

2004098, Lewis Cox, Stress Analysis, ACHA5015 P1 of 6

$$A = 2$$

$$B = 0$$

$$C = 0$$

$$D = 4$$

$$E = 0$$

$$F = 9$$

$$G = 8$$

1i) $\epsilon_x = 28 \times 10^{-6}$ tensile

$$\epsilon_y = -19 \times 10^{-6} \text{ compress}$$

$$\nu = 0.28$$

$$E = 170 \times 10^9 \text{ Pa}$$

$$\begin{aligned}\sigma_x &= \frac{E}{1-\nu^2} (\epsilon_x + \nu \epsilon_y) \Rightarrow \frac{(170 \times 10^9)}{1-0.28^2} (28 \times 10^{-6} + 0.28 \times -19 \times 10^{-6}) \\ &= (1.844 \times 10^{11}) (2.268 \times 10^{-5}) \\ &= +4.18 \text{ MPa, tensile.}\end{aligned}$$

$$\begin{aligned}\sigma_y &= \frac{E}{1-\nu^2} (\epsilon_y + \nu \epsilon_x) \Rightarrow \frac{(170 \times 10^9)}{1-0.28^2} (-19 \times 10^{-6} + 0.28 \times 28 \times 10^{-6}) \\ &= (1.844 \times 10^{11}) (-1.489 \times 10^{-5}) \\ &= -27.45 \text{ Pa, tensile}\end{aligned}$$

ii) $\sigma_x = 89 \times 10^6 \text{ Pa}$

$$\frac{1}{170 \times 10^9} = 5.88 \times 10^{-12}$$

$$\sigma_y = 48 \times 10^6 \text{ Pa}$$

$$\begin{aligned}\epsilon_x &= \frac{1}{E} (\sigma_x - \nu \sigma_y) \Rightarrow \frac{1}{170 \times 10^9} (89 \times 10^6 - 0.28(48 \times 10^6)) = 4.44 \times 10^{-4} \text{ tensile} \\ \epsilon_y &= \frac{1}{E} (\sigma_y - \nu \sigma_x) \Rightarrow \frac{1}{170 \times 10^9} (48 \times 10^6 - 0.28(89 \times 10^6)) = -1.35 \times 10^{-4}\end{aligned}$$

2004098, Lewis Cox, Stress Analysis, ACHA5015, P2 of 6

2i) $\sigma_x = 100 \times 10^6 \text{ Pa}$
 $\sigma_y = 209 \times 10^6 \text{ Pa}$
 $\tau_{xy} = 125 \times 10^6 \text{ Pa}$

~~$\sigma_x = \sigma$~~ $\sigma_1 = \frac{(\sigma_x + \sigma_y)}{2} + \frac{\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}}{2}$

$\sigma_1 = \frac{(100 \times 10^6 + 209 \times 10^6)}{2} + \frac{\sqrt{(100 \times 10^6 - 209 \times 10^6)^2 + 4(125 \times 10^6)^2}}{2}$

$\sigma_1 = \frac{(-104.5 \times 10^6)}{2} + \frac{\sqrt{(-109 \times 10^6)^2 + (6.25 \times 10^6)^2}}{2}$

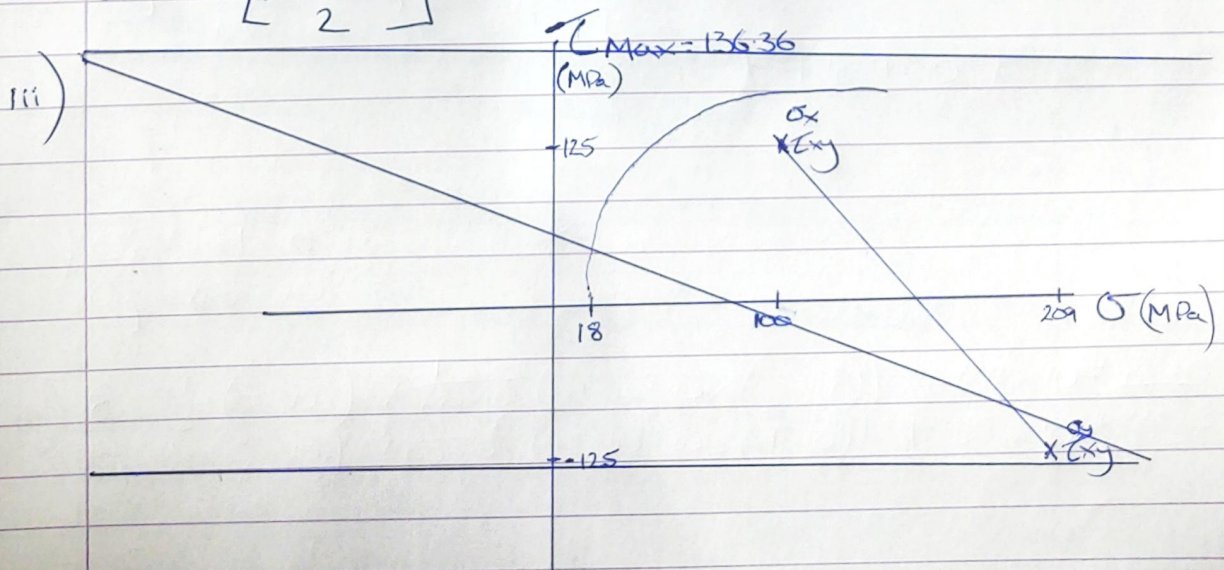
$\sigma_1 = \frac{(-104.5 \times 10^6)}{2} + (272.72 \times 10^6 / 2)$
 $\sigma_1 = 290.86 \times 10^6 \text{ Pa}$

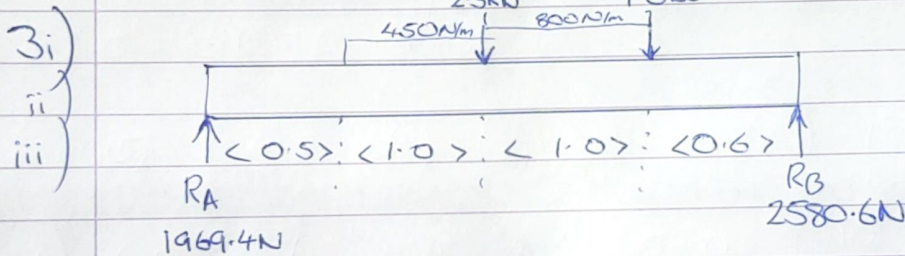
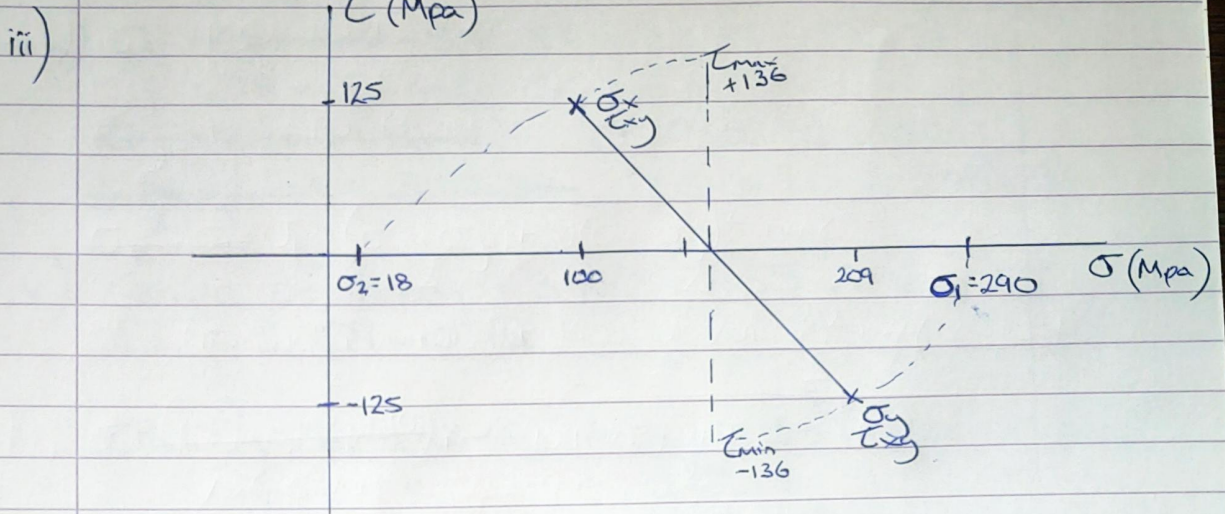
$\sigma_2 = (154.5 \times 10^6) - (272.72 \times 10^6 / 2) = 18.14 \times 10^6 \text{ Pa}$

ii) $\tau_{\max} = \frac{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}{2} = \frac{\sigma_1 - \sigma_2}{2} = \frac{(290.86 \times 10^6) - (18.14 \times 10^6)}{2}$

$\tau_{\max} = 136.36 \times 10^6 \text{ Pa}$

$\tau_{\min} = -\left[\frac{\sigma_1 - \sigma_2}{2}\right] = -136.36 \times 10^6 \text{ Pa}$





$$EI \frac{d^2y}{dx^2} = 1969.4[x] - 450[x-0.5]^2 + 450[x-1.5]^2 - (2.3 \times 10^3)[x-1.5] - 800[x-1.5]^2 + \frac{800^2}{2}[x-2.5]^2 - 1000[x-2.5]$$

$$EI \frac{dy}{dx} = 1969.4[x]^2 - \frac{450}{6}[x-0.5]^3 + \frac{450}{6}[x-1.5]^3 - \frac{2300}{2}[x-1.5]^2 - \frac{800}{6}[x-1.5]^3 + \frac{800^2}{6}[x-2.5]^3 - \frac{1000}{2}[x-2.5]^2 + A$$

$$EI y = \frac{1969.4}{6}[x]^3 - \frac{450}{24}[x-0.5]^4 + \frac{450}{24}[x-1.5]^4 - \frac{2300}{6}[x-1.5]^3 - \frac{800}{24}[x-1.5]^4 + \frac{800^2}{24}[x-2.5]^4 - \frac{1000}{6}[x-2.5]^3 + Ax + B$$

iv) Boundary Conditions, $(x=0, y=0)$ $(x=3.1, y=0)$
 $EI y = 0$ "Brackets negative" $+ 0A + B \therefore B = 0$

$$0 = \frac{1969.4}{6}[3.1+5]^3 - \frac{450}{24}[3.1-0.5]^4 + \frac{450}{24}[3.1-1.5]^4 - \frac{2300}{6}[3.1-1.5]^3 - \frac{800}{24}[3.1-1.5]^4 + \frac{800^2}{24}[3.1-2.5]^4 - \frac{1000}{6}[3.1-2.5]^3 + A(3.1) + 0$$

20040918, Lewis Cox, Stress Analysis, ACHASOIS, PH of 6

3iv cont.)
$$0 = \frac{9778.39}{3.1} - 856.83 + 122.88 - 1570.13 - 218.45 + 4.32 - 36 + (3.1A)$$

$$0 = \frac{1209.77}{3.1} + (3.1A) \quad 0 = 7651.36 + (3.1A)$$

$$\frac{1209.77}{3.1} - A = 390.24 \quad 7651.36/3.1 = A = 2468.34$$

v) Centre of beam = $3.1/2 = 1.55 \therefore x = 1.55$
 $EI = 1.09 \times 10^6 \text{ Pa}$

$$\begin{aligned} (1.09 \times 10^6)y = & \frac{1969.4[1.55+5]^3}{6} - \frac{450[1.55-0.5]^4}{24} + \frac{450[1.55-1.5]^4}{24} \\ & - \frac{2300[1.55-1.5]^3}{6} - \frac{800[1.55-1.5]^4}{24} + \frac{800[1.55-2.5]^4}{24} \\ & - \frac{1000[1.55-2.5]^4}{24} + 2468.34(1.55) + 0 \end{aligned}$$

$$(1.09 \times 10^6)y = \frac{1222.29 \cdot 22.79}{2468.34(1.55)} + (1.17 \times 10^4) \cdot 0.0479 \cdot 0.0002 +$$

$$(1.09 \times 10^6)y = 5024.94$$

$$y = 5024.94 / (1.09 \times 10^6) = 4.61 \times 10^{-3} \text{ m}$$

4i) Thin walled

$$d = 0.28 \text{ m}$$

$$L = 0.60 \text{ m}$$

$$n_L = 59\%$$

$$n_C = 68\%$$

$$P_{max} = 14.0 \text{ Bar}, 14 \times 10^5 \text{ Pa}$$

$$\sigma_{max} = 150 \times 10^6 \text{ Pa}$$

$$\sigma_C = pd / 2t n_L \Rightarrow t = pd / 2\sigma_C n_L$$
$$t = (14 \times 10^5)(0.28) / 2(150 \times 10^6)(0.59)$$
$$t = 2.2 \times 10^{-3} \text{ m}$$

$$\text{ii) } t = pd / 4\sigma_C n_C \Rightarrow t = (14 \times 10^5)(0.28) / 4(\cancel{0.59})(150 \times 10^6)$$
$$t = 9.6 \times 10^{-4} \text{ m}$$

$$\text{iii) Min Thickness } \therefore 2.2 \times 10^{-3} \text{ m}$$

5.)

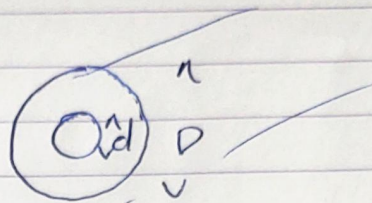
$$L = 1.39 \text{ m}$$

$$D = 19.05 \times 10^{-3} \text{ m}$$

$$d = 16.65 \times 10^{-3} \text{ m}$$

$$E = 200 \times 10^9 \text{ Pa}$$

$$\text{thickness} = 1.2 \times 10^{-3} \text{ m}$$



$$A = \frac{\pi}{4} (D^2 - d^2) = \frac{\pi}{4} ((19.05 \times 10^{-3})^2 - (16.65 \times 10^{-3})^2)$$

$$= \frac{\pi}{4} (3.62 \times 10^{-4} - 2.77 \times 10^{-4})$$

$$= 0.667 \text{ m}^2$$

$$I = \frac{\pi}{64} (D^4 - d^4) = \frac{\pi}{64} (1.31 \times 10^{-7} - 7.68 \times 10^{-8})$$

$$= \frac{\pi}{64} (5.42 \times 10^{-8})$$

$$= 2.66 \times 10^{-9} \text{ m}^4$$

$$K = \sqrt{\frac{EI}{A}} = \sqrt{(2.66 \times 10^{-9}) / 0.667} = 6.3 \times 10^{-5} \text{ m}$$

ii)

$$F_{cr} = \frac{n \pi^2 EI}{L^2} \Rightarrow n = \frac{F_{cr} L^2}{\pi^2 EI}$$

~~Both ends fixed~~ $n = 0.25$ $n = 1$, one free end, one fixed

$$F_{cr} = \frac{1}{0.25} \times \pi^2 \times (200 \times 10^9) \times (2.66 \times 10^{-9}) / 1.39^2$$

$$= 1312.65 / 1.93 = 5250.62 / 1.93$$

$$= 679.39 \text{ N} = 2720.52 \text{ N}$$

iv)

$$S.R. = L / K \quad K \text{ calculated in "Si" as } (6.3 \times 10^{-5} \text{ m})$$

$$= 1.39 / 6.3 \times 10^{-5}$$

$$= 22063$$

$$K \sqrt{\frac{EI}{A}} = \sqrt{(2.66 \times 10^{-9} / 0.667)}$$

S.R. > 120 when made of steel, confirmed.