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Lesson study in prospective mathematics teacher education: didactic and paradidactic technology in the post-lesson reflection

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Abstract This paper presents a detailed analysis of the post-lesson reflection, carried out in the context of eight cases of lesson study conducted by teams of Danish, lower secondary prospective teachers and their supervisors. The participants, representing different institutions, were all new to the lesson study format. Nevertheless, it is demonstrated how their interaction shape the development of discourse about mathematical learning. The anthropological theory of the didactic is employed as the theoretical approach to analyse the mathematical and primarily didactical praxeologies developed and discussed during the meetings. The study investigates what happens when lesson study, a well-established Japanese "system" for professional teacher development, is transposed to another educational and cultural context, with the aim of enhancing prospective teacher learning during the practicum of a teacher education programme. The findings highlight significant different positions in the discourse during the post-lesson reflection. Specific practice-related knowledge is developed, to the benefit of prospective teachers, educators and researchers alike. This kind of knowledge is of interest and concern to the whole profession of mathematics teachers and the analysis adds to our insight into the potential of lesson study in prospective education as a meeting place where pertinent actors contribute to the expansion and dissemination of shared professional knowledge.

Keywords Lesson study · Reflective practice · Mathematical praxeology · Anthropological theory of the didactic · Paradidactic infrastructure · Didactic praxeology

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Introduction and research question

The study of how teachers learn from practice, and subsequently develop their practice, is an ongoing interest of educational research. Many prospective teacher education programmes seek to build foundations for lifelong development of the teachers' own practice, and in some institutions, this is followed up by induction programmes in the first years of the teachers' career (Howe 2006). Indeed, it is an important challenge of all teacher education programmes, to facilitate prospective teachers' first meetings with teaching practice. This paper deals with issues related to the first formalized encounter with the field of live teaching, the so-called practicum. I investigate an experimental setting for the first practicum period in Danish mathematics teacher education, as it was reorganized based on elements from the Japanese format of lesson study. In order to gain insight into the generation of shared teacher knowledge, I focus on the post-lesson reflection which is part of the lesson study format. In Japan, lesson study is a common format both in prospective education (e.g. Elipane 2012) and in regular teachers' professional development (Fernandez and Yoshida 2004). Many countries support initiatives which try to import this method or "system" for enhancing the professional education and development of teachers, and the use of lesson study in prospective education poses some unique challenges. Murata and Pothen (2011) describe how they have implemented lesson study in an American context to make prospective teachers see the connections between the teaching and the learning of pupils. McMahon and Hines (2008) report how these connections are easily overlooked as prospective teachers often focus on the technical aspects of implementing their lessons. Moreover, meaningful discussions and support of knowledgeable advisors have been identified by Fernández (2010) as crucial elements of lesson study in prospective education. In the Danish experiment studied here, none of the cooperating teachers or prospective teachers involved had any prior experience with lesson study, which makes the reported findings of interest to all who seek to incorporate lesson study in educational and cultural settings where it is not yet an established practice. Lesson study is, very roughly put, a process in which a lesson is planned by a team of teachers or prospective teachers and then taught by a member of the team while observed by a number of other people and the rest of the planning team. This larger group then reflects on what happened, based on specific questions and hypotheses developed in the planning phase. This leads to the overall research question: When the lesson study format is unfamiliar to the participants, what characterize the post-lesson discussion? This research question will be sharpened in the subsequent sections, but first I need to present the background for this study, relating it to the research literature on teacher education practicum and lesson study. Then I give a brief account of one of the eight cases, which I shall use as a paradigmatic case throughout the article to exemplify both the presented theoretical model, and then in the findings section, to compare and contrast it with the other seven cases observed. The anthropological theory of the didactic is introduced to model and analyse the research question, before I finally present the methods and findings of the study.

Background

Prospective teacher practicum is generally portrayed as having an enormous influence on teacher education as a whole (Bergsten et al. 2009) and on prospective teachers' opinion of mathematics in itself, and as a teaching discipline (Grootenboer 2006). Much of the research on practicum experience focuses on the notion of reflectivity; that is, in order to

learn from things done, the learner has to reflect on what has been experienced in practice (see, for instance, Ghaye 2010; York-Barr et al. 2005 for numerous references). The settings for doing, facilitating and promoting these reflections are many: keeping individual diaries, video recording in classrooms, collective supervision, interviews and lesson study. A main point for prospective teachers and educators is to set aside time for individual and collective reflection on the practical experience. To develop shared knowledge of general interest to the mathematics teacher profession, special attention is naturally given to settings where the parties can meet and reflect collectively on the practices of teaching they seek to improve. In prospective teacher education, a number of formats exist where the prospective teacher meets other persons who are knowledgeable of the profession. The post-lesson interview, where the prospective teacher and a cooperating teacher meet to discuss issues arising from recently carried out lessons, is a common format that can be both directed and prescriptive, but also reflective and theoretical (Chaliès et al. 2004). These interviews may take place immediately after lessons taught by the prospective teacher or they can be postponed, e.g. with a recording of the lesson available for later review. Delayed interview is sometimes reported to improve the quality of prospective teacher reflection (Watson and Williams 2004). Another common tool to improve the reflection, and thus the learning and development of professional knowledge, is to include other "experts" in the group of people who join in reflecting on a particular lesson. A Scandinavian example of this is the so-called tripartite conversation, where the prospective teacher and cooperating teacher is joined by the prospective teacher's professor from the University College (hereafter abbreviated UC)[1] (Gloppen 2013; Rasmussen 2007).

An even wider group of people is involved in the joint reflecting on a lesson just observed within the Japanese format of lesson study and the setting I focus on in this paper. Lesson study is a diverse phenomenon and can be seen to involve many distinct aspects (Fernandez and Yoshida 2004). More specifically in this article, I focus on one important element of lesson study: the post-lesson reflection. This meeting, focused on collective reflection, is termed "hanseikai" in Japanese (see e.g. Peterson 2005 for a short description).

Lewis et al. (2006) identified three critical needs for research on the functions of lesson study: a need to expand the descriptive knowledge base, a need to explicate the innovation mechanism and a need to do design-based research cycles. This article aims to contribute to the first two needs. Regarding the first, I describe a lesson study experiment situated in special circumstances and I focus on the discourse observed during hanseikai. With regard to the second need, Lewis et al. (2006) conjectured that: lesson study strengthens three pathways to instructional improvement: teachers' knowledge, teachers' commitment and community, and learning resources. I will analyse eight instances of hanseikai for evidence of these pathways, as they are expressed through shared discourse about a commonly observed lesson. This shared discourse of hanseikai has been described by Miyakawa and Winsløw (2013), as "expressed knowledge and logic behind didactic techniques" (i.e. didactic technology, cf. theoretical section below).

[1] University Colleges are the institutions in charge of pre-service teacher education programmes in Denmark (for primary and lower secondary school).

The context and description of one example case

The eight cases of lesson study took place in 2012 and 2013. I will refer to them as LS1 to LS8, and the research lessons were held in grades 8,6,7,8,3,6,5 and 5, respectively. Each instance was centred on one 45-min lesson, planed collectively by groups of 2–4 lower secondary prospective teachers, who attended regular teacher education at one of two different UCs. The teacher education programme consists mostly of coursework, but with 24 weeks of practicum spread over the programmes total duration of 4 years. The practicum took place at schools affiliated with the UC, and the school follows certain guidelines set together with the UC, to regulate the prospective teachers' rights and obligations when in practicum. The depth of the cooperation (formal or otherwise) between UCs and schools varies and depends much on the individual choices of the UC professor and cooperating school teacher. The cooperating school teacher is usually the main interlocutor of the prospective teachers during practicum. The eight cases took place during the prospective teachers' practicum period, in their second year, on the initiative of mainly the UC professors. The prospective teachers were obliged to participate as part of their training, whereas the cooperating school teachers, UC professors and "knowledgeable others", i.e. resource persons from an associated university participated on a semi-voluntary basis. Prospective teachers in their second year are still somewhat inexperienced and have two more years to familiarize themselves with two other disciplines besides mathematics, which is the only teaching discipline they study during the first 2 years.

The plan for each research lesson was generated by the groups themselves with guidance from the UC professor and the cooperating school teacher. The lesson plans were to be quite elaborate in order to make them helpful to all participants, including those who would only participate in observation and reflection on the research lesson. While it is common practice in Japan to prepare lesson plans which are readily understandable to persons not involved in the planning itself, this is generally not the case in the practice of Danish teachers or prospective teachers. To compensate for this fact and for the unfamiliarity with lesson study in general, a brief guide and lesson plan template (see appendix, Tables 1 and 2) were developed and made available to those taking part in the lesson study. The prospective teachers also attended a 2-h lecture on the nature of lesson study, given by an outside expert, but otherwise no common preparation was given. Therefore, the eight cases can be regarded as a pilot study to shed light on what can be accomplished with a minimal implementation of preparation.

In order to broaden the context and introduce the theoretical model, I now describe the plan and content of LS8, which will be used as a recurring example. LS8 was chosen because it clearly exemplified issues common to all cases, but other cases could also have been used. The goal for the pupils in the fifth-grade class was to develop a formula for the area of right-angled triangles. They were to do that by experimenting with the geoboard.[2] First, the prospective teacher presented the agenda for the lesson and then recalled earlier work the class had done on the area of rectangles. This was done as a "teacher led" dialogue. Then the prospective teacher led a whole class discussion about what right-angled triangles are; the discussion was supported by a PowerPoint, displaying right-angled triangles in different shapes, sizes, colour and orientation (see also Fig. 2 below). Next a "problem" was introduced where the pupils were to make a number of right-angled triangles on the geoboard. The produced triangles were "recorded" by the pupils by

[2] The geoboard is a physical board with a certain number of pegs placed in a grid, around which rubber bands can be wrapped to form plane geometrical shapes.

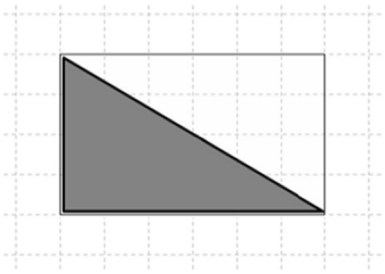


Fig. 1 Picture shown to pupils, indicating a way to find the area of a rectangle by superimposing it on a net of unity squares. A triangle is placed suggestively inside the rectangle, alluding a way to derive the triangle area formula from the rectangle area formula

copying them onto paper prepared with unity squares. The overall task was to find a "rule" that expressed how to calculate the area. A way to find the area of rectangles by counting the squares inside, and also a relation to triangles were subtly indicated in a figure from the problem presentation (Fig. 1). The pupils worked on the problem in pairs, and finally, their different solutions were discussed and contrasted to each other in yet another whole class discussion. (Details of this final discussion are described in the beginning of "Discourse in the hanseikai and the sharpened research question" section and further elaborated in "Didactic praxeologies of whole class discussions" section).

Theoretical model and analytical strategy

The anthropological theory of the didactic (hereafter ATD) employs an epistemological and institutional approach to analyse "didactic phenomena", i.e. instances of teaching and learning (Bosch et al. 2006). Didactic phenomena are conceived and given meaning by the institutional context in which they appear. These phenomena are considered as more or less contingent; no matter how "natural" or "eternal" they may seem. Indeed, it is the obligation of the epistemologist to question the perceived obviousness of the world as it is presented by individuals or organizations. Below, I explain how ATD models all human activity, in particular those of a didactical and mathematical nature, as praxeologies. Praxeologies consist of tasks, techniques, technologies and theories, all related to another.

Tasks in ATD are understood very broadly, but may be explicitly stated by someone, e.g. when the prospective teacher in LS8 asks the pupils: "Which of these shapes are right triangles?" The pupils are thus given a task (denoted by $\mathbf{t}_1$ in the caption of Fig. 2). Generally, tasks are something humans are faced with as part of life. They need not be stated by anyone nor consciously recognized as such, even though that is usually the case in school contexts. The response to a task is achieved through the use of techniques to cope with the task. In the example from LS8, a technique may be for the pupil to grab the

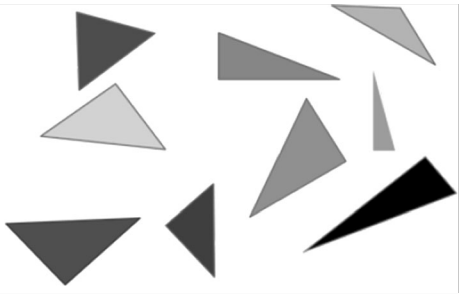


Fig. 2 Picture accompanying the specific task ($\mathbf{t}_1$) of finding the right-angled triangles among this collection of triangles

protractor, place it correctly on the largest angle, read of the scale and conclude whether the triangle has a right angle or not. This amount to a technique τ_1 for the type of task $\mathbf{T}_1$: determine whether a given triangle is right angled. The pair of type of task and technique ($\mathbf{T}_1$,τ_1) co-define each other and is referred to as praxis. It involves some "doing", physical or mental, which might be more or less mechanical, that is, without considering how the task is recognized, why the task is relevant in the situation, why the technique is appropriate, how or why the technique produces a valid answer. It is then obvious, that this pair of task type and technique can never truly exist alone, there has to be some discourse or reasoning regarding the technique, at least some consideration of how to recognize a task to which the technique applies. This explicit consideration of task and technique is called technology (often denoted by θ) meaning "techne-logos" or "the discourse on the technique".[3] Technology may, or may not, be deliberately produced, but it is always present along with human praxis. Proposing an example from LS8, a pupil may be able to tell her seatmate how to use the protractor and that colour is irrelevant. Her seatmate may confess that she is uncertain whether only triangles with a 90° vertex pointing to the right side may be called a right triangle. Such deliberations are observable technology. According to Durand-Guerrier et al. (2010),

> … it is an important characteristic of human activity to allow for coherent discourse about tasks and techniques (called technology), and in some cases to organise these discourses in theories that make explicit the understandings and justifications underlying technology and techniques. For instance, an instruction on how to perform a multiplication [of] integers belongs to a technology, while the systematic discussion of why the multiplication methods works is within the domain of theoretical discourse (Durand-Guerrier et al. 2010).

This quote also introduces the last component of a praxeology, namely the theory (often denoted by Θ), which elaborates the meaning of discourse to encompass the whole network of understandings and justifications which are used to account for technology itself and its relation to other technologies. It is a perhaps trivial consequence that accounting for the technology and theory of pupils in regard to a given task can be quite extensive. Nevertheless, short suggestive statements may be identified as crucial elements of a theory; for example in LS8, the theory of right-angled triangles may encompass a clear definition which helps to distinguish these shapes from other triangles.

Discourse in the hanseikai and the sharpened research question

What is discussed in the hanseikai is didactic praxeologies (Bosch and Gascón 2002). Didactic praxeologies can be related to the mathematical praxeologies as they begin with tasks for the mathematics teacher, or as in this paper, the prospective teacher. The generic didactic task is to support pupil's learning of a mathematical praxeology. For instance, in LS8 the prospective teacher notices pupils' diverse and imprecise language, which makes it difficult for the whole class to participate in a common discussion. Clarifying pupils' discourse becomes a task for the prospective teacher. She reformulates pupils' assertions and draws illustrations on the blackboard to clarify the meaning. These techniques can be used in relation to teaching different praxeologies, but its specific use depends on the mathematical praxeology to be learned as well as other aspects of the situation.

[3] It is crucial **not** to confuse the ATD meaning of technology with the more every day sense of the word.

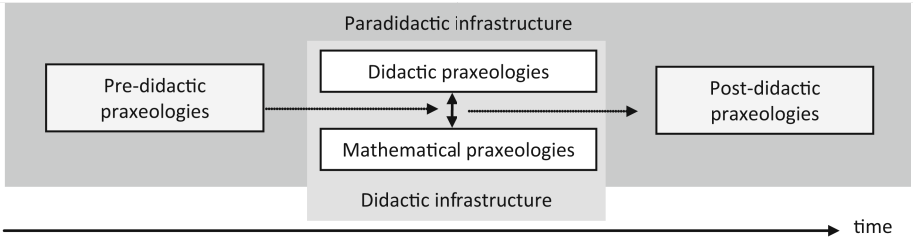


Fig. 3 Didactic and paradidactic infrastructure conditioning teachers development of practice and knowledge inside and outside teaching situations. The lesson study format is a form of paradidactic infrastructure, an in particular "hanseikai" is a collection of post-didactic praxeologies (Model adapted from Miyakawa and Winsløw 2013)

Teachers' didactic technology and theory is more difficult to extract from classroom observation. But my analytical strategy (Andersen 2003) is that this will be visible during the hanseikai phase of lesson study.

Besides the notion of praxeologies, I need two other theoretical constructs from ATD. The didactic infrastructure (Chevallard 2009) and the paradidactic infrastructure (Winsløw 2011; Miyakawa and Winsløw 2013), see Fig. 3.

The didactic infrastructure consists of all conditions for the teacher's work when teaching. The physical classroom, for example, is itself part of the didactic infrastructure, as is the duration of lessons, syllabi and overall pedagogical principles of the school. As mentioned above, lesson study holds the promise to enhance instruction by improving upon teaching resources. I here take teaching resources to be part of the didactic infrastructure which either limits or helps the teacher in teaching situations. The didactic infrastructure conditions the possible didactic praxeologies, which again condition the mathematical praxeologies actually realized in teaching situations. Didactic and mathematical praxeologies are co-determined, meaning they influence each other and they are influenced by institutional levels outside the school (for a more in depth explanation of co-determination, see Chevallard 2004).

Not all teacher work takes place in teaching situations; preparation and professional development are good examples. Everything which conditions teacher work outside teaching situations is part of the paradidactic infrastructure. The lesson study process is a paradidactic infrastructure, as it provides a set of conditions under which teachers, or as in this paper, prospective teachers, has the opportunity to plan and reflect on a commonly observed lesson. The paradidactic infrastructure of lesson study can be further specified to condition pre-didactic praxeologies employed before the teaching activity and post-didactic praxeologies after (Miyakawa and Winsløw 2013).

I am now in position to reformulate the research question precisely using ATD: *Which post-didactic praxeologies arises in the unfamiliar paradidactic infrastructure of hanseikai and which didactic praxeologies are elaborated and developed during the hanseikai?*

Methodology and the collection of data

This paper focuses on the hanseikai, but as this is intimately connected to the planning and observation of the lesson itself, data were obtained from a variety of sources: field notes were taken during the research lessons and during the hanseikais. I attended these as a non-participating observer except in LS6 and LS8 where I substituted in the role of university

resource person (c.f. context section above). The eight hanseikais were audio recorded, and these recordings comprise the main data source for the analysis. In addition, the following background material was collected: materials created by the prospective teachers for use in the lessons, e.g. presentations, worksheets and of course the lesson plans.

The audio recordings were time indexed and inventoried minute by minute to produce an initial overview of issues debated. This resulted in a large number of coherent passages which were given a short description, or memo, to characterize the content. Then two types of data review were undertaken: one where the passages were grouped into those concerning the same content, e.g. didactic tasks, mathematical tasks, didactic infrastructures. And one where the passages were grouped according to the way, and tone, in which the utterances were made, e.g. as questions, appraisal, rebukes. The first shed light on the developed didactic praxeologies, whereas the latter illuminated the post-didactic praxeologies. This time-consuming, iterative process entailed numerous re-listening of the audio material, especially as the discourse tended to be fragmented, with one statement referring to another several minutes before, and also when a multitude of discourse trajectories was pursued at the same time. This methodology is inspired by grounded theory (see e.g. Glaser and Strauss 2009) in as much as no didactic or post-didactic praxeology were hypothesized beforehand, and the process involved successive rewriting of "memos" on what described the above-mentioned groupings of data. ATD does not describe or recommend any particular method be used in the gathering or structuring of data, but it provides the analytical unit (i.e. praxeologies), and as a consequence, it was inevitable that the described method would end in findings that could be described in these same terms. This is not the case for research based on pure grounded theory. ATD holds that praxeologies are institutionally situated; therefore, it is pertinent to look for differences and similarities in discourse according to institutional categories. In this way, ATD guides one in data analysis. In order to present this in a clear way, characteristic quotations from the groups of audio recordings has been selected for presentation in the findings ("Didactic praxeologies of whole class discussions" section). The quotations, marked with LS1 to LS8 from which they belong, and the pertinent time index, were transcribed and then translated from Danish by the author.

Findings

Following the sharpened research question, I first present the post-didactic praxeologies observed in the hanseikai phase ("Post-didactic technology in the paradidactic infrastructure of hanseikai" section). Then I take a closer look at the didactic praxeologies that turned out to be discussed in the hanseikai ("The technology of didactic praxeologies" section).

Post-didactic technology in the paradidactic infrastructure of hanseikai

Let me recapitulate the need to expand the descriptive knowledge base regarding lesson study. When you try to transpose an element of paradidactic infrastructure, a number of things happen that you are unable to anticipate based on praxeologies commonly found in the institutional setting which gave birth to this element in the first place. What is seen as a natural and therefore unquestioned convention might turn out to be a vital characteristic of the successful functioning of the element. Therefore by trying to adapt lesson study to a new institutional context, something can be learned which expands our knowledge of the "original" lesson study format. One first realization is that lesson study; as the "invading"

paradidactic infrastructure will be entwined with existing infrastructure, the consequences of which will be shown below. The overall paradidactic task type ($\mathbf{T}_1$) facing the participants of the hanseikai is to make comments regarding the lesson just seen. ($\mathbf{T}_1$: express useful observations related to the lesson plan, lesson observed and questions for discussion posed by the prospective teachers) Several techniques can be employed in this endeavour, which I will not elaborate on, but instead jump straight to looking at five post-didactic technologies related to $\mathbf{T}_1$, which featured prominently in the hanseikai. The first I could term "performance evaluation".

$\boldsymbol{\theta}_{1,1}$ Post-didactic technology that evaluates the personal performance of the executive prospective teacher:

As mentioned above, nearly the only formal introduction to the new paradidactic infrastructure was the brief guide on lesson study written by a university researcher participating in the experiment. This document now needs further commentary. It is made up of three pages of description (Table 1 in the appendix) in addition to a one-page expandable standard form in which to fill in the lesson plan. The standard form consists of two sections (See Table 2 in the appendix): one to give the gist of the lesson and one to specify the temporal progression of the lesson. Looking first at the description, I summarize the proposed characteristics: (1) lesson study is described as a repeatable process of planning, observing and reflecting, in which the participating prospective teachers and cooperating teachers refine a lesson plan. (2) One of these cycles has a special observation and reflection session to which a wider audience is invited, which gives this part of the lesson study process some likeliness to the "open lesson" format (Miyakawa and Winsløw 2013). (3) It is emphasized that in (2), the discussion must be centred on concrete observations of the lesson, primarily formulated as questions, never as critique of the executive prospective teacher. This last recommendation resonates well with the ubiquitous quality of all descriptions of lesson study in Japanese institutional settings: "a critical factor is that the *lesson* be criticized, not the teacher" (Padilla and Riley 2003, my italics) This point turned out to be difficult to achieve in more than half of the observed instances (LS2,4,5,6,7), especially LS4 exposition very clear examples of this tendency:

> …, but I thought that; couldn't' you just have told it to them, or set the time longer… (One prospective teacher to executive prospective teacher, LS4, Time index 11:30–11:37).

> You use a lot of time with certain groups… [others] have raised hands, it is too bad if they should loose momentum… but that said, I think you interact well with the pupils (One prospective teacher to executive prospective teacher, LS4, Time index 13:40–14:08).

> Could we try to direct our comments more towards pupil learning in what we observed instead of specific comments to each other? (Cooperating UC professor, who moderates the discussion, LS4, time index 14:41–14:51).

The two first quotations are just a few examples of discourse directed at evaluating the performance of the prospective teacher who did the actual teaching of the lesson. The all-pervasive use of "you" in second person singular form gives the impression of a non-collaborative effort behind the lesson. The flow of the lesson and the interaction with the pupils appears as the executive prospective teacher's personal problem. In the third quote, the moderator of the discussion tries to change track, but the "verbal assault" continues

and the executive prospective teacher is pressed to repeat the moderator's request to focus on pupil learning:

> But what about the purpose?… It's just with regard to what [the moderator] said; it [the comments] should be more in connection to that! (Executive prospective teacher, LS4, Time index 17:15–17:25).

It is not just the prospective teachers themselves who use discourse that seek to evaluate personal performance; it is also common among the cooperating school teachers:

> I'll begin by saying that you connect very well with the children. I can feel this, and they respect you, and they like to ask you about things. If you did not have this, and they did not believe in you, they would not ask. And I think that is very good! (Cooperating school teacher to executive prospective teacher, LS5, Time index 10:20–10:35).

Discourse about how the prospective teacher interacts on a personal level with the pupils (teacher–pupil relation, classroom management, seeing the whole class, giving clear directions) is not just a peculiarity of Danish teachers. Comparing with the work of the "guiding teacher", attached to teachers in their first year of induction in Japan, we see a similar emphasis on "understanding pupils". Japanese guiding teachers regard the communication and relationship between the teacher and the pupils as the most important factor, without which a new teacher will make no progress (Padilla and Riley 2003).

$\boldsymbol{\theta}_{1,2}$ Post-didactic technology that challenges and probes pre-didactic and didactic praxeologies of the prospective teacher team:

This technology is something akin to exam-like evaluation and is especially prevalent among the cooperating UC professors:

> Now I am going to say something which I did not plan on saying, but I'll say it anyway… when I read this lesson plan at home, I got no idea about what was going to take place, there were no details at all, so I thought: "They have completely misunderstood everything (Cooperating UC professor, LS1 36:08–36:27).

Here the UC teacher clearly perceives that this comment might not be appropriate for the post-lesson discussion, but it highlights a (perhaps natural) beginner's difficulties in writing a lesson plan suitable for a lesson study. As mentioned in "The context and description of one example case" section, it is generally not a common practice of Danish teachers or prospective teachers to share lesson plans with others outside the team. As a consequence, most lesson plans are written in an implicit manner which is not easily accessible to outsiders. Nevertheless, exam-like discourse is common throughout all eight cases, e.g. the following sentence which is nearly ubiquitous at the end of all traditional Danish oral examinations:

> Is there something you think you haven't had comments to, which you would like, or is there anything else you would like to add? [Long pause] (Cooperating UC professor to group of prospective teachers, LS3 51:00–51:09).

This prompting of prospective teachers to supply answers is a usual element of UC professor talk, putting focus on something the prospectives supposedly should know:

> But to adhere to what [another UC professor] said, we lack other ways to organise the end of class discussion. Do you have any good suggestions for that? (Cooperating UC professor to group of prospective teachers, LS6 33:36–33:44).

The exam-like discourse is characterized by appraisal and by directly addressing the prospective teachers in ways that will make them expose what they know, and perhaps especially reveal the limit of their knowledge.

$\boldsymbol{\theta}_{1,3}$ Post-didactic technology where desirable avenues of reflection are suggested408 to the prospective teacher team:

Exam-like discourse in the hanseikai was often combined with instances of what may be termed "guidance", in which a UC professor attempted to direct the prospective teachers' attention in certain directions. Something which they should consider in future work or something they should just "think about". Guidance discourse is often linked to specific suggestions about how to handle or change a situation seen in the lesson:

> …yes, but just try to understand what I am thinking. It is about this: You cannot speak with each single pupil here at the end [of the lesson]. And then it is I am thinking: How can you obtain knowledge about pupils thinking? This is what I want to challenge, and want you to think about. Can we produce some new ways in which to capstone the lesson, where we obtain the knowledge that you are interested in… There is no simple answer, that's not what I'm saying, but that's what I think we should talk about (Cooperating UC professor, LS6 28:48–29:17).

The guiding discourse is given by UC professors both when solicited by the prospective teachers and when not, whereas knowledgeable others, mostly from the universities, only did so on explicit request. The role of UC professors as guides for the prospective teachers are common to all ordinary prospective teacher education in Denmark; indeed, the time allocated to UC professors' direct interaction with prospective teachers is divided in two categories which are termed in the following way: teaching time with whole classes and guidance time with individual or smaller groups of prospective teachers. It is understandable that such well-established paradidactic infrastructure will significantly influence the discursive actions taken. Practicum is in itself a paradidactic infrastructure in Denmark, and the demands on what it should support have changed over the years. One current requirement is to provide a "space" for investigations into one's own and others' practice (Undervisningsministeriet 2011). Practicum is furthermore a component of the education programme which the prospective teachers have to pass, in some kind of examination managed by the cooperating teachers from the school and the UC. The examination focuses on didactics, classroom management and "relational work". This precipitates the close linkage between $\boldsymbol{\theta}_{1,2}$ and $\boldsymbol{\theta}_{1,3}$.

$\boldsymbol{\theta}_{1,4}$ Post-didactic technology concerning didactic praxeologies of a general nature:

There is a tendency, especially among the "knowledgeable others", to frame their comments on the basis of an observed didactic praxeology, and then quickly rephrase it into a general didactic concern, which the observed occurrence is an example of. This can be seen in the following quotation where the aim of the lesson is to "understand" the multiplication principle in combinatorics:

> We couldn't care less how many menus can be made of something in some fictitious burger bar. How is it we get the problem to be the problem we actually want to work with? […] There is a fundamental recurring challenge in everything we do of this kind; we ask concrete questions, but what we really want to work with is something else (University resource person, LS4 27:57–28.23).

Pointing out general issues can be productive, if the rest of the participants can relate to the shift in abstraction, and the discussion does not turn into detached philosophies of education. Indeed, a shift to more general aspects regarding the issues debated has a tendency to bring out the understated institutional understandings, which is particularly clear in the following exchange regarding pupils' use of mathematical terminology:

Prospective teacher	Of course you put it [the words] in their mouth, you are bound to, since it is not natural for them to use it, but anyway I think…
UC professor (interrupting)	Even the fact that they do say it [, is a good thing…]
Prospective teacher	I think there was several who said it, but it is difficult; it is still very new to them, these here things…
University resource person	I have the desire to ask what it is. What is so good about it? Exactly that they say the word itself?
UC professor (surprised)	They have to acquire the mathematical language, and if they only hear it, then they will be able to recognize it in situations where they hear it. But if they have to learn to use it, then it also requires the teaching to build upon use in class
University resource person	Yes, I can also see in the lesson plan that you [prospective teachers] put a lot of emphasis on pupil assertions, that is, like a "sign", that if they use some of the earlier terms, then they are on the right track

(LS2: Time index 26:40–27:39)

It is clear from the whole exchange that the school and UC institutions tacitly agree on certain assumptions on the value of using mathematical terminology; this often appears as unquestioned didactic theory. When they are questioned, a moment of tension arises, and a longer exchange on the general issues of mathematical language takes place. Questions that challenge what is taken for granted in one institutional context can be taken as an annoyance, or an opportunity to reconsider usual didactic praxeologies, particularly at the level of didactic theory. This is definitely a great potential of the lesson study hanseikai.

$\boldsymbol{\theta}_{1,5}$ Post-didactic technology concerning the lesson study format:

This last technology is endemic to the experimental nature of the lesson study. Interspersed between talk directly related to the observed didactic praxeologies, I find a discussion regarding lesson plans and hanseikai in general. Firstly, the lesson plan template engendered extensive debate in some of the post-lesson reflections (LS2, LS3, LS4 and LS5). The nature of a temporally tight lesson was of concern to the prospective teachers. Some had great difficulties judging how long time each teaching activity would last, making them apologetic in the post-lesson discussion:

> There were some [pupils] who did not have time to finish one task before I gave them the next. Some of them could have used the entire lesson on the first task (laughing nervously) (Executive prospective teacher, LS4 8.36–8.46).

> I wish that there were more time for them [the pupils] to try measuring triangles put together.
> … but again shortage of time; time-pressure! I had prepared more triangles in order for them the get a deeper understanding, which was not so smart considering the shortage of time (Executive prospective teacher, LS5 4:40–4:45 and 32:30–32.41).

This was recognized by the participating resource person, providing yet another example of $\boldsymbol{\theta}_{1,4}$:

> This discussion illustrates after all, that even though we start from something concrete, we quickly get into more general points of interest to the teacher, a lot of different ones actually. For example this about how to make the "didactic time" work, in order to provide challenge and make it clear where the lesson is heading (University resource person, LS5 42.10–42:35).

It clearly exposes a general difficulty of many prospective teachers. They are unaccustomed to making a very precise plan for a single lesson. It is evident that prospective teachers could benefit from repeated experiences of the same lesson. Having opportunity to make several "test-runs" as recommended in the given lesson study guide would counter this difficulty and make it less prominent as an object for discussion in the hanseikai. The fact that few prospective teacher teams had test-run opportunity makes the challenge of didactic time less attributable to the prospective teachers themselves. Meanwhile, the other participants in the hanseikai had a more relaxed view of the strictness and merits of trying to estimate how long teaching activities would last:

> It is difficult to carry out a tight schedule with minutes indicated as you have; they never fit, do they? So I think it is good that you just say: Ok, then I give a little here and take a little there (University resource person to prospective teacher team, LS2 14.51–15:04).

> To me it matters little if you overshoot by 2 min or undershoot by one. I believe the schedule is more a factor while preparing; you really consider deeply what this particular activity contains, and how long you imagine it will take (UC professor to prospective teacher team LS2 15:17–15:33).

Other characteristic talk about the lesson study format concerns the hanseikai itself, and this was typically seen in the start or end of the discussion. We see again the clash with the existing post-didactic practice of giving guidance to the prospective teachers, something which the participating UC professors are in fact quite aware of, even if they are prone to lapse back into $\boldsymbol{\theta}_{1,2}$ and $\boldsymbol{\theta}_{1,3}$ themselves, as we have seen above.

> This conversation is really about the teaching and the learning of the pupils, and not as a usual practicum guidance session about whether Sakin [the executive prospective teacher] was standing "in the right way" at the blackboard, writing in an ugly fashion or that kind of thing [laughs] (UC professor LS6: 2:04–2:17).

UC professors give introductory remarks to set the scene for how the hanseikai should proceed and how the post-didactic technology should be:

> When we make comments, focus is on the teaching objectives; that is, to what degree did the [actual] teaching live up to the expectations? […] and of course, all comments should be phrased constructively, because it is not our role to judge in any way, but to give feedback which the team can use in the rest of their practicum (UC professor, LS2 4.48–5:27).

Even though the stance of judgement was intended to be minimized, we have seen it occur ($\boldsymbol{\theta}_{1,2}$), and the prospective teachers were somewhat unsure of the agenda for the post-lesson reflection:

> I have been anxious to know what it was we were going to evaluate, how it was going to take place, because it is really cool to have someone else look at what we are doing; so I think is has been nice, […] it's not that I imagined that I would be "slaughtered", but I was uncertain how much you would dig into the mathematics and didactics (Executive prospective teacher, LS1 56:02–56.50).

It is doubtless that all five described technologies contribute to addressing vital challenges facing the prospective teacher in practicum, but are they all desirable in lesson study? While feedback on prospective teachers' personal performance ($\boldsymbol{\theta}_{1,1}$), as well as guidance ($\boldsymbol{\theta}_{1,2}$), may be much desired by the prospective teachers themselves, there seems to be nothing that warrants the use of speech which alludes to examination ($\boldsymbol{\theta}_{1,3}$). If the aim of lesson study is to assist prospective teachers become more knowledgeable about didactic praxeologies common to the entire mathematics teacher profession, then the general inquisitive discourse ($\boldsymbol{\theta}_{1,4}$) appears desirable. It is noteworthy that participants from a given type of institution do indeed seem to have prevalence for certain types of discourse. In that respect it is valuable, that the kind of lesson study experimented here brings them all into considering a shared teaching experience, providing a nexus of perspectives that would otherwise not exist.

The five kinds of post-didactic technology show that there are still some way to go before having lesson study in prospective teacher education that revolves closely around lesson plans which try to tackle didactical problems of general concern to the mathematics teacher profession. One of the main reasons for having lesson study in Japanese teacher education is to "facilitate the transition of prospective mathematics teachers from being students in methods courses into undergoing actual teaching practice in schools" (Elipane 2012). This means Japanese prospective teachers are to prepare themselves for engaging in lesson study throughout their careers. This is obviously not (yet) a common practice in Danish schools, so what benefits in the short term can be reaped from lesson study in the practicum setting? I propose the ideal to strive for is the ability of prospective teachers to identify key didactical problems which could otherwise be hidden behind all the personal challenges of becoming a teacher and to realize that some challenges are faced by everyone in the profession and that these challenges can be taken up in collaboration between schools, UCs and universities. In the following section, I will take a closer look at the praxeologies that emerged from the teaching experiment as general issues for the profession of mathematics teachers.

The technology of didactic praxeologies

It is obvious that a large number of didactic praxeologies were discussed if one looks at all eight cases of lesson study. Every mathematical praxeology that the prospective teachers

sought to help the pupils acquire gave rise to numerous enacted didactic praxeologies, and even more were suggested as alternatives in the hanseikai. With praxeologies being defined by tasks and further differentiated by a multitude of possible techniques, technology and theory, it is impossible here to give a comprehensive account of all eight cases together. Instead, I present three didactic praxeologies from LS8, and special emphasis is given to the technology part, which is readily observed during hanseikai. The observations presented were also prominent in the other cases and can therefore be considered of general importance to prospective mathematics teachers.

Didactic praxeologies of whole class discussions

It is a mainstay of all the eight observed lessons that the prospective teachers engage themselves in a discussion with the whole class after the pupils have worked for some time on a designated mathematical task. That is, the prospective teachers undertake the didactic task type $\mathbf{T}_2$:

$\mathbf{T}_2$ Orchestrate a common session of discussion with the class, to top off individual work.

In a Japanese lesson, this type of task would take place in a phase called neriage (Shimizu 1999). In general, numerous techniques can be employed to solve this, depending on the purpose of the common session. In LS8, the specific technique used can be described as follows.

τ_2 The prospective teacher calls on a pupil from mainly one group to describe in words how they propose to find the area of the triangle and to write it as a formula. The prospective teacher draws on the blackboard and asks clarifying questions: "Was this what you meant?" etc.

The common session was discussed during hanseikai, and the discourse can be collected in the following two technologies which relate to pros, cons, alternatives to, or expansions of the chosen technique. Each technology is illustrated by a short citation from the audio recording of the LS8 hanseikai:

$\theta_{2,1}$ Draw attention to important mathematical points and share these points with the whole class:

> I was concerned whether all [pupils] were paying attention during your [summarizing] review, because their minds tend to wander when they have been engaged in [individual] work… but on the other hand I consider the review to be important; it is very important to get it [the mathematical knowledge] (UC professor, LS8 25:13–25:32).

$\theta_{2,2}$ Expounding (only) a finite number of pupils ideas and being selective about them:

> There is a suggestion [from a pupil] where you have a right angled triangle, and a line cutting, around which you flip [part of the triangle]… what are the possibilities in this? It does not fit readily into the progression towards understanding area…. We skip this saying "that was a nice suggestion, does anyone have another" (University resource person, LS8 26:10–26:34).

The citations naturally express only singular points in the discourse, while the technology stands for the totality of utterances in relation to the technique, which can be

collected into the two described technologies. These two technologies express knowledge about how and why the teacher directs the class towards the intended learning goals of the lesson. They elaborate what is regarded as viable and valuable courses of action in the concluding phase of the lesson.

Didactic praxeologies of supporting pupils independent explorative work

All the lessons in the experiment involved a phase where pupils were to investigate some problem, either in groups or individually. In Japanese lessons, this phase is called kikan-shido, in which the teacher mainly circulates among the pupils, taking note of their suggested solutions and clarifying the problem if necessary (Stigler and Hiebert 1999). However in the case context, the prospective teacher tries to help pupils more actively in the problem-solving process. This is a common characteristic of such phases in Danish mathematics teaching. In our eight cases of lesson study, this phase covered at least one-third of the lesson time, frequently more, and always more than anticipated by the prospective teachers. The more "open ended" or explorative the problem was, the longer the phase became. The prospective teachers wanted as many of their pupils as possible to find some solution. They wanted to tailor the instruction to the individual needs of each pupil, or small group of pupils, making the teaching much differentiated (Carlgren et al. 2006).[4] Therefore, a complicated didactic task $\mathbf{T}_3$ was faced by the prospective teachers in the Danish equivalent of the kikan-shido:

$\mathbf{T}_3$ Actively support individual pupils while they do explorative work.

The techniques outlined below all take into account the tacit requirement that the support must not give away an answer to the problem, thereby ruining the learning potential of the problem-solving situation. Again these techniques can be varied, but in the context of LS8, one example is the following:

$\boldsymbol{\tau}_3$ The prospective teacher enters into a dialogue with the pupil who is either "stuck" in the problem solving process, or who needs further challenge.

The quality and nature of this supporting dialogue lends itself well to debate during hanseikai, and the prospective teachers are most interested in ideas about what good practice could be. The support dialogue in LS8 was successful, so the following excerpts mostly praise the displayed practice:

> You are really good at posing questions which make them [the pupils] think… to make them think mathematically, also to make them express themselves mathematically, without you telling them what to say (ASTE prospective teacher (specially invited prospective teacher), LS8 14:30–14.44).

> I am impressed by today's prospective teacher's ability to deny pupils direct answers, to let the group themselves decide when an answer is correct. Thereby letting the mathematics be the authority, and not just the teacher saying "this is how it is (University resource person, LS8 17.28–17.45).

[4] "Differentiated teaching (*undervisningsdifferentiering* in Danish) is a fundamental principle written into the Danish national curriculum, and the principle permeates all teaching and thinking about teaching at primary and secondary level in Denmark.

These utterances are exponents of didactic technology, respectively:

$\boldsymbol{\theta}_{3,1}$ *Use of questions* posed to pupils makes them think more deeply, and more mathematically, about the exploration.

$\boldsymbol{\theta}_{3,2}$ *Denial of direct answers* promotes more independent and self-reliant pupils.

It be could argued that the italicized parts belong to didactic technique, but the articulation in the hanseikai takes them into the realm of didactic technology (note also that the cited utterances are examples of the post-didactic technology $\boldsymbol{\theta}_{1,1}$).

Didactic praxeologies of fostering pupils mathematical communication

In connection to $\boldsymbol{\theta}_{1,4}$ it was seen that the development of proper mathematical language among pupils was regarded as important practice by the UC professors, and how tension arose when the importance was questioned. In this section, I take a closer look at the related didactic technology defined by $\mathbf{T}_4$:

$\mathbf{T}_4$ Support the pupils' development of, and use of, mathematical language.

A multitude of techniques were used if looking at all cases, but again I focus on LS8. LS8 distinguished itself with respect to this task, because here the prospective teachers actually wrote down a question closely associated with $\mathbf{T}_4$ as one they wanted to discuss in the hanseikai: "Are mathematical terms used by the pupils while working in pairs on the problem?" (from LS8 lesson plan, pp.3) And this question also featured prominently in the discussion. The technique used to handle $\mathbf{T}_4$ can be expressed as follows:

$\boldsymbol{\tau}_4$ The prospective teacher should consistently use mathematical terms herself, appreciate pupils mathematical statements, even in colloquial terms, and re-voice these statements using the mathematical terms.

An example of immediate re-voicing can be seen in the following quotation, recounting an exchange between the prospective teacher and a pupil:

> Even though the pupil says something about squares, then you [the prospective teacher] are able to put in rectangles, asking: "yes, did you say rectangle?" and then pupil respond by: "No, I actually said square (University resource person, LS8 18.45–18.51).

The didactic technology to justify and explain the technique revolves around the notion of "appreciative communication", see, for example, (Bae 2012), which is not specific to mathematics teaching, but deemed specifically useful to help pupils use mathematical language where the exact terms carry more precise meaning. The idea is to appreciate the fact that statements of a mathematical nature is at all put forward. This will in turn lead more pupils to dare enter into communicating about mathematics. And finally, the teacher's consistent use of precise words will make the pupils adopt these.

> I noticed that many of the mathematical terms and explanations only appear [in pupil communication] when someone comes and asks [them] a question… your way of speaking to and with the pupils is very appreciative, and this really helps [them feel comfortable using mathematical language]… e.g. [pupil says:] "we use these pencil-strokes", and then [prospective teacher says] "yes, we use these pencil-strokes" (Cooperating school teacher, LS8, 11:02–11:44).

Only later will pencil strokes be re-voiced as straight lines.

The lesson progression for pupils to make triangles, first on the geoboard, then on paper and then formulate a rule and write it down was identified as essential. These transitions between representing the triangles and rectangles in different media helps pupils acquire the intended mathematical praxeology:

> You do something with the rubber band, and then you transfer it to the [paper] and look at it anew. Just like you said, one thing is to find a calculation rule, but in the moment you have to write it down, it will be internalized in another fashion. In the same way I think [the change in media] is important to retain (observing prospective teacher, LS8 22:40–22.57).

The importance of identifying subtle quality differences in pupils' mathematical language were also displayed as technology for τ_4:

> I heard somebody say: "length and width and then we just take half of it", really that is perfect. And there was someone who found the truly perfect: "Multiply length and width and then we divide by two" all by themselves and that was fantastic because this was what we wanted (observing prospective teacher, LS8, 23:00–23:20).

Below is an attempt to phrase all this scattered discourse in one dense technology. This is not easy, but it just reflects the richness of the didactic praxeology developed during the lesson study process in general and in the hanseikai in particular:

$\boldsymbol{\theta_4}$ Appreciative communication and re-voicing will enhance pupil participation in mathematical discourse and make this more precise. Work in pairs, use of different media in which to represent the mathematical objects and the requirement of a written rule, is conductive of their oral and written mathematical language.

Conclusion

This article adds to the descriptive knowledge base of lesson study, c.f. the first need as stated by Lewis et al. (2006) by focussing on the hanseikai phase of lesson study situated in prospective teacher education. Even though discourse is difficult to capture due to its multifaceted and scattered appearance, this paper shows that the context of hanseikai is remarkably well suited to provide information on the technology of didactic and paradidactic practices. It can be concluded from "Post didactic technology in the paradidactic infrastructure of hanseikai" section that the post-didactic praxeologies of hanseikai compete with existing infrastructures in schools ($\boldsymbol{\theta}_{1.1}$,) and in teacher education ($\boldsymbol{\theta}_{1.2}$ and $\boldsymbol{\theta}_{1.3}$). The inclusion of resource persons from universities is an important dimension of the paradidactic practice of hanseikai ($\boldsymbol{\theta}_{1.4}$), thus explicating part of the innovative mechanism behind lesson study (second need stated by Lewis et al.). With this added dimension, seemingly natural institutional tenets can be questioned, opening up a deeper consideration of these to the benefit of each institutional perspective. The identification and reflection upon points of contention between the institutions is valuable because it works in favour of theorizing praxis and utilizing theory, that is, to strengthen the connections between the praxis part and theory part of mathematical and didactical praxeologies. The result is that different institutional preferences in the discussions are a source of new insight for all participants. This finding corroborate other research into lesson study in prospective teacher education (Fernandez and Zilliox 2011; Yu 2011).

It is furthermore evident that the unfamiliar infrastructure of lesson study in general and hanseikai in particular becomes a prominent object for reflection ($\boldsymbol{\theta}_{1.5}$). Guidelines and preparatory introductions are insufficient to facilitate a clear transition from existing paradidactic infrastructures. This may in hindsight be unsurprising, but it is significant to all who engage in lesson study, that the hanseikai also serves to evaluate and develop the technique and technology of lesson study as a paradidactic practice. The greatest drawback, from the unfamiliar infrastructure and the above-mentioned institutional preference, is the lack of a common discursive direction in the hanseikai. This could reasonably be ameliorated by improvements in the prepared lesson plans, but until firm pre-didactic praxeologies for preparing lesson plans, suitable for lesson study, are established, it may be difficult for prospective teachers to extract didactic knowledge of pertinence to the teacher profession in general.

Looking at the pathways which Lewis et al. (2006) claims are strengthened by lesson study, it is seen how didactic praxeologies related to mathematics teaching are developed ("The technology of didactic praxeologies" section). In several of the eight hanseikais, they tend to be obscured by paradidactic challenges, but they are still there. Three of these were presented in the context of LS8 where they did stand out clearly: how to carry out common sessions which sum up the individual mathematical work ($\mathbf{T}_2$), how to support mathematical exploration ($\mathbf{T}_3$) and how to foster mathematical language ($\mathbf{T}_4$). The praxeologies these tasks engender are connected to teaching that has mathematical problem solving as its focus: mathematical problems should be explored by the pupils themselves and they must be able to communicate their exploits among each other in a precise manner. The exploration should lead to common understandings shared by the whole class. The hanseikai exhibits discourse, described by the didactic technologies $\boldsymbol{\theta}_{2.1}$, $\boldsymbol{\theta}_{2.2}$, $\boldsymbol{\theta}_{3.1}$, $\boldsymbol{\theta}_{3.2}$ and $\boldsymbol{\theta}_4$, that justifies the techniques employed by the prospective teacher to resolve the three mentioned tasks. Using the framework of ATD, it is demonstrated that didactic technology expounded in the hanseikai provides the prospective teacher and the participants in general, with mathematics teacher knowledge, which is understandable, accessible and useful due to its grounding in collectively observed teaching practice.

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Appendix

See Tables 1 and 2.

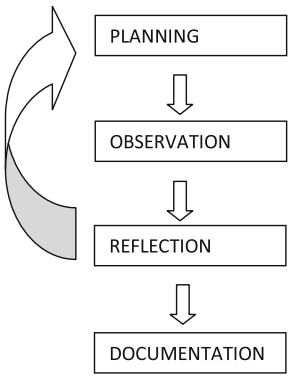
Table 1 Translation of guide to lesson study

A SHORT GUIDE TO THE LESSON STUDY PROCESS WITH RESPECT TO PROSPECTIVE TEACHERS IN THE ASTE PROJECT

A complete lesson study consists of four phases, of which 1–3 may be repeated several times. This document explains the case where the team planning a lesson is a group of prospective teachers:

1. PLANNING (Group meetings, individual studies)
   - •Selecting a topic for the lesson (selected from a collection of lessons that corresponds to a chapter in the textbook and/or curriculum)
   - •Study of the textbook, curriculum and possibly existing lesson plans
   - •Formulation of specific objectives for the lesson
   - •Construction of lesson plan (see template)
2. OBSERVATION (in the classroom with pupils)
   - •The lesson is taught by one or more members of the group
   - •The other members participate as observers (taking notes) and other observers also participate, see below
3. REFLECTION (Group meeting shortly after the lesson)

Presentation of concrete observations from the lesson and reflections regarding the connection between goals/questions and observations. (These can form the basis for a review of the lesson plan in a following new phase 1)



4. DOCUMENTATION

This part shall be agreed between prospective teachers and UC professors(s)

The following are some guidelines for 1.–3. within the framework of teacher education where the lesson is planned and implemented by prospective teachers under the guidance of an experienced cooperating teacher and one or more UC professors. It is suggested to let an initial test run of lesson, observation and reflection take place in a smaller group—for example, only consisting of cooperating teacher and prospective teachers (meaning the description of phase 2 and phase 3 below will only be partly usable). Of course one can only test a lesson once in the same class, so several passes of phases 2 and 3 will require several parallel classes which can meaningfully be subjected to the lesson

PLANNING

- •In cooperation with UC professor and cooperating teacher: Select a specific topic from the school discipline. The topic can be wide or narrow but should preferably contain some special challenges for pupils, and thus for the prospective teacher
- •Formulate these challenges, more precisely, supported by a detailed study of what curricula, textbooks and literature (e.g. from the course work or from own search) says about them.
- •Select a small number (2-3) of these specific challenges which the lesson should help the pupils with
- •Design a problem (e.g. an open task) that the lesson is to introduce to the pupils and which at the same time
  - •is sufficiently rich, so the solution will involve overcoming the selected challenges and
  - •is designed so that pupils have resources to provide at least partial solutions—and preferably with different approaches appropriate for the pupils
- •This is a very central part of the lesson so you should take the time to fine tune the problem!
- •Plan the lesson in detail using the lesson plan template. Note that the lesson plan must be useable as a tool for observation and therefore should contain explicit, comprehensible hypotheses and questions that observers can focus on—and which mainly relates to the illumination of pupil learning
- •Lesson plan will be discussed at length in the team internally as well as with the cooperating teacher and UC professor—if possible on several joint meetings before the first observation

OBSERVATION

The event takes place in a regular classroom, where pupils participate in the lesson carried out by one prospective teacher. The observation is usually attended by representatives from various stakeholders in teacher education and practicum school, including at least persons 1–5 below:

Table 1 continued

1. The group of prospective teachers who have planned the lesson together, and one of which conducts the lesson in the role of teacher. Unless otherwise agreed, it is only the latter that communicate with pupils in the lesson
2. The cooperating teacher who is the regular class teacher and who guides the group of prospective teachers
3. The UC professor who teaches the group of pre-service teachers (and has guided them in the preparation of the lesson plan)
4. A "Resource Person" ("knowledgeable other") with relevant knowledge from other institutions, such as universities, textbook publishers or municipality
5. Other teachers and UC professors at invitation (at least 1 in addition to those mentioned above—acting as moderator during the following discussion)
6. Other prospective teachers at invitation

Observers are usually standing at the back of the class or along the walls to the side and interfere as little as possible. During periods in which pupils work alone or in groups, they circulate quietly in class in order to gather information about pupils' work in relation to the hypotheses under investigation. Pupils are prepared beforehand that there will be observers present and they have to work as usually under the guidance of the prospective teacher

All observers have pre-read the lesson plan before the observation begins. The three main observers (2. 3. and 4. above) agrees to a gross partition of questions from the lesson plan, so that they do not all focus on the same during the observation

REFLECTION

Immediately after the lesson, all participants (1–6 above) attend a joint meeting where people share observations and reflections concerning the lesson

The meeting is led by a moderator, usually an experienced teacher or UC professor (from group 5 above). The meeting begins with a panel that speaks, and sits at a table, in the order given below. (Usually, all participants sit at the same table, but in case of many participants, the panel sit in front of the others.)

The panel consists of:

1A. the prospective teachers who have taught
1B. other prospective teachers from the planning group
2. Cooperating teacher
3. UC professor responsible for prospective teacher guidance
4. The resource person
5. The moderator

The moderator normally do not enter into the conversation, but makes sure it remains focused on the questions and hypotheses raised in the lesson plan, and that the conversation always takes place in a constructive and collegial tone, focusing on the lesson plan and pupil learning

The moderator gives a welcome speech and briefly reminds everybody of the principles and process of the meeting. He gives the floor to the panel in the order above (each, approx. 3 min):

1A: provides some personal reflections regarding lesson in relation to the lesson plan
1B: present two to three observations from working to prepare the lesson plan
2: present two to three of observations based on his experience with the class and in relation to the lesson plan questions
3: present two to three observations relevant to the lesson plan questions
4: present two to three observations relevant to the lesson plan questions

Note that in this initial phase, no complicated or concluding reflections should be ventured

The moderator then organizes a discussion of the issues raised by the lesson plan, where all observers have the opportunity to contribute and comment. These interventions should be short, concise and constructive—usually in form of *questions* posed to the pre-service teachers, and always based on *concrete observations of the lesson* (and *never* "knowingly such-and-such would I have done"). Between each intervention 1A and 1B is given opportunity to comment or ask clarifying questions to the observations. This "free" part of the discussion takes approximately 25 min (the moderator takes care to stop this part approximately 5 min before the meetings' scheduled end time)

Finally, the resource person is given the floor for approximately 5 min, intended to sum up an important point from the observation and discussion. The resource person generally acts more observant than active in the preceding discussion, in order to make this summarization from an "external perspective"

Table 2 Translation of lesson plan template

Lesson Plan

School and class:
Date:
Teacher:
Lesson Study Group:
Mathematics topic:
Title:
Goal:

Attainment targets (in accordance with national course of study):

Background Information:

- Why we have chosen this topic for lesson study?
- Why is it important for pupil learning to have this lesson now?
- Why we have chosen the following main activities?
- What teaching strategies are needed for this lesson?

Teacher Academic goals:

- Which teaching strategies, methods, etc.., would we like to explore and try out in this lesson?
- How would we like to explore and test these strategies?
- What ideas / hypotheses do we make regarding the use of strategies in relation to pupil learning?

Lesson Description:

Learning activities	Expected duration	Expected pupil responses	Teacher support	Evaluation Signs - and data

Evaluation:

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