

(Q2)

$$n) \quad 1) \quad \int \frac{2x}{(4+3x^2)} dx \rightarrow \int 2x(4+3x^2)^{-1} dx$$

$$\text{Int. by parts} \quad \int u \frac{dv}{dx} = uv - \int \frac{du}{dx} v$$

$$u = 2x \quad \frac{dv}{dx} = (4+3x^2)^{-1}$$

$$\frac{du}{dx} = 2 \quad v = \ln(4+3x^2)$$

$$= 2x \cdot \ln(4+3x^2) - \int 2 \cdot \ln(4+3x^2)$$

$$- 2 \int \ln(4+3x^2)$$

$$- 2 \cdot (4+3x^2) \ln(4+3x^2) - (4+3x^2) + C$$

$$= 2x \ln(4+3x^2) - 2(4+3x^2) \ln(4+3x^2) + (4+3x^2) + C$$

$$11) \quad \int_0^4 e^{\frac{1}{2}x} dx$$

$$\left[2e^{\frac{1}{2}x} \right]_0^4 \quad (2e^{\frac{1}{2}(4)} + C) - (2e^{\frac{1}{2}(0)} + C)$$

$$= 2e^2 - 2e^0 = 2e^2 - 2 = 12.778$$

$$111) \quad \int_0^4 \frac{5x+13}{(2x+1)(x+4)} dx \rightarrow \int \frac{A}{2x+1} + \frac{B}{x+4} dx$$

$$5x+13 = A(x+4) + B(2x+1)$$

$$x = -4 \quad x = -\frac{1}{2}$$

$$5(-4)+13 = B(2(-4)+1) \quad 5(-\frac{1}{2})+13 = A(-\frac{1}{2}+4)$$

$$-7 = B(-7)$$

$$\frac{21}{2} = A(\frac{7}{2})$$

$$-1 = B$$

$$3 = A$$

iii cont.) $\int \frac{3}{2x+1} + \frac{-1}{x+4} dx$

$$= 3 \ln(2x+1) - \ln(x+4)$$

$$= \ln(2x+1)^3 - \ln(x+4)$$

$$= \ln \frac{(2x+1)^3}{(x+4)}$$

b) $\int \frac{4}{x(1+4\ln x)^2} dx$

$$= 4 \ln(x(1+4\ln x)^2)$$

c) 1) $\int_{\ln 2}^{\ln 3} \frac{3e^{2x}}{\sqrt{e^x-1}} dx \rightarrow \int_{\ln 2}^{\ln 3} 3e^{2x} (\sqrt{e^x-1})^{-1}$

$$u = 3e^{2x} \quad \frac{du}{dx} = (\sqrt{e^x-1})^{-1}$$

$$\frac{du}{dx} = 6e^{2x} \quad v = \ln(\sqrt{e^x-1})$$

$$3e^{2x} \cdot \ln(\sqrt{e^x-1}) - \int 6e^{2x} \cdot \ln(\sqrt{e^x-1})$$

$$v) y = (x + \sin 2x)$$

$$\frac{dy}{dx} = 1 + 2\cos 2x$$

$$vi) y = \frac{1-10x}{(2x-1)^2}$$

$$u = 1-10x$$

$$\frac{du}{dx} = -10$$

$$v = (2x-1)^2$$

$$\frac{dv}{dx} = 4(2x-1)$$

$$= 8x-4$$

$$\frac{dy}{dx} = \frac{-10(2x-1)^2 - (1-10x)(8x-4)}{(2x-1)^4}$$

$$= \frac{4x^2 - 4x + 1 + 80x^2 + 48x - 4}{(2x-1)^4}$$

$$= \frac{84x^2 + 44x - 3}{(2x-1)^4}$$

$$(1-10x)(8x-4)$$

$$8x-4-80x^2+40x$$

$$-80x^2+48x-4$$

$$(2x-1)(2x-1)$$

$$4x^2-4x+1$$

$$b) y = x^{-1} + \cos 2x$$

$$\frac{dy}{dx} = -x^{-2} - 2\sin 2x$$

$$\frac{d^2y}{dx^2} = 2x^{-3} - 4\cos 2x$$

$$\frac{d^3y}{dx^3} = -6x^{-4} + 8\sin 2x$$

$$\frac{d^4y}{dx^4} = 24x^{-5} + 16\cos 2x$$

$$\frac{d^5y}{dx^5} = -120x^{-6} - 32\sin 2x$$

$$c) y = \ln(\operatorname{cosec} x - \cot x)$$

$$\frac{dy}{dx} = \frac{1}{\operatorname{cosec} x - \cot x} \cdot (-\operatorname{cosec} x \cot x + \operatorname{cosec}^2 x)$$

$$= \frac{-\operatorname{cosec} x \cot x + \operatorname{cosec}^2 x}{\operatorname{cosec} x - \cot x}$$

$$\operatorname{cosec} x (-\cot x + \operatorname{cosec} x)$$

$$(\operatorname{cosec} x - \cot x)$$

$$= \operatorname{cosec} x$$

$$\operatorname{cosec} x (-\cot x + \operatorname{cosec} x)$$

$$= \operatorname{cosec} x$$

$$= \operatorname{cosec} x - \cot x$$

Q1

$$a) \quad i) \quad y = \frac{x^2}{2x+1} \quad u = x^2 \quad v = 2x+1$$

$$\frac{du}{dx} = 2x \quad \frac{dv}{dx} = 2$$

$$\frac{dy}{dx} = \frac{2x(2x+1) - x^2 \cdot 2}{(2x+1)^2}$$

$$= \frac{4x^2 + 2x - 2x^2}{(2x+1)^2}$$

$$= \frac{2x^2 + 2x}{(2x+1)^2}$$

$$= \frac{(2x+1)(2x+1)}{(2x+1)^2}$$

$$ii) \quad y = \sqrt{x^2+1} \quad > \quad y = (x^2+1)^{\frac{1}{2}} \quad u = x^2+1$$

$$y = u^{\frac{1}{2}} \quad \frac{du}{dx} = 2x$$

$$\frac{dy}{dx} = \frac{1}{2} u^{-\frac{1}{2}} \cdot 2x$$

$$\frac{dy}{dx} = \frac{1}{2} u^{-\frac{1}{2}} \cdot 2x$$

$$= \frac{2x}{2(x^2+1)^{\frac{1}{2}}} = x(x^2+1)^{-\frac{1}{2}}$$

$$iii) \quad y = 3x^{\frac{3}{2}} + \frac{3}{x^2} + 6x$$

$$= 3x^{\frac{3}{2}} + 3x^{-2} + 6x$$

$$\frac{dy}{dx} = \frac{9}{2}x^{\frac{1}{2}} - 6x^{-3} + 6$$

$$iv) \quad y = \sec^3 4x$$

$$\frac{dy}{dx} = 3 \sec^2 4x \cdot \tan 4x \cdot 4$$

$$= 12 \sec^2 4x \cdot \tan 4x$$