Max_Modified_625_Redesign = 726.8048
```

```
Bolt_Stress_Mean_Modified_625_Redesign = (Bolt_Stress_Max_Modified_625_Redesign + Bolt_Stress_Min_Modified_625_Redesign)/2 % N/mm^2
```

```
Bolt_Stress_Mean_Modified_625_Redesign = 680.6061
```

```
%% PART F - MODIFIED RPM - MODIFIED MATERIALS - REDESIGN
```

```
Allowable_Range_Modified_625_Redesign = (2)*(Final_Bolt_End_Lim_625)*((1-(Bolt_Stress_Mean_Modified_625_Redesign)/Bolt_UTS_Modified_625)) % N/mm^2
```

```
Allowable_Range_Modified_625_Redesign = 336.2565
```

```
Fatigue_Safety_Factor_Modified_625_Redesign = (Allowable_Range_Modified_625_Redesign)/(Bolt_Stress_Range_Modified_625_Redesign) % Ratio
```

```
Fatigue_Safety_Factor_Modified_625_Redesign = 3.6392
```

9.4 Appendix Four - Question Four MATLAB Script

LOADING OF CONSTANTS

```
Load_Constants = ☐; % TICK TO LOAD IN CONSTANTS

if Load_Constants == 1
    Constants_And_Cylinder_Data
end
```

CONSTANT VALUES

```
%% MASSES
Recip_Mass_Crank = (Mass_Piston_Total) + (1/3*(Mass_Conrod)); % Kg
Rot_Mass_Crank = (Mass_Conrod_Total) - (1/3*Mass_Conrod); % Kg
Recip_Mass_Piston = (Mass_Piston_Total) + (1/3*Mass_Conrod_Total); % Kg
```

%% CRANK DIMENSIONS

```
L2 = 33.41; % mm
AB = 20.75; % mm
D1 = 43.97; % mm
D2 = 48.00; % mm
b = ((D1) + (D2))/2 % mm
```

```
b = 45.9850
```

```
D = 20.50; % mm
I = ((b)*(D^3))/12 % mm^4
```

```
I = 3.3014e+04
```

```
Y = (0.5)*(AB) % mm
```

```
Y = 10.3750
```

```
Surface_Area = ((pi)*(D1)^2)/4 % mm^2
```

```
Surface_Area = 1.5185e+03
```

ANGULAR VELOCITY

```
% 1000rpm
W_1000rpm = (2*pi*1000)/60; % rads/sec
```

CRANK INERTIA FORCE

```
% 1000rpm
Inertia_Force_1000rpm = ((Recip_Mass_Crank)*(Crank_Throw_R)*(W_1000rpm)^2*(1+(Crank_Throw_R/Rod_Length_L))) + ((Rot_Mass_Crank)*(Crank_Throw_R)*(W_1000rpm)^2); % K
```

NET FORCE

```
% 1000rpm
Net_Force_1000rpm = (Gas_Force_1000rpm)-(Inertia_Force_1000rpm); % N

[Max_Net_Force_1000rpm,I_Max_Net_1000rpm] = max(Net_Force_1000rpm); % N
[Min_Net_Force_1000rpm,I_Min_Net_1000rpm] = min(Net_Force_1000rpm); % N
```

BENDING MOMENT

```
% 1000rpm
Max_Bending_Moment_1000rpm = ((Max_Net_Force_1000rpm/2)*(L2)); % N/mm
Min_Bending_Moment_1000rpm = ((Min_Net_Force_1000rpm/2)*(L2)); % N/mm
```

BENDING STRESS

```
% 1000rpm
Max_Bending_Stress_1000rpm = ((Max_Bending_Moment_1000rpm)*(Y))/(I); % N/mm^2
Min_Bending_Stress_1000rpm = ((Min_Bending_Moment_1000rpm)*(Y))/(I); % N/mm^2
```

SHEAR STRESS

```
% 1000rpm
Max_Shear_Stress_1000rpm = (Max_Net_Force_1000rpm/2)/(Surface_Area); % N/mm^2
Min_Shear_Stress_1000rpm = (Min_Net_Force_1000rpm/2)/(Surface_Area); % N/mm^2
```

COMBINED STRESS

```
% 1000rpm
Max_Combined_Stress_1000rpm = sqrt(Max_Bending_Stress_1000rpm^2+(2*Max_Shear_Stress_1000rpm)^2); % N/mm^2
Min_Combined_Stress_1000rpm = sqrt(Min_Bending_Stress_1000rpm^2+(2*Min_Shear_Stress_1000rpm)^2); % N/mm^2
```

OVERALL MAXIMUM STRESSES

```
Max_Compressive_Stress = Max_Combined_Stress_5700rpm % N/mm^2
```

```
Max_Compressive_Stress = 175.3910
```

```
Max_Tensile_Stress = (Min_Combined_Stress_6000rpm)*(-1) % N/mm^2
```

```
Max_Tensile_Stress = -76.6969
```

```
Maximum_Tensile_Stress_Proportional = ((Inertia_Force_6000rpm)/(Max_Net_Force_5700rpm))*(Max_Compressive_Stress)*(-1); % N/mm^2
```

```
Mean_Stress = (Max_Compressive_Stress + Max_Tensile_Stress)/2 % N/mm^2
```

```
Mean_Stress = 49.3471
```

```
Range_Stress = (Max_Compressive_Stress - Max_Tensile_Stress) % N/mm^2
```

```
Range_Stress = 252.0878
```

STRESS CONCENTRATION FACTOR DATA

```
t = 0.52; % mm
```

```
H = 7.43; % mm
```

```
r = 1.50; % mm
```

```
d = 44.01; % mm
```

```
R_over_d = (r)/(d)
```

```
R_over_d = 0.0341
```

```
t_over_r = (t)/(r)
```

```
t_over_r = 0.3467
```

```
C1 = (0.966) + (1.056)*(sqrt(t_over_r)) - (0.022)*(t_over_r)
```

```
C1 = 1.5801
```

```
C2 = (-0.192) - (4.037)*(sqrt(t_over_r)) + (0.674)*(t_over_r)
```

```
C2 = -2.3353
```

```
C3 = (0.808) + (5.321)*(sqrt(t_over_r)) - (1.231)*(t_over_r)
```

```
C3 = 3.5142
```

$$C4 = (-0.567) - (2.364)*(\text{sqrt}(t_over_r)) - (0.566)*(t_over_r)$$

$$C4 = -2.1551$$

$$KT = (C1) + (C2)*(2*t/d) + (C3)*(2*t/d)^2 + (C4)*(2*t/d)^3$$

$$KT = 1.5269$$

$$\text{Final_Crankshaft_End_Lim} = (\text{Crankshaft_End_Lim})/(KT) \quad \% \text{ N/mm}^2$$

$$\text{Final_Crankshaft_End_Lim} = 221.0392$$

ALLOWABLE RANGE

$$\text{Allowable_Range} = (2)*(\text{Final_Crankshaft_End_Lim})*((1-(\text{Mean_Stress}/\text{Con_Rod_UTS}))) \quad \% \text{ N/mm}^2$$

$$\text{Allowable_Range} = 412.9914$$

FATIGUE SAFETY FACTOR

$$\text{Fatigue_Safety_Factor} = (\text{Allowable_Range})/(\text{Range_Stress}) \quad \% \text{ Ratio}$$

$$\text{Fatigue_Safety_Factor} = 1.6383$$