

Harry Williams 2004075
Analytical methods exam 18/5/21

Question two complex numbers

a) $z_1 = 2+3j$ $z_2 = 1+5j$ $z_3 = 3-j$

i) $z_1 + z_2$
 $(2+3j) + (1+5j)$
 $2+3j + 1+5j$
 $3 + 8j$

ii) $z_1 - z_3$
 $(2+3j) - (3-j)$
 $2+3j - 3+j$
 $-1 + 4j$

iii) $\frac{z_1}{z_2} = \frac{(2+3j)}{(1+5j)}$

Conjugate

~~$\frac{2+3j}{1+5j}$~~ $1-5j$

$$\frac{(2+3j)}{(1+5j)} \times \frac{(1-5j)}{(1-5j)}$$

$$\frac{2 + 10j + 3j - 15j^2}{1 + 8j + 8j - 1} \quad \leftarrow j^2 = -1$$

$$\frac{2 + 10j + 3j - 15(-1)}{1 + -1}$$

$$\frac{2 + 10j + 3j - 14}{1 + -1}$$

$$\frac{2 + 13j - 14}{2}$$

$$\frac{-12 + 13j}{2}$$

$$\frac{-12}{2} + \frac{13j}{2}$$

iv) $z_1 - z_2$
 $(2 + 3j) - (1 + 5j)$
 $2 + 3j - 1 - 5j$
 $1 - 2j$

✓

$$b) \quad \underset{a}{5}x^2 + \underset{b}{3}x + \underset{c}{12} = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(3) \pm \sqrt{(3)^2 - 4(5)(12)}}{2(5)}$$

$$x = \frac{3 \pm \sqrt{9 - 240}}{10}$$

$$x = \frac{3 \pm \sqrt{240}}{10}$$

$$x = \frac{3 \pm 15.77}{10}$$

$$x = \frac{3 - 15.77}{10}$$

$$x = \frac{3 + 15.77}{10}$$

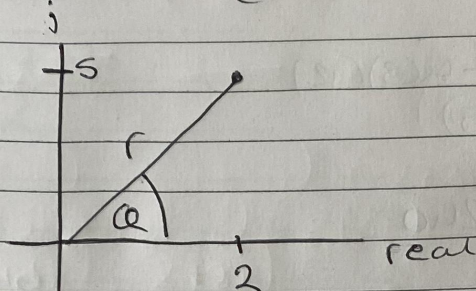
$$x = \frac{-12.77}{10}$$

$$x = \frac{18.77}{10}$$

c) Polar form de moires theorem

i $(2 + 5j)^6$

theorem = $r^n (\cos \theta + j \sin \theta)$



$$r^2 = 2^2 + 5^2$$

$$r = \sqrt{4 + 25}$$

$$r = \sqrt{29}$$

$$r = \underline{\underline{5.38}}$$

$$\tan \alpha = \frac{5}{2}$$

$$\alpha = \tan^{-1}\left(\frac{5}{2}\right)$$

$$\alpha = \underline{\underline{68.1985}}$$

Put into equation

Polar form

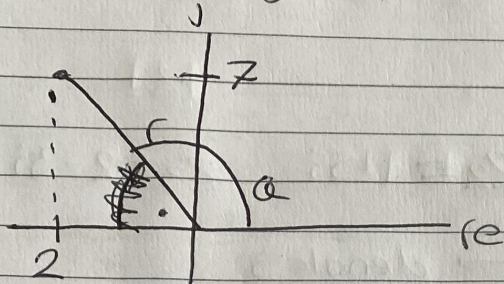


$$5.38^6 (\cos 6 \times 68.1985 + j \sin 6 \times 68.1985)$$

$$24248.98 (\cos 409.141 + j \sin 409.141)$$

~~24248.98~~

ii $(-2 + 7j)^4$



$$r^2 = \cancel{2^2}^2 + 7^2$$

$$r^2 = \sqrt{4 + 49}$$

$$r^2 = 53$$

$$r = \sqrt{53}$$

$$r = 7.2801$$

$$\theta = \tan^{-1} \frac{7}{2}$$

$$\theta = \tan^{-1} \left(\frac{7}{2} \right)$$

$$\theta = 74.05^\circ$$

$$4\theta = 180 - 74.05$$

$$\theta = 105.95$$

Put into equation

$$7.2801^{(4)} (\cos 4 \times 105.95 + j \sin 4 \times 105.95)$$

$$2808.48 (\cos 423.8 + j \sin 423.8) \leftarrow \text{Polar form}$$

D = value of j^2

$$j^2 = -1$$

e) $z_1 = 5 + 4j$ $z_2 = 1 + 5j$ $z_3 = 2 - 3j$

$z_1 z_2 \overline{z_3}$ ← denotes

$$\begin{aligned} & (5 + 4j) \times (1 + 5j) \times (2 + 3j) \\ & (5 + 25j + 4j + 20j^2) (2 + 3j) \quad j^2 = -1 \\ & (5 + 48j) (2 + 3j) \\ & 10 + 15j + 96j + 144j^2 \quad j^2 = -1 \\ & 10 + 254j \end{aligned}$$

f) $\overline{\overline{x}} = \underline{\underline{\frac{1}{x}}}$ or $\overline{\overline{xe^{-1}}}$

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question 3 integration

a

i) $\int (x^2 - x)(x^3 + 6x) dx$
 $\int x^5 + 6x^3 - x^4 + 6x^3 - 6x^2 dx$
 $\frac{x^6}{6} - \frac{x^5}{5} + \frac{3x^4}{2} - 2x^3$ can be written

$$\frac{x^3(5x^3 - 6x^2 + 15x - 60)}{30} + C$$

ii) $\int (e^x + 5)(e^x + 6) dx$
 $\int e^{2x} + 6e^x + 5e^x + 30 dx$
 $\int e^{2x} + 11e^x + 30 dx$
 $\rightarrow \frac{e^{2x}}{2} + 11e^x + 30x + C$

iii) $\int_0^3 (x+6)(x-1) dx$
 $x^2 - x + 6x - 6$
 $\int x^2 + 5x - 6 dx$
 $\rightarrow \frac{x^3}{3} + \frac{5}{2}x^2 - 6x + 6$

let $x=3$

$$\frac{(3)^3}{3} + \frac{5(3)^2}{2} - 6(3) = 13.5$$

$$\text{iv) } \int_1^5 (2x+3)(x+3) dx$$

$$= 2 \int_1^5 x^2 dx + 3 \int_1^5 x dx + 21 \int_1^5 1 dx$$

$$\int x^2 dx$$

$$= \frac{x^3}{3}$$

$$\int 1 dx = x$$

$$x \left(\frac{4x^2 + 39x + 126}{6} \right) + C$$

b)

i) $\int 2x \cos x \, dx$

$$2 \int x \cos(x) \, dx$$

integrate by Parts

$$x \sin(x) - \int \sin(x) \, dx$$

$$= 2x \sin(x) + 2 \cos(x)$$

$$= \underline{\underline{2x \sin(x) + 2 \cos(x) + C}}$$

ii

$$\int_1^2 5xe^x \, dx$$

remove constant

$$= 5 \int_1^2 xe^x \, dx$$

integration by Parts

$$= 5 [e^x x - \int e^x \, dx]_1^2$$

$$\int e^x \, dx = e^x = 5 [e^x x - e^x]_1^2$$

calculating for boundaries

$$= \underline{\underline{5e^2}}$$

c) ~~4~~ integrate following

iii) $\int \frac{2x}{x^2} dx$

$$= 2 \cdot \int \frac{x}{x^2} dx$$

$$= 2 \cdot \int \frac{1}{x} dx$$

$$= 2 \ln x$$

$$= \underline{\underline{2 \ln x + C}}$$

ii) $\int \frac{9}{7x+10} dx$

$$= 9 \cdot \int \frac{1}{7x+10} dx$$

$$= 9 \cdot \int \frac{1}{7u} du$$

$$= 9 \cdot \frac{1}{7} \cdot \int \frac{1}{u} du$$

$$= 9 \cdot \frac{1}{7} \ln 7x+10$$

$$= \frac{9}{7} \ln u$$

$$= 9 \cdot \frac{1}{7} \ln 7x+10$$

$$D) \int 5 \sin(x) \cos(x) dx$$

$$= 5 \int \sin(x) \cos(x) dx$$

$$= 5 \frac{u^{1+1}}{1+1}$$

$$= 5 \frac{\sin^{1+1}(x)}{1+1}$$

$$= \frac{5}{2} \sin^2(x)$$

$$= \frac{5}{2} \sin^2(x)$$

$$= \frac{5}{2} \sin^2(x) + C$$

$$e) \int 3x^2 \sqrt{x^3} dx \quad u = x^3$$

$$= 3 \cdot \int \frac{\sqrt{u}}{3} du$$

$$= 3 \cdot \frac{1}{3} \cdot \int \sqrt{u} du$$

$$= 3 \cdot \frac{1}{3} \cdot \int u^{\frac{1}{2}} du$$

$$= 3 \cdot \frac{1}{3} \cdot \frac{(x^3)^{\frac{1}{2}+1}}{\frac{1}{2}+1} = \frac{2}{3} x^{\frac{9}{2}}$$

$$= \frac{2}{3} x^{\frac{9}{2}}$$

$$= \frac{2}{3} x^{\frac{9}{2}} + C$$