

1. Research Design:

1.1 - Theory:

A vibrating cantilever is a horizontal structural element that is unsupported at one end and fixed on the other. It is capable of bending when an external force is placed on it. When a load is applied, it transfers that load to the fixed end by bending. Since it is a rigid body, the forces that act on the free ends are counteracted by the fixed end, and thus any forces acting on the cantilever will not cause it to move or accelerate in any direction. This phenomenon is seen in everyday objects such as diving boards, road signs and bridges.

Throughout this investigation I plan to understand how the length of the cantilever impacts its motion when a constant force is applied and explore how the material's properties and elasticity (young's modulus) influence the oscillation period.

1.2 Research Question

How does the variation of length of a cantilever affect the period of oscillation of the cantilever while keeping the mass attached constant?

1.3 - Background Information:

When an external force is applied to a cantilever beam it will bend, oscillate and behave according to simple harmonic motion. The oscillation period depends on: the cantilever's length (L) as well as the attached mass (M), which creates an external force. The period of oscillation can be derived and found using the physical properties of a cantilever.

For a cantilever with a constant mass (M) placed on the free end. The length of the cantilever (L) and the period of oscillation (T) are related through the equation:

$$E = \frac{16\pi^2 ML^3}{bd^3 T^2} \quad (1) \quad (\text{Scribd, 2017})$$

Where:

E - Young Modulus of the vibrating cantilever (Pa) or (N/m^2) or ($kgm^{-1}s^{-2}$)

M - mass attached to the free end of the of vibrating cantilever (kg)

L - length of the vibrating cantilever beam (m)

b - width of the vibrating cantilever beam (m)

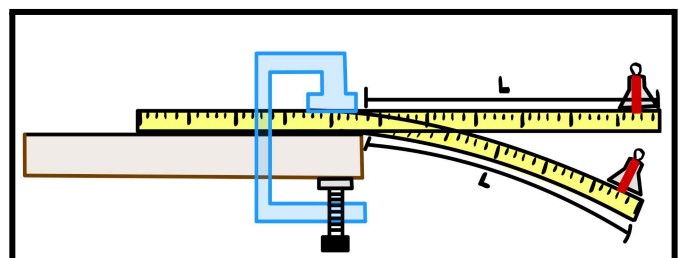
d - depth of vibrating cantilever beam (m)

T - period of a single oscillation (s)

Hence, we can rearrange the equation (1) for the young modulus (E) in the format $y = mx$ to predict the relationship between the independent variable (length of the cantilever) and the dependent variable (period of oscillation)

$$T^2 E b d^3 = 16\pi^2 M L^3 \quad (2)$$

Diagram of Vibrating Cantilever (Ruler) Oscillation: (Figure 1):



$$T^2 = \frac{16\pi^2 M}{Ebd^3} \cdot L^3 \quad (3)$$

Hence equation (1) is now in the format to display the relationship between the variables:

$$y = mx$$

Where:

y - is the period of a single oscillation (T^2) in seconds (s) - dependent variable:

m - is ($\frac{16\pi^2 M}{Ebd^3}$) gradient to the linear equation:

x - is the length of the cantilever cubed (L^3) in meters (m) - independent variable:

Thus rearranging the equation in the format of $y = mx$, allowing to plot (T^2) on the y axis and on the x-axis (L^3), the gradient - m of the linear equation can be utilised, using the known dimensions of the wooden ruler (vibrating cantilever): width (b), depth (d) and the attached mass (m) the young modulus (E) can be calculated:

$$\frac{T^2}{L^3} = \frac{16\pi^2 M}{Ebd^3} \quad (4)$$

Where:

- $\frac{T^2}{L^3}$ is equal to the gradient of the curve ($\frac{16\pi^2 M}{Ebd^3}$)

Hence the equation can be re-written to experimentally determine the young's modulus (E), thus qualifying the wooden ruler (cantilever) stiffness:

$$E = \frac{16\pi^2 M \cdot (\frac{1}{\text{gradient}})}{bd^3} \quad (5)$$

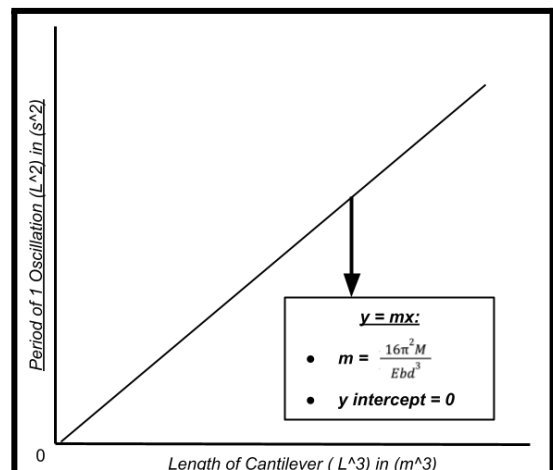
1.4 - Hypothesis:

I hypothesise that increasing the length of the cantilever whilst keeping the mass attached to the free end constant will result in a longer period of oscillation, following simple harmonic principles. According to the derived equation $T^2 = \frac{16\pi^2 M}{Ebd^3} \cdot L^3$ (3) in the format ($y = mx$). I predict that the relationship between them is:

$$T^2 \propto L^3$$

This indicates as the length of the cantilever increases, the square of the period of oscillation will increase in direct proportion to the cube of the length. Therefore, I predict when plotting the period of oscillation squared (T^2) against the length of the cantilever cubed (L^3) there will be a linear relationship that passes through the origin. As seen in figure 2 above:

Predicted relationship in form $y = mx$: (Figure 2)



To further understand this relationship between the two variables, we can view the relationship as a logarithmic form. (This relationship is displayed in the [Appendices](#).)

1.5 - Variables:

Table 1 – showcasing and explaining the independent and dependent variables in the experiment:

Manipulated and measured variables:			
Variables:		Explanation:	Apparatus:
Independent variable	The length of cantilever	{0.30m,0.350m,0.400m,...,1.000m} The distance from the fixed end to the free end of the cantilever.	Wooden Ruler ($\pm 0.001\text{m}$) (Measured the length of the cantilever, hence cubing the value to obtain the independent variable)
Dependent variable	The Period of time for a single oscillation	The period of time for a single oscillation was measured from rest until the cantilever returned to its starting position. This process was repeated 5 times for each length	Stopwatch ($\pm 0.01\text{s}$)

Table 2 - the control variables of the experiment and how to keep them constant to ensure a fair experiment

Controlled variables:		
Variable:	Significance:	Means of control:
Material of the Cantilever:	The material of the cantilever must remain constant as different materials have varying elastic properties and young's modulus values which affect oscillation period.	The same wooden ruler was used as my cantilever beam through the experiment.
Mass Attached to Free End of Cantilever (kg)	The mass attached to the free end of the cantilever must be constant as the mass can affect the period of oscillation according to equation (3): $T^2 = \frac{16\pi^2 M}{Ebd^3} \cdot L^3$	Fixed mass (0.2kg) attached to the free end of the ruler with duct tape throughout the experiment.
Attachment point of the mass	Changes in attachment point of mass can vary the effective length of the cantilever.	The mass is attached at a fixed point near the free-end of the wooden ruler.

Number of Oscillations Recorded:	The number of oscillations directly impacts the accuracy of the period measurement. A larger number of oscillations helps to reduce the timing errors and uncertainty.	The number of oscillations will be kept constant at 10 to ensure that all measurements of the oscillation period will be consistent.
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Table 3 - uncontrolled variables throughout the experiment:

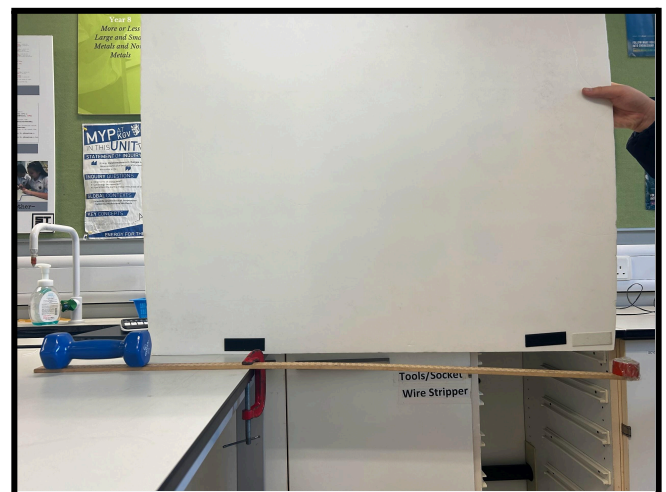
Uncontrolled variables:	
Variables:	Significance:
Inconsistencies in the cantilver's material	microscopic cracks and density variations may cause small changes in the oscillation behaviour hence impacting accuracy of period measurements.
Environmental Conditions	Air resistance can affect the amplitude of oscillations, thus affecting the oscillation period by dissipating energy level. Furthermore temperature fluctuations can cause variations in cantilever properties young's modulus and stiffness

1.6 - Apparatus:

Diagram of Apparatus (Figure 4):

- Cantilever (Wooden Ruler)
- Masses (0.2kg) ($\pm 0.1\text{kg}$)
- Stopwatch ($\pm 0.01\text{s}$)
- Ruler (Measuring Tape) ($\pm 0.0005\text{m}$)
- Vernier Caliper
- G-Clamp
- Measuring Scale (Balance) ($\pm 0.01\text{g}$)
- Tape
- Marker

The apparatus are shown in the diagram on the left in **figure 4**:



1.7 - Method:

1. To start: using a vernier calliper measure the width (b) and depth (d) of the wooden ruler. These measurements will be used to calculate the young's modulus.
2. Secure the end of the cantilever to a flat surface above ground level using a G-Clamp ensuring the cantilever is clamped firmly to avoid any hazards.
3. Attached the fixed mass (0.2kg) to the free end of the wooden ruler (cantilever) using duct tape to maintain a consistent attachment point
4. Use a ruler to measure the length of the cantilever from where it leaves the surface to the free-end. Start with the longest length of (1.00m) decreasing the length with 0.05m increments (e.g 0.95m, 0.90m ...) until 0.30m for each different trial.

- For each length, pull the cantilever down slightly to create a small displacement and initial force, then release it to allow it to begin oscillating. Then use a stopwatch to measure the time for the cantilever to complete 10 full oscillations.
- For each length, repeat this measurement for 10 full oscillations five times to ensure the reliability of the experiment and then average the results by dividing total time by 50.
- Record all the measurements of (L - length of cantilever and T - time period) , including the depth(d), width(b) and mass(M) of the cantilever, in a data file in logger pro.

1.8 Preliminary Trails:

Throughout the preliminary trails, I identified issues with my methodology and optimized my procedure. I initially planned to measure the time for 5 oscillations but this increased to 10 oscillations in the final experiment to reduce percentage uncertainty when timing. Additionally I switched from using a 0.1kg to a 0.2kg mass to help increase the average oscillation period time for more precise timing.

1.9 - Risk Assessment:

Table 4: Potential Hazards, Risks and precautions to take throughout the experiment:

Hazard:	Risk:	Precautions:
Cantilever (wooden ruler) falling off clamp	The cantilever could possibly fall off the g-clamp on the table and hit someone.	Ensure that the cantilever is clamped securely to the table. I will check on the cantilever and g-clamp at every length interval.
Mass falling off cantilever (wooden ruler)	The mass attached to the free end of the cantilever could fall on someone.	Before the experiment starts ensure that the masses are attached securely to the cantilever and I will check the masses position after each length interval.

2. Analysis:

Firstly, the theoretical values of the period of oscillation for each length were calculated using the formula (3) acquired in background information and the young modulus value for “Maple Sugar (Green)” which is a common wood type utilised to produce rulers:

- Young’s Modulus of Maple Sugar (Green): 10.7 GPa or $1.07 \times 10^{10} \text{ kgm}^{-1} \text{ s}^{-2}$
- Uncertainty of the Young Modulus Value according to (Amesweb) is 22%:

All sample calculations were conducted for the length of cantilever of 0.9m + 0.0005m:

- Calculating the theoretical values of the period of oscillation and period of oscillation squared:

$$T^2 = \frac{16\pi^2 M}{Ebd^3} \cdot L^3 \quad (3)$$

$$T^2 = \frac{16L^3 \pi^2 M}{Ebd^3}$$

The formula can be expressed as:

$$T = \sqrt{\frac{16L^3 \pi^2 M}{Ebd^3}}$$

Plugging in 0.90m into each equation:

These are the constants that will be required in the following calculations:

- $b = 0.0270m \pm 0.0001m$
- $d = 0.0072m \pm 0.0001m$
- $M = 0.20kg \pm 0.01kg$

$$T^2 = \frac{16(0.90)^3 \pi^2 (0.20)}{(1.07 \times 10^{10})(0.0270)(0.0072)^3} = 0.22s^2$$

$$T = \sqrt{\frac{16(0.90)^3 \pi^2 (0.20)}{(1.07 \times 10^{10})(0.0270)(0.0072)^3}} = 0.47s$$

2. Calculating the percentage uncertainty of the theoretical value of period of oscillation and period of oscillation squared:

For the length of the cantilever percentage uncertainty I will utilise the median/mean value of the length (0.65m) to get a percentage uncertainty that is representative:

To find the uncertainty of the period of oscillation squared I will sum up the uncertainty from each component:

$$\% \Delta T^2 = 3\% \Delta L + \% \Delta M + \% \Delta E + \% \Delta b + 3\% \Delta d$$

$$\% \Delta T^2 = 3(100(\frac{\Delta L}{L(\text{mean})})) + 100(\frac{\Delta M}{M}) + (\% \Delta E) + 100(\frac{\Delta b}{b}) + 3(100(\frac{\Delta d}{d}))$$

$$\% \Delta T^2 = 3(100(\frac{0.001}{0.65})) + 100(\frac{0.10}{2.0}) + (22) + 100(\frac{0.0001}{0.0270}) + 3(100(\frac{0.0001}{0.0072})) \approx \pm 32\%$$

Whilst for the period of oscillation, I will multiply the period of oscillation squared uncertainty by half:

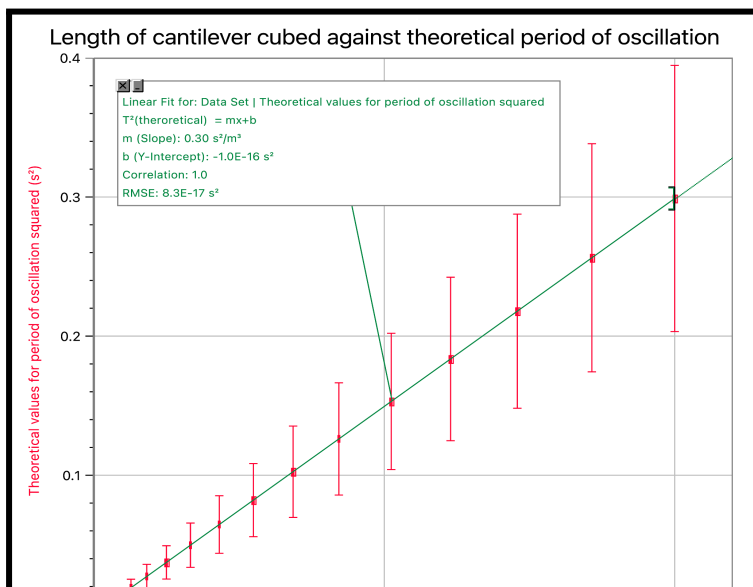
$$\% \Delta T = \frac{1}{2} \cdot (3(100(\frac{\Delta L}{L(\text{mean})})) + 100(\frac{\Delta M}{M}) + (\% \Delta E) + 100(\frac{\Delta b}{b}) + 3(100(\frac{\Delta d}{d})))$$

$$\% \Delta T = \frac{1}{2} \cdot (3(100(\frac{0.001}{0.65})) + 100(\frac{0.10}{2.0}) + (22) + 100(\frac{0.0001}{0.0270}) + 3(100(\frac{0.0001}{0.0072}))) \approx \pm 16\%$$

Table 5: The results are presented in the [Appendices](#):

3. Plotting the data:

(Graph 1): Length of cantilever cubed against theoretical period of oscillation squared:



The equation $T^2 = \frac{16\pi^2 M}{Ebd^3} \cdot L^3$ (3), helps to predict the linear relationship between T^2 and L^3 suggesting that T^2 is directly proportional to L^3 . Hence indicating that as the length of the cantilever increases, the period of oscillation will increase proportionally. The theoretical data presented above follows this trend

helping to confirm the proportionality of $T^2 \propto L^3$

2.1 Quantitative Data:

Table 6: Includes the raw data collected through my experiment

Length (L) of cantilever (m) $\pm 0.001\text{m}$	Period for 10 oscillations of the cantilever (s) $\pm 0.1\text{s}$				
	Trial #1 T(1)	Trial #2 T(2)	Trial #3 T(3)	Trial #4 T(4)	Trial #5 T(5)
1.000	5.5	5.6	5.5	5.5	5.6
0.950	5.3	5.2	5.2	5.2	5.2
0.900	4.9	4.8	4.9	4.8	4.8
0.850	4.3	4.3	4.3	4.5	4.3
0.800	4.0	4.0	4.0	4.1	4.0
0.750	3.6	3.6	3.6	3.5	3.6
0.700	3.3	3.3	3.3	3.3	3.2
0.650	2.9	2.9	2.9	2.8	2.8
0.600	2.6	2.5	2.5	2.6	2.5
0.550	2.3	2.2	2.2	2.1	2.2
0.500	1.7	1.8	1.9	1.9	2.0
0.450	1.7	1.6	1.6	1.6	1.6
0.400	1.4	1.4	1.3	1.4	1.4
0.350	1.1	1.1	1.1	1.2	1.2
0.300	0.9	1.0	0.9	0.9	0.9

2.2 Data Processing:

1. The uncertainty of the length of the cantilever can be calculated using the formula:

$$\Delta x = \text{smallest increment (Deanza, n.d)}$$

Since I utilised a ruler for measuring the length of the cantilever, which has a smallest increment of 0.1cm/0.001m. Substituting 0.001m into the formula below, I found the absolute uncertainty as:

$$\Delta L = 0.001 = \pm 0.001m$$

2. To identify the uncertainty for a period of 10 oscillations I calculated the range from the 5 repeats and used it as the absolute uncertainty.

For example: for the period of 10 oscillations at length of 0.90m:

$$\Delta T = \max(T) - \min(T)$$

$$\Delta T = 4.9s - 4.8s = \pm 0.1$$

As the range is $\pm 0.1s$ for the majority of the lengths of the cantilever, I will take the period of 10 oscillations uncertainty as: $\pm 0.1s$:

All the data processing from this point will be presented in the form of a formula and a sample calculation for the length of the cantilever at 0.90m:

3. Calculating the average period of time of a single oscillation, the following equation is used as each trial contained 10 oscillations and to find period of a single oscillation, the sum of each of the 5 trials must be divided by 50:

$$T(\text{avg}) = \frac{T(1)+T(2)+T(3)+T(4)+T(5)}{50}$$

$$T(\text{avg}) = \frac{4.9 + 4.8 + 4.9 + 4.8 + 4.8}{50} = 0.48s$$

4. Calculating the uncertainty of the average time period for a single oscillation:

To calculate the percentage uncertainty of a single oscillation, I will divide the absolute uncertainty for 10 oscillations by median time for 10 oscillations at each length, then multiplying by 100.

For example: The uncertainty calculation of 1 oscillation at length of 0.90 meters:

$$\% \Delta T = \left(\frac{\Delta T_{10}}{T_{10}} \right) \cdot 100$$

$$\% \Delta T = \left(\frac{0.1}{4.8} \right) \cdot 100 = 2\%$$

Since uncertainties are proportional, this percentage can be utilised for a single oscillation.

Table 7: The results of the calculations are above are presented - displayed in [Appendices](#):

5. Cubing the values of the length of the cantilever:

$$L^3 = L \cdot L \cdot L$$

$$L^3 = (0.900) \cdot (0.900) \cdot (0.900) = 0.73m^3$$

6. Finding the uncertainty of the length of cantilever cubed utilising the formula below:

$$\frac{\Delta X}{X} = |n| \cdot \frac{\Delta A}{A} \text{ (Australian National University, n.d)}$$

Using the mean length of 0.65m to get a uncertainty that is representative:

$$\frac{\Delta L^3}{L^3} = |3| \cdot \frac{\Delta L}{L(\text{mean})}$$

$$\frac{\Delta L^3}{0.65^3} = |3| \cdot \frac{0.001}{0.65}$$

$$\Delta L^3 = \pm 0.001 m^3$$

7. Squaring the values of the average period of a single oscillation:

$$T(avg)^2 = T \cdot T$$

$$T(avg)^2 = (0.48) \cdot (0.48) = 0.23 s^2$$

8. Finding the uncertainty of the average period of a single oscillation squared:

This will be the sum of each period of oscillation component uncertainty:

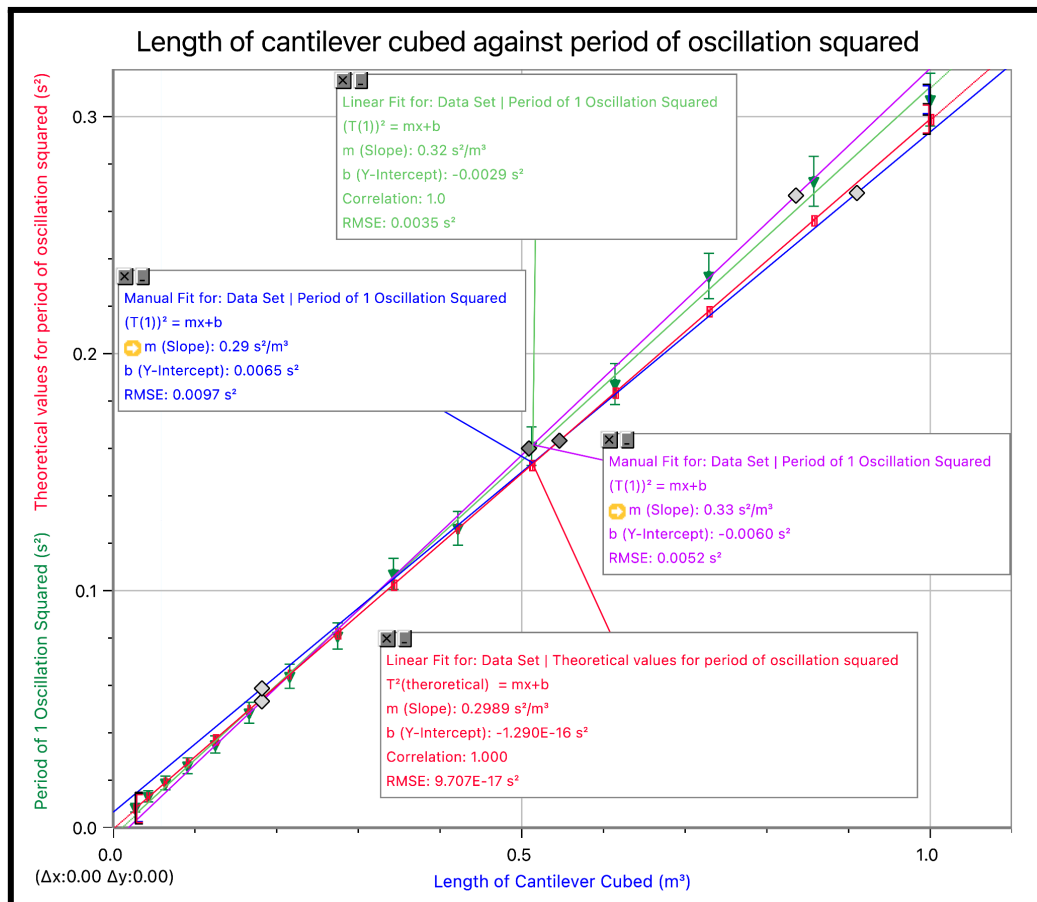
$$\% \Delta T(avg)^2 = \% \Delta T(avg) + \% \Delta T(avg)$$

$$\% \Delta T(avg)^2 = 2\% + 2\% = 4\%$$

Table 8: The results of the calculations are above are presented - displayed in [Appendices](#):

9. Plotting the data on the graph using Logger-Pro and determining the correlation between: length of cantilever cubed and period of a single oscillation squared as predicted in my hypothesis:

Graph 2: Length of cantilever cubed against period of oscillation squared:



The value of r - Pearson correlation coefficient of the data is equal to $1.0 \approx (0.9994)$ according to (Socscistatistics), thus this suggests that there is a positive linear correlation between length of the cantilever cubed and period of a single oscillation squared.

The error bars of the theoretical values of period of oscillation are larger than the experimental values due to the high young's modulus uncertainty (22%) of the literature value (Amesweb)

10. Calculating the young modulus of elasticity values for each line plotted above:

Using the equation (5) $E = \frac{16\pi^2 M \cdot (\frac{1}{\text{gradient}})}{bd^3}$ from the hypothesis:

- $b = 0.0270m \pm 0.0001m$
- $d = 0.0072m \pm 0.0001m$
- $M = 0.20kg \pm 0.01kg$

Original line: (*gradient*: $0.32 s^2/m^3$)

$$E = \frac{16\pi^2 \cdot (0.2) \cdot (\frac{1}{0.32})}{(0.02670)(0.0072)^3} = 1.0 \times 10^{10} kgm^{-1}s^{-2}$$

Maximum line : (*gradient*: $0.33 s^2/m^3$)

$$E = \frac{16\pi^2 \cdot (0.2) \cdot (\frac{1}{0.33})}{(0.0270)(0.0072)^3} = 0.99 \times 10^{10} kgm^{-1}s^{-2}$$

Minimum line : (*gradient*: $0.29 s^2/m^3$)

$$E = \frac{16\pi^2 \cdot (0.2) \cdot (\frac{1}{0.29})}{(0.02690)(0.00716)^3} = 1.1 \times 10^{10} kgm^{-1}s^{-2}$$

8. Calculating the gradient and young's modulus of elasticity uncertainty:

Uncertainty of the gradient can be calculated using the following:

$$\text{absolute uncertainty} = \frac{\text{max.gradient} - \text{min.gradient}}{2} \text{ (UPenn, n.d)}$$

$$\text{absolute uncertainty} = \frac{0.33 - 0.29}{2} = \pm 0.02 s^2/m^3$$

Calculating the gradient percentage uncertainty utilising the median value the gradient:

$$\% \Delta \text{gradient} = \frac{0.02}{0.32} \times 100 = \pm 6\%$$

Uncertainty of the young's modulus can be calculated using the following:

$$\Delta x = \frac{x_{\text{max}} - x_{\text{min}}}{2} \text{ (UPenn, n.d)}$$

$$\Delta E = \frac{1.1 \times 10^{10} - 0.99 \times 10^{10}}{2} = \pm 0.1 \times 10^{10} kgm^{-1}s^{-2}$$

Calculating the young's Modulus percentage uncertainty utilising the median value of young's modulus.

$$\% \Delta E = \frac{0.1 \times 10^{10} kgm^{-1}s^{-2}}{1.0 \times 10^{10} kgm^{-1}s^{-2}} \times 100 = \pm 1.0\%$$

3. Conclusion:

The statistical analysis of the data showcases a strong positive linear correlation between length of the cantilever cubed and the period of a single oscillation squared. This supports the hypothesis (in figure 2) as well as the theoretical equations discussed above. The results indicate that as the length of the cantilever increases, the period for oscillation also increases.

The equation $T^2 = \frac{Ebd^3}{16\pi^2 M} \cdot L^3$ (3) rearranged in the form: $y = mx$, allows for a linearized graphical representation producing a best-fit line passing almost perfectly through the origin with a Pearson correlation coefficient of $r = 1.0$ (0.9994), demonstrating a strong positive correlation.

The experiment findings allowed for the determination of the young modulus (stiffness) of the wooden cantilever ruler. Using the gradient obtained from the derived equation: $E = \frac{16\pi^2 M \cdot (\frac{1}{\text{gradient}})}{bd^3}$, The young's modulus values were calculated as follows:

Original Line: $1.0 \times 10^{10} \text{ kgm}^{-1} \text{ s}^{-2}$ or $10 \times 10^9 \text{ kgm}^{-1} \text{ s}^{-2}$

Maximum line: $0.99 \times 10^{10} \text{ kgm}^{-1} \text{ s}^{-2}$ or $9.9 \times 10^9 \text{ kgm}^{-1} \text{ s}^{-2}$

Minimum line: $1.1 \times 10^{10} \text{ kgm}^{-1} \text{ s}^{-2}$ or $11 \times 10^9 \text{ kgm}^{-1} \text{ s}^{-2}$

These values correspond to the reference data (AmesWeb) for common wood types: The calculated young modulus values suggest that the material falls within the range of Ash White (Green) - 9.90 GPa, Fir Balsam (Green) - 10.0 GPa and Spruce Black (Green) - 11.1 GPa.

The experiment's precision and accuracy was determined by analyzing the uncertainty in the calculated young's modulus values. The percentage uncertainty of the gradient was calculated to be: 6%, which is relatively low, thus ensuring that the theoretical value of the young's modulus fit within the experimental uncertainty. Hence, the hypothesis in section (1.4) was correct in stating that the length of the period of oscillation squared is directly proportional to the length of the cantilever cubed.

4. Evaluation:

4.1 Uncertainty:

The uncertainties of the experiment mainly arose due to measurement, human reaction time and setup inconsistencies, leading to the absolute uncertainty for the length of the cantilever of $\pm 0.0005\text{m}$, whilst the period of oscillation had a uncertainty of $\pm 0.1\text{s}$, for 10 oscillations. Hence in turn, each of these uncertainties affected their cubic and squared uncertainties.

Variations and errors in results were primarily due to random error such as: slight inconsistencies in forces when releasing the cantilever and the reaction time when utilising a stopwatch. Furthermore systematic errors may have occurred during the experiment for example: a slight change in the placement of the mass or changes in the position when the g-clamp was secured thus changing the effective length of the cantilever.

4.2 Comparison:

The experimental errors for each length of the cantilever can be calculated by comparing the theoretical values (calculated in section 2) to the values obtained through the experiment for a period of oscillation.

$$\text{Experimental error (\%)} = \left| \frac{\text{experimental value} - \text{expected value}}{\text{expected value}} \right| \cdot 100\% \text{ (CalculatorSoup, 2023)}$$

Calculating the experiment error for length of cantilever of 0.8m:

$$\text{Experimental error (\%)} = \left| \frac{0.40 - 0.39}{0.39} \right| \cdot 100\% = 2.6\%$$

Table 9: The results of the calculations are above are presented:

Experimental error of the period of oscillation for lengths of the cantilever: T/s ± 0.1s															
L(m)	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95	1.0
(%) error	1.4	3.1	3.3	1.0	2.6	0.1	2.2	0.7	0.5	1.3	3.1	2.1	0.7	1.6	2.2

Hence calculating the average percentage error throughout the experiment, to be found as:

$$\text{Average percentage experimental error} = \frac{1.4+3.1+3.3+1.0+2.6+0.1+2.2+0.7+0.5+1.3+3.1+2.1+0.7+1.6+2.2}{15} = 1.34\% \approx 1\%$$

Hence, the overall experimental error was 1%, The error was the greatest for the length of the cantilever of 0.40m (3.3%). Whilst the least error was for the length of cantilever at 0.55m being 0.1%.

4.3 Weaknesses and limitations

Table 10: The random errors are presented below:

Source of error:	Impact of result:	Improvement:
Human reaction time in timing oscillations periods	Timing of oscillations with a stopwatch introduces human errors due to changes in reaction times, thus affecting accuracy levels in measuring the period. Since the average human reaction time is on average 0.2 seconds (HumanBenchMark, 2025), hence the period of oscillation, percentage uncertainty should vary an additional 5% on average.	Use an motion sensor or a high speed camera with frame analysis, thus allowing to obtain more precise and accurate results of oscillation periods
Damping effects due to air resistance.	Air resistance over the time for 10 oscillations can gradually reduce the amplitude of oscillation, which can affect the average single period measurement over multiple repeats.	Conduct the experiment in a closed environment with minimal air flow level and use a heavier cantilever to reduce the effect of air resistance.

Measurement of Length of Cantilever:	The wooden ruler utilised throughout this experiment measures the length of the cantilever has a small uncertainty ($\pm 0.001\text{m}$), thus it can introduce a small variations in the effective length of the cantilever	Use a laser measuring device to get a more precise measurement of where the wooden ruler cantilever leaves the table surface, to minimize the measurement uncertainty
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Table 11: The systematic errors are presented below:

Source of error:	Impact of result:	Improvement:
Attachment Point of the Mass	Slight changes in the placement of the mass throughout the experiment could affect the effective length of the cantilever and hence the oscillation period.	The current use of duct tape to help hold the mass in place can be improved by using adhesive putty and a specialised clamp to ensure the attachment point will stay constant.
Clamping Position of the Cantilever	Small variations in how the wooden ruler cantilever is secured to the table surface with the G-clamp change the effective length.	An improvement would be to ensure that the cantilever is clamped to the same position on the table surface by utilising alignment markers.
Limited Sample size of lengths tested:	Throughout this experiment, I only tested specific length increments of 0.05m from (1.0m to 0.3m), thus it may not fully capture the relationship between length of the cantilever and period of oscillation.	Increase the number of length variations by possibly having increments of 0.025m thus increasing sample lengths from 15 to 30.
Young's modulus assumption of the wooden cantilever:	The young's modulus used for theoretical calculations, was based on reference values that were collected from (Amesweb), which may differ from the actual properties of the material in the wooden ruler used.	Measure the young's modulus of the wooden ruler utilising other methods e.g stress and strain testing - $E = \frac{\text{stress}}{\text{strain}}$

Table 12: The strengths of the procedure are presented in the [Appendices](#):

4.4 Extension:

An extension to this experiment: would be to investigate the effect of different materials of the cantilever on the period of oscillation. The experiment conducted was utilising a wooden ruler as the cantilever, but cantilevers made of different materials (aluminum, steel and plastic) would allow for a comparison of the different young's modulus values effects on period of oscillation.

Word Count: 2998

Bibliography:

References

Department of Physics & Astronomy Lab Manual Undergraduate Labs Managing Errors and Uncertainty. (n.d.). Managing Errors and Uncertainty.

<https://www.physics.upenn.edu/sites/default/files/Managing%20Errors%20and%20Uncertainty.pdf>

geol.lsu.edu/jlorenzo/geophysics/uncertainties. (2000, July 1st).

geol.lsu.edu/jlorenzo/geophysics/uncertainties.

<http://www.geol.lsu.edu/jlorenzo/geophysics/uncertainties/Uncertaintiespart2.html#powers>

Johnson, R., & Alsud, H. (n.d.). *Cantilever | Definition, Structure & Examples - Lesson.* Study.com.

Retrieved February 16, 2025, from

<https://study.com/learn/lesson/cantilever-structure-design-examples.html>

M, K. (2024, December 17). *Calculating Uncertainties* | *DP IB Physics Revision Notes 2023*. Save

My Exams. Retrieved February 16, 2025, from

<https://www.savemyexams.com/dp/physics/ib/23/hl/revision-notes/tools/processing-uncertainties/calculating-uncertainties/>

Vibrations of Cantilever Beams: Deflection, Frequency, and Research Uses. (1999, April 23rd).

mechanics-pages -scott-whitney.

<https://emweb.unl.edu/mechanics-pages/scott-whitney/325hweb/beams.htm>

YOUNG'S MODULUS (MODULUS OF ELASTICITY) OF WOOD. (2016).

Materials/Youngs-Modulus-of-Wood.

https://amesweb.info/Materials/Youngs-Modulus-of-Wood.aspx#google_vignette

<https://www.deanza.edu/faculty/lunaeduardo/documents/MeasurementsUncertainties2A.pdf>

https://www.mso.anu.edu.au/pfrancis/labskills/uncertainties/Propagation_Equations.pdf

<https://www.toppr.com/guides/physics/electronics/oscillations-definition-and-factors-that-affect-periods-of-oscillation/>