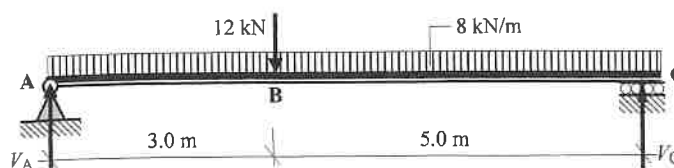


4.1.7 Solutions: Statically Determinate Beams – Shear Force and Bending Moment

Solution

Topic: Statically Determinate Beams – Shear Force and Bending Moment
 Problem Number: 4.1
 Page No. 1

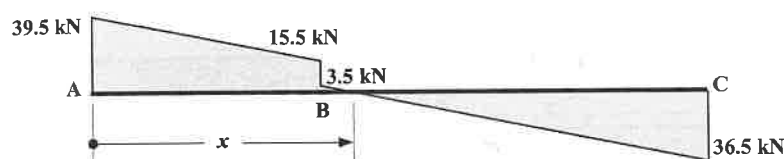
**Support Reactions**

Consider the rotational equilibrium of the beam: $+ve \curvearrowright \Sigma M_A = 0$

$$+ (12.0 \times 3.0) + (8.0 \times 8.0)(4.0) - (V_C \times 8.0) = 0 \quad \therefore V_C = +36.5 \text{ kN} \uparrow$$

Consider the vertical equilibrium of the beam: $+ve \uparrow \Sigma F_y = 0$

$$+ V_A - 12.0 - (8.0 \times 8.0) + V_C = 0 \quad \therefore V_A = +39.5 \text{ kN} \uparrow$$

Shear Force Diagram

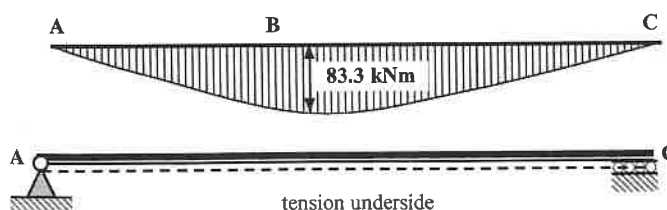
Position of zero shear force $x = [3.0 + (3.5/8.0)] = 3.438 \text{ m}$
 (This corresponds with the position of the maximum bending moment in the beam.)

Bending Moment Diagram

$$M_x = + (39.5 \times 3.438) - (8.0 \times 3.438^2/2.0) - (12.0 \times 0.438) = +83.3 \text{ kNm}$$

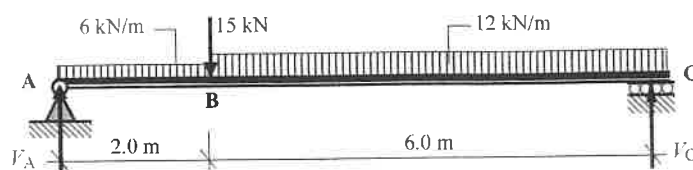
Alternatively, calculating the area under the shear force diagram:

$$M_x = + [0.5(39.5 + 15.5)(3.0)] + (0.5 \times 0.438 \times 3.5) = +83.3 \text{ kNm}$$



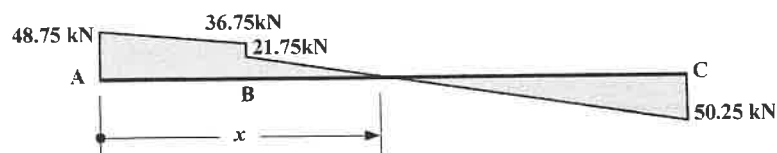
Solution

Topic: Statically Determinate Beams – Shear Force and Bending Moment
Problem Number: 4.2 **Page No. 1**

**Support Reactions**

Consider the rotational equilibrium of the beam: $+ve \curvearrowright \Sigma M_A = 0$
 $+ (6.0 \times 2.0)(1.0) + (15.0 \times 2.0) + (12.0 \times 6.0)(2.0 + 3.0) - (V_C \times 8.0) = 0$
 $\therefore V_C = + 50.25 \text{ kN} \uparrow$

Consider the vertical equilibrium of the beam: $+ve \uparrow \Sigma F_y = 0$
 $+ V_A - (6.0 \times 2.0) - 15.0 - (12.0 \times 6.0) + V_C = 0$
 $\therefore V_A = + 48.75 \text{ kN} \uparrow$

Shear Force Diagram

Position of zero shear force $x = [2.0 + (21.75/12.0)] = 3.813 \text{ m}$
 (This corresponds with the position of the maximum bending moment in the beam.)

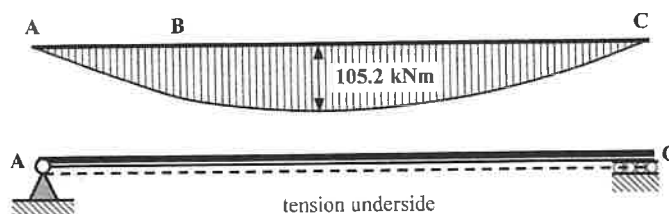
Bending Moment Diagram

$$M_x = + (48.75 \times 3.813) - (6.0 \times 2.0)(3.813 - 1.0) - (15.0 \times 1.813) - (12.0 \times 1.813^2/2)$$

$$= + 105.2 \text{ kNm}$$

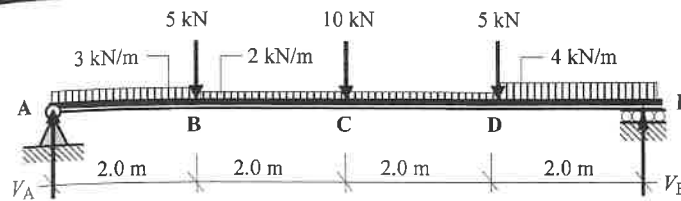
Alternatively, calculating the area under the shear force diagram:

$$M_x = + [0.5(48.75 + 36.75)(2.0)] + (0.5 \times 1.813 \times 21.75) = + 105.2 \text{ kNm}$$



Solution

Topic: Statically Determinate Beams – Shear Force and Bending Moment
Problem Number: 4.3 **Page No.** 1

**Support Reactions**

Consider the rotational equilibrium of the beam: $+ve \curvearrowright \Sigma M_A = 0$

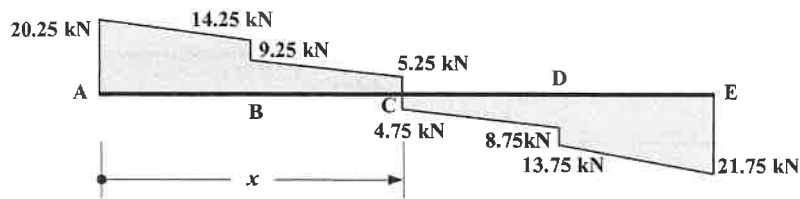
$$+ (3.0 \times 2.0)(1.0) + (5.0 \times 2.0) + (2.0 \times 4.0)(4.0) + (10.0 \times 4.0) + (5.0 \times 6.0) + (4.0 \times 2.0)(7.0) - (V_E \times 8.0) = 0$$

$$\therefore V_E = +21.75 \text{ kN} \uparrow$$

Consider the vertical equilibrium of the beam: $+ve \uparrow \Sigma F_y = 0$

$$+ V_A - (3.0 \times 2.0) - 5.0 - (2.0 \times 4.0) - 10.0 - 5.0 - (4.0 \times 2.0) + V_E = 0$$

$$\therefore V_A = +20.25 \text{ kN} \uparrow$$

Shear Force Diagram

Position of zero shear force $x = 4.0 \text{ m}$

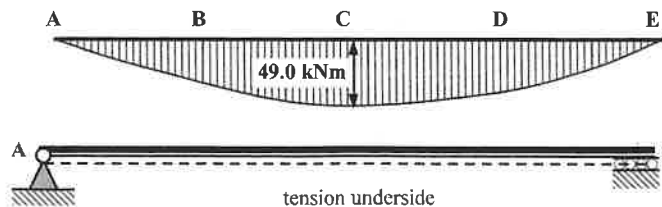
(This corresponds with the position of the maximum bending moment in the beam.)

Bending Moment Diagram

$$M_x = + (20.25 \times 4.0) - (3.0 \times 2.0)(3.0) - (5.0 \times 2.0) - (2.0 \times 2.0)(1.0) = +49.0 \text{ kNm}$$

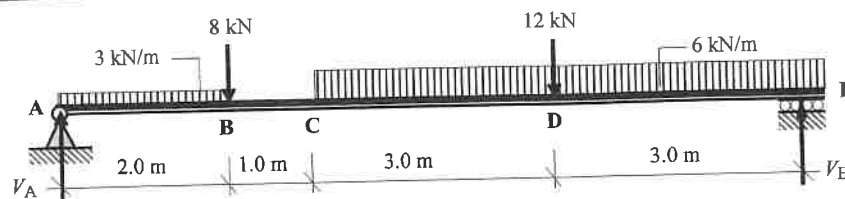
Alternatively, calculating the area under the shear force diagram:

$$M_x = + [0.5(20.25 + 14.25)(2.0)] + [0.5(9.25 + 5.25)(2.0)] = +49.0 \text{ kNm}$$



Solution

Topic: Statically Determinate Beams – Shear Force and Bending Moment
Problem Number: 4.4 **Page No. 1**

**Support Reactions**

Consider the rotational equilibrium of the beam: $+ve \curvearrowright \Sigma M_A = 0$

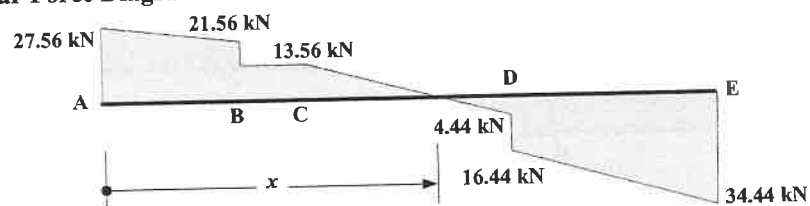
$$+ (3.0 \times 2.0)(1.0) + (8.0 \times 2.0) + (6.0 \times 6.0)(6.0) + (12.0 \times 6.0) - (V_E \times 9.0) = 0$$

$$\therefore V_E = +34.44 \text{ kN} \uparrow$$

Consider the vertical equilibrium of the beam: $+ve \uparrow \Sigma F_y = 0$

$$+ V_A - (3.0 \times 2.0) - 8.0 - (6.0 \times 6.0) - 12.0 + V_E = 0$$

$$\therefore V_A = +27.56 \text{ kN} \uparrow$$

Shear Force Diagram

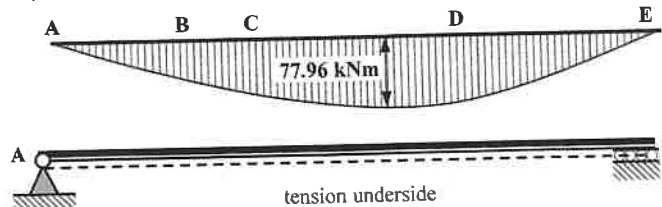
Position of zero shear force $x = [3.0 + (13.56/6.0)] = 5.26 \text{ m}$ (3.74 m from E)
 (This corresponds with the position of the maximum bending moment in the beam.)

Bending Moment Diagram

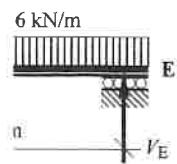
$$M_x = + (34.44 \times 3.74) - (6.0 \times 3.74^2/2) - (12.0 \times 0.74) = +77.96 \text{ kNm}$$

Alternatively, calculating the area under the shear force diagram:

$$M_x = + [0.5(34.44 + 16.44)(3.0)] + (0.5 \times 0.74 \times 4.44) = +77.96 \text{ kNm}$$



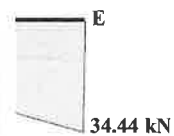
ending Moment age No. 1



$$-(V_E \times 9.0) = 0$$

$$\therefore V_E = +34.44 \text{ kN} \uparrow$$

$$\therefore V_A = +27.56 \text{ kN} \uparrow$$



74 m from E)
ment in the beam.)

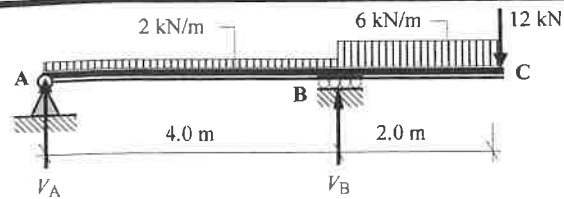
kNm

kNm



Solution

Topic: Statically Determinate Beams – Shear Force and Bending Moment
Problem Number: 4.5
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Support Reactions

Consider the rotational equilibrium of the beam: $+ve \curvearrowright \Sigma M_A = 0$

$$+(2.0 \times 4.0)(2.0) + (6.0 \times 2.0)(5.0) + (12.0 \times 6.0) - (V_B \times 4.0) = 0$$

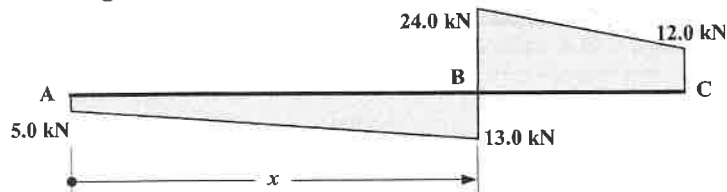
$$\therefore V_B = +37.0 \text{ kN} \uparrow$$

Consider the vertical equilibrium of the beam: $+ve \uparrow \Sigma F_y = 0$

$$+V_A - (2.0 \times 4.0) - (6.0 \times 2.0) - 12.0 + V_B = 0$$

$$\therefore V_A = -5.0 \text{ kN} \downarrow$$

Shear Force Diagram



Position of zero shear force $x = 4.0 \text{ m}$

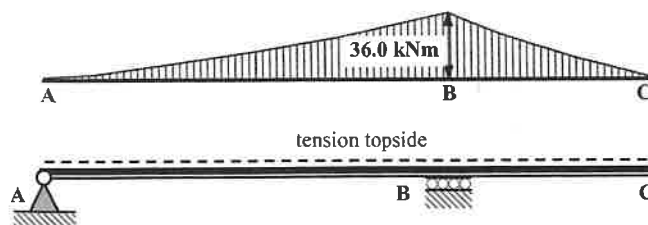
(This corresponds with the position of the maximum bending moment in the beam.)

Bending Moment Diagram

$$M_x = -(5.0 \times 4.0) - (2.0 \times 4.0^2/2) = -36.0 \text{ kNm}$$

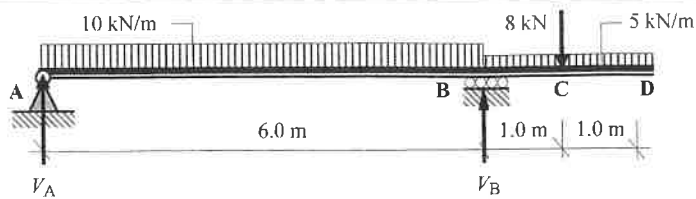
Alternatively, calculating the area under the shear force diagram:

$$M_x = -[0.5(5.0 + 13.0)(4.0)] = -36.0 \text{ kNm}$$



Solution

Topic: Statically Determinate Beams – Shear Force and Bending Moment
Problem Number: 4.6 **Page No. 1**

**Support Reactions**

Consider the rotational equilibrium of the beam: $+ve \curvearrowright \Sigma M_A = 0$

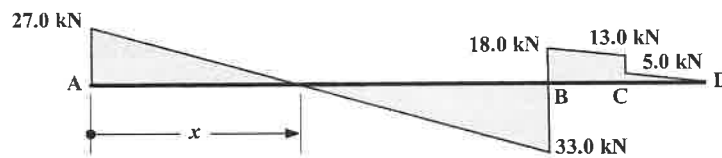
$$+ (10.0 \times 6.0)(3.0) + (5.0 \times 2.0)(7.0) + (8.0 \times 7.0) - (V_B \times 6.0) = 0$$

$$\therefore V_B = +51.0 \text{ kN} \uparrow$$

Consider the vertical equilibrium of the beam: $+ve \uparrow \Sigma F_y = 0$

$$+ V_A - (10.0 \times 6.0) - (5.0 \times 2.0) - 8.0 + V_B = 0$$

$$\therefore V_A = +27.0 \text{ kN} \uparrow$$

Shear Force Diagram

Positions of zero shear force: $x = (27.0 / 10.0) = 2.7 \text{ m}$ and $x = 6.0 \text{ m}$
 (These correspond with the positions of the maximum bending moments in the beam.)

Bending Moment Diagram

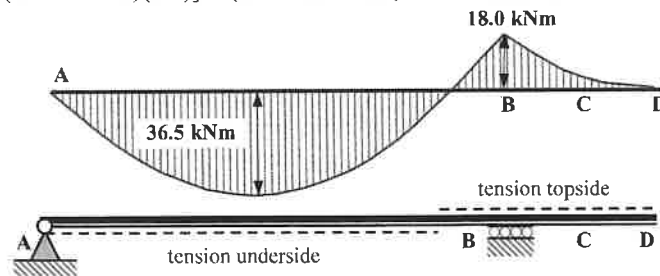
$$M_x = + (27.0 \times 2.7) - (10.0 \times 2.7^2 / 2) = +36.5 \text{ kNm}$$

$$M_B = - (5.0 \times 2.0)(1.0) - (8.0 \times 1.0) = -18.0 \text{ kNm}$$

Alternatively, calculating the area under the shear force diagram:

$$M_x = - (0.5 \times 2.7 \times 27.0) = +36.5 \text{ kNm}$$

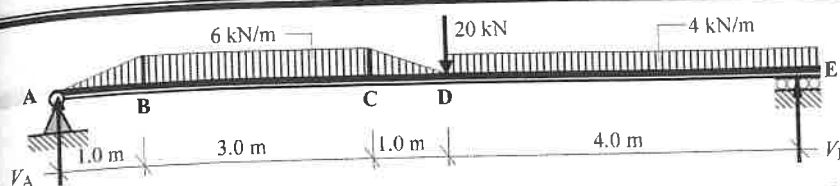
$$M_B = - [0.5(18.0 + 13.0)(1.0)] + (0.5 \times 1.0 \times 5.0) = +18.0 \text{ kNm}$$



Solution

Topic: Statically Determinate Beams – Shear Force and Bending Moment
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Load between A and B = $(0.5 \times 1.0 \times 6.0) = 3.0$ kN: centre of gravity is 0.67 m from A
Load between B and C = $(6.0 \times 3.0) = 18.0$ kN: centre of gravity is 2.50 m from A
Load between C and D = $(0.5 \times 1.0 \times 6.0) = 3.0$ kN: centre of gravity is 4.33 m from A

Support Reactions

Consider the rotational equilibrium of the beam: $+ve \curvearrowright \Sigma M_A = 0$

$$+ (3.0 \times 0.67) + (18.0 \times 2.5) + (3.0 \times 4.33) + (20.0 \times 5.0) + (4.0 \times 4.0)(7.0) - (V_E \times 9.0) = 0$$

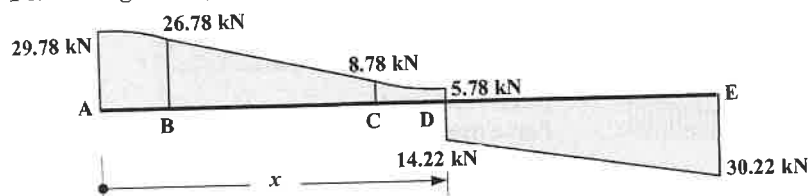
$$\therefore V_E = +30.22 \text{ kN} \uparrow$$

Consider the vertical equilibrium of the beam: $+ve \uparrow \Sigma F_y = 0$

$$+ V_A - 3.0 - 18.0 - 3.0 - 20.0 - (4.0 \times 4.0) + V_E = 0$$

$$\therefore V_A = +29.78 \text{ kN} \uparrow$$

Shear Force Diagram (Note: the diagram is curved from A to B and from C to D)



Position of zero shear force $x = 5.0$ m

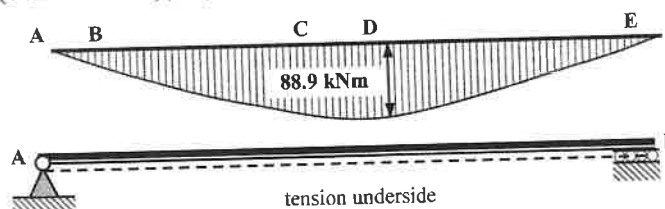
(This corresponds with the position of the maximum bending moment in the beam.)

Bending Moment Diagram: (consider the right-hand side)

$$M_x = + (30.22 \times 4.0) - (4.0 \times 4.0^2/2) = +88.9 \text{ kNm}$$

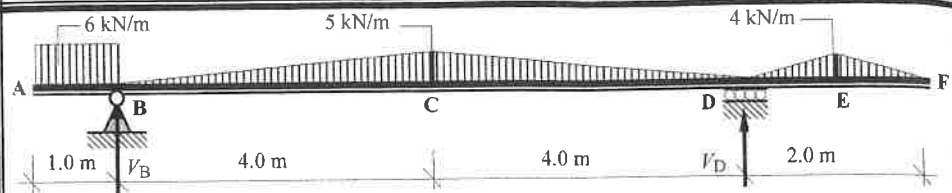
Alternatively, calculating the area under the shear force diagram:

$$M_x = +0.5(14.22 + 30.22)(4.0) = +88.9 \text{ kNm}$$



Solution

Topic: Statically Determinate Beams – Shear Force and Bending Moment
Problem Number: 4.8 **Page No. 1**

**Support Reactions**

Consider the rotational equilibrium of the beam: $+ve \curvearrowright \Sigma M_B = 0$

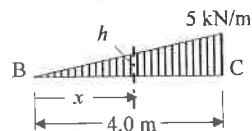
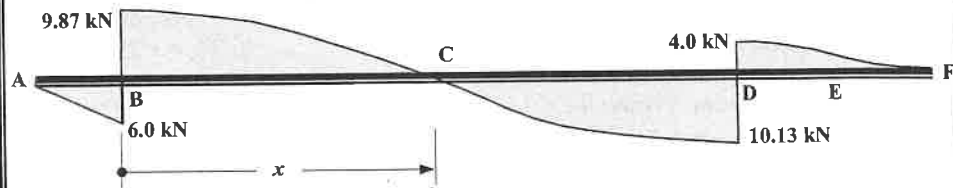
$$-(6.0 \times 1.0)(0.5) + (0.5 \times 8.0 \times 5.0)(4.0) + (0.5 \times 2.0 \times 4.0)(9.0) - (V_D \times 8.0) = 0$$

$$\therefore V_D = +14.13 \text{ kN} \uparrow$$

Consider the vertical equilibrium of the beam: $+ve \uparrow \Sigma F_y = 0$

$$-(6.0 \times 1.0) + V_B - (0.5 \times 8.0 \times 5.0) - (0.5 \times 2.0 \times 4.0) + V_D = 0$$

$$\therefore V_B = +15.87 \text{ kN} \uparrow$$

Shear Force Diagram

$$(h/x) = (5.0/4.0) \therefore h = 1.25x$$

$$\text{Force over length } x = (0.5 \times x \times 1.25x) = 0.625x^2$$

This force must equal 9.78 for zero shear at x

$$\text{Position of zero shear force } x: 9.78 = 0.625x^2 \therefore x = 3.956 \text{ m from B}$$

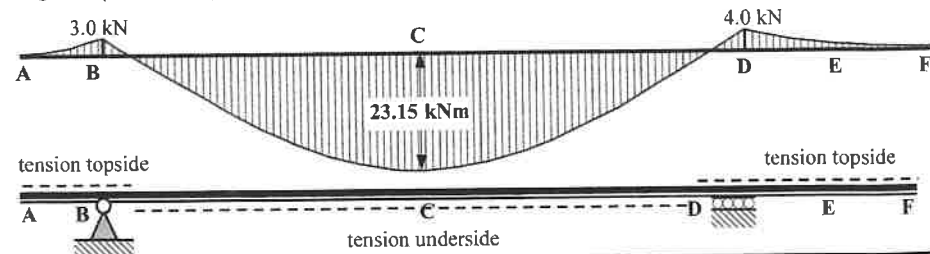
$$\therefore h = (1.25 \times 3.956) = 4.945$$

Bending Moment Diagram

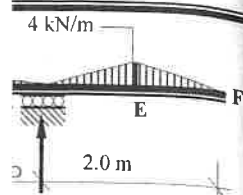
$$M_x = -(6.0 \times 1.0)(4.456) + (15.87 \times 3.956) - [(0.625 \times 3.956^2)(3.956/3.0)]$$

$$= +23.15 \text{ kNm}$$

$$M_B = -(6.0 \times 1.0^2)/2 = -3.0 \text{ kNm}; \quad M_D = -(0.5 \times 2.0 \times 4.0)(1.0) = +4.0 \text{ kNm}$$



ending Moment age No. 1

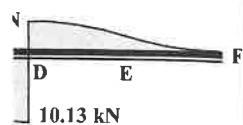


0

$$-(V_D \times 8.0) = 0$$

$$\therefore V_D = +14.13 \text{ kN} \uparrow$$

$$\therefore V_B = +15.87 \text{ kN} \uparrow$$



$$) = 0.625x^2$$

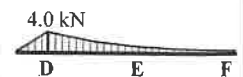
near at x

om B

$$956) = 4.945$$

$$3.956/3.0)]$$

$$0) = +4.0 \text{ kNm}$$



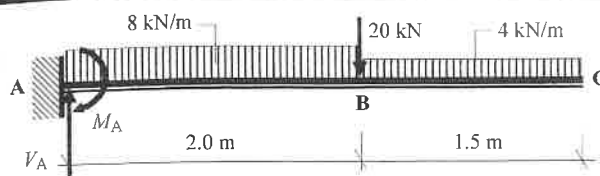
tension topside



Solution

Topic: Statically Determinate Beams – Shear Force and Bending Moment
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Support Reactions

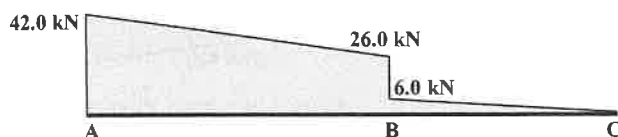
Consider the rotational equilibrium of the beam: $+ve \curvearrowright \Sigma M_A = 0$

$$M_A + (8.0 \times 2.0)(1.0) + (20.0 \times 2.0) + (4.0 \times 1.5)(2.75) = 0 \quad \therefore M_A = -72.5 \text{ kNm}$$

Consider the vertical equilibrium of the beam: $+ve \uparrow \Sigma F_y = 0$

$$+V_A - (8.0 \times 2.0) - 20.0 - (4.0 \times 1.5) = 0 \quad \therefore V_A = +42.0 \text{ kN} \uparrow$$

Shear Force Diagram



Bending Moment Diagram

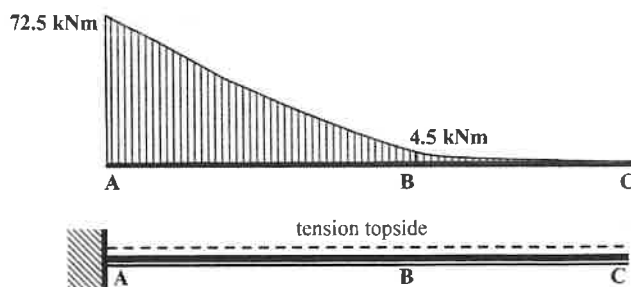
$$M_A = -72.5 \text{ kNm}$$

$$M_B = -(4.0 \times 1.5)(0.75) = -4.5 \text{ kNm}$$

Alternatively, calculating the area under the shear force diagram:

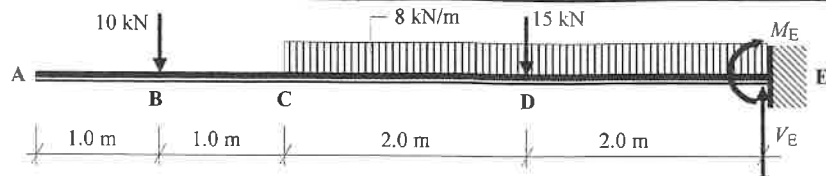
$$M_A = -[0.5(42.0 + 26.0)(2.0)] - (0.5 \times 1.5 \times 6.0) = -72.5 \text{ kNm}$$

$$M_B = -(0.5 \times 1.5 \times 6.0) = -4.5 \text{ kNm}$$



Solution

Topic: Statically Determinate Beams – Shear Force and Bending Moment
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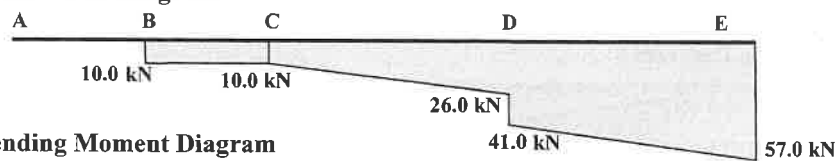
**Support Reactions**

Consider the rotational equilibrium of the beam: $+ve \curvearrowright \Sigma M_E = 0$

$$-(10.0 \times 5.0) - (8.0 \times 4.0)(2.0) - (15.0 \times 2.0) + M_E = 0 \quad \therefore M_E = +144.0 \text{ kN} \curvearrowright$$

Consider the vertical equilibrium of the beam: $+ve \uparrow \Sigma F_v = 0$

$$-10.0 - (8.0 \times 4.0) - 15.0 + V_E = 0 \quad \therefore V_E = +57.0 \text{ kN} \uparrow$$

Shear Force Diagram**Bending Moment Diagram**

$M_A = M_B = \text{zero}$

$$M_C = -(10.0 \times 1.0) = -10.0 \text{ kNm}$$

$$M_D = -(10.0 \times 3.0) - (8.0 \times 2.0^2/2) = -46.0 \text{ kNm}$$

$$M_E = -144.0 \text{ kNm}$$

Alternatively, calculating the area under the shear force diagram:

$$M_C = -(10.0 \times 1.0) = -10.0 \text{ kNm}$$

$$M_D = -(10.0 \times 1.0) - [0.5(10.0 + 26.0)(2.0)] = -46.0 \text{ kNm}$$

$$M_E = -(10.0 \times 1.0) - [0.5(10.0 + 26.0)(2.0)] - [0.5(41.0 + 57.0)(2.0)] = -144.0 \text{ kNm}$$

