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Design & Analysis of Extended Surfaces

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Abstract

This report investigates the application, evolution and optimisation of extended surfaces (fins) for enhanced thermal management. Using analytical methods, temperature distributions and heat loss were evaluated for four rod materials under varying fin conditions. The effects of fin tip boundary assumptions were evaluated, comparing original rod lengths to both optimum and infinite fin lengths.

The optimum fin length was determined using the adiabatic tip condition, while the infinite fin model was applied to assess the minimum required length for accurate temperature prediction. Results show that optimum lengths reduce material usage without compromising performance, whereas infinite fin lengths offer more accurate thermal profiles at the cost of increased size.

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1 Introduction

Efficient thermal management is a critical aspect of engineering design, influencing the performance and longevity of various mechanical and electronic systems. One widely used method to enhance heat dissipation is the addition of extended surfaces, commonly known as fins to increase the surface area available for heat transfer by convection (Roetzel, et al., 2020). The effectiveness of fins depends on multiple factors, including the material properties, geometry and boundary conditions.

This report explores the application and evolution of fin technology in automotive design, highlighting key historical advancements and the role of extended surfaces in modern microprocessors. Additionally, recent innovations in fin design are discussed focussing on the perceived advantages of these. In addition, the relationship between the fin parameter mL and heat transfer efficiency is analysed using developed calculations in MS Excel to determine the optimum fin length. Furthermore, a detail examination of temperature distribution and heat loss in rods of different materials is conducted to compare theoretical predictions with optimised solutions.

1.1 Aims & Objectives

1.1.1 Aim

This reports aims to analyse the evolution, applications and optimisation of extended surfaces in heat transfer. With a focus on automotive and microprocessor cooling and to evaluated fin performance for enhanced thermal management.

1.1.2 Objectives

This report has the following objectives:

- 1) Analyse the historical development of fins in automotive design and their modern applications in microprocessor cooling.
- 2) Evaluate the impact of different fin tip boundary conditions on heat transfer and assess heat loss variations across various materials and find lengths.
- 3) Use MS Excel to model and determine the optimal fin length based on the mL parameter and compare temperature distributions for different fin materials.
- 4) Optimise fin performance by identifying the optimum fin length for enhanced heat dissipation and improving thermal prediction using the infinite fin approximation.

2 Applications and Advancements of Extended Surfaces

2.1 Overview of Extended Surfaces

Heat transfer is a fundamental aspect of engineering design, particularly in systems where efficient thermal management is required. Extended surfaces, commonly referred to as fins are widely used to enhance heat dissipation by increasing the available surface area for heat exchange (Saha, et al., 2019). Since the early 20th century, fins have played a critical role in the thermal management of both large scale systems, such as automotive engines and also small scale devices such as microprocessors.

The principle extended surfaces is based on Fourier's law of heat conduction and Newton's law of cooling. Heat is conducted along the fin from the base where it is attached to a heat source and then dissipated into the fluid via convection along its surface. By extending the surface, a larger interface is provided for heat exchanging and therefore improves the system's ability to remove thermal energy (Bergman, et al., 2011).

2.2 Importance of Fins in Thermal Management Systems

Fins are essential components in thermal management systems due to their ability to enhance heat transfer by increasing the effective surface area for convection. In modern engineering often devices are designed to be more compact yet generate higher thermal loads, requiring more efficient cooling to maintain performance (Lasance, 2005). Fins help to prevent overheating which can lead to reduced efficiency, component degradation or at times system failure. In automotive applications, fins are vital for cooling engines, radiators, batteries and other critical part of an automobile. In a similar way, fins have also become critical in microprocessors being widely used as heat sinks in both microprocessors and power modules where stable operations are required despite rising power densities (Garimella, et al., 2008).

The importance of fins become even more apparent in applications where space and weight are constrained, such as electric vehicles, portable electronic devices and aerospace systems. Their ability to provide effective thermal management without significantly increasing the size or mass of components make them critical (Zhao & Wang, 2016). This is illustrated in Figure 1, which shows a schematic of a compact narrow heat sink.

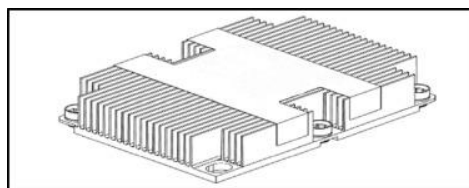


Figure 1: Schematic of a compact heat sink (Gore & Dhoble, 2023).

2.3 Historical Development of Fins in Automotive Design

The use of fins in automotive design dates back to the early 20th century where engineers sought effective methods to manage increasing thermal loads in vehicle engines. The primary goal was to enhance heat dissipation from engine components such as radiators, exhaust systems and power trains (Incropera, et al., 2007). In the 1920s and 1930s, the automotive industry began experimenting with extended surfaces such as finned radiators. The design improved the heat transfer by increasing the surface area for heat exchange, the introduction allowed for vehicles to operate more efficiently and reduce the risk of engine overheating (Gosman & Gaskell, 1992). The development allowed engines to run at higher temperatures, boosting the efficiency and also power output which could be achieved.

As time has progressed the emphasis has shifted more towards fuel efficiency and the use of smaller engines. Fins have now began to be integrated into more compact cooling systems such as turbocharger intercoolers and radiators. These fins have now become not only for the regulation of engine heat but also to regulate the heat generated by the increasing advancements in automotive electronics. A significant advancement has been the introduction of electrical vehicles, further emphasising the role of fins as battery thermal management becomes a crucial (Zhao & Wang, 2016).

2.4 Application of Extended Surfaces in Microprocessor Cooling

As microprocessors have evolved in performance, the heat they generate has posed increasing challenges for thermal management. The efficient dissipation of heat is critical as excessive temperatures can degrade performance, reduce lifespan and lead to component failure. Extended surfaces, in particular heat sinks have become critical to microprocessor cooling strategies (Saber & Maref, 2018). These heat sinks now enable devices to handle growing thermal loads as a result of increased computational power.

The principle behind heat sinks is that heat is transferred from the microprocessor to the extended surface of the heat sink through conduction. The larger surface area of the fins promotes heat dissipation into the surrounding air via convection (Bergman, et al., 2011). In recent years, the pursuit for more effective thermal management in microprocessors has led to the adoption of more advanced materials such as graphene, composites and carbon nanotubes (Mahmoud, et al., 2020). The increased use of 3D printing has also led to the ability to produce highly optimised and complex heat sink geometries, which would have been previously difficult or impossible with traditional methods which has led to improved cooling performance (Leung, et al., 2021).

2.5 Recent Advances in Extended Surface Design

The design of extend surfaces have advanced over the past decade, driven by the growing demand for efficient, compact and lightweight thermal management solutions. One advancement is the use of optimised fin geometries inspired by natural heat dissipation systems. These designs improve thermal performance by promoting more effective air flow across extended surfaces (Wang, et al., 2018). Figure 2 shows an example of a fin geometry which is a bio-inspired fin structure. The branching patterns increase the surface area while maintaining compactness leading to enhanced heat dissipation.

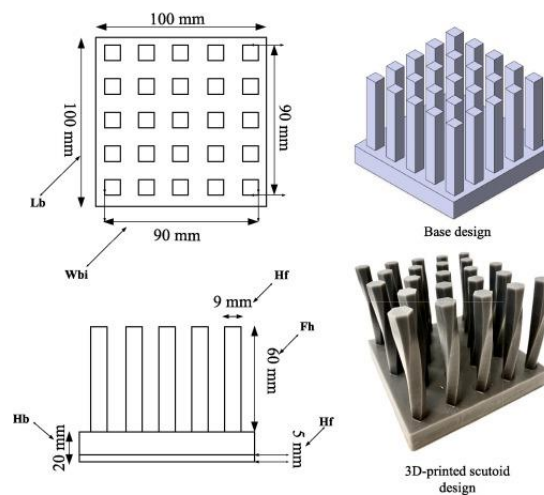


Figure 2: Optimised fin geometry with a bio-inspired fin structure (Harris, et al., 2024).

One further advancement is the use of surface micro-texturing which is where micro-grooves and surface roughness are engineered to disrupt boundary layers. This improves the convective heat transfer and delays flow separation (Mochizuki, et al., 2015). Combined with advanced manufacturing techniques, such as laser surface structuring and electrochemical etching, these approaches have been applied to compact heat exchangers and electronic cooling systems.

3 Optimising Fin Length with Adiabatic Tips

In thermal management, understanding the impact of fin length on heat transfer efficiency is crucial for optimising designs. One important parameter in this context is the fin parameter mL , which characterises the fin's geometry and thermal performance. For fins with an adiabatic tip boundary condition, it is generally considered that the fin parameter mL should not exceed approximately 2.65 for optimal heat transfer (Incropera, et al., 2007). Figure 3 illustrates the effect of mL on the heat transfer ratio, where the mL parameter is plotted on the x-axis ranging from 0 to 4 in increments of 0.1. This plot helps visualise the heat transfer

ratio changes with varying fin lengths and demonstrates the point at which the optimum fin length occurs at 2.65. Beyond this length, increasing the fin's length does not significantly improve heat transfer.

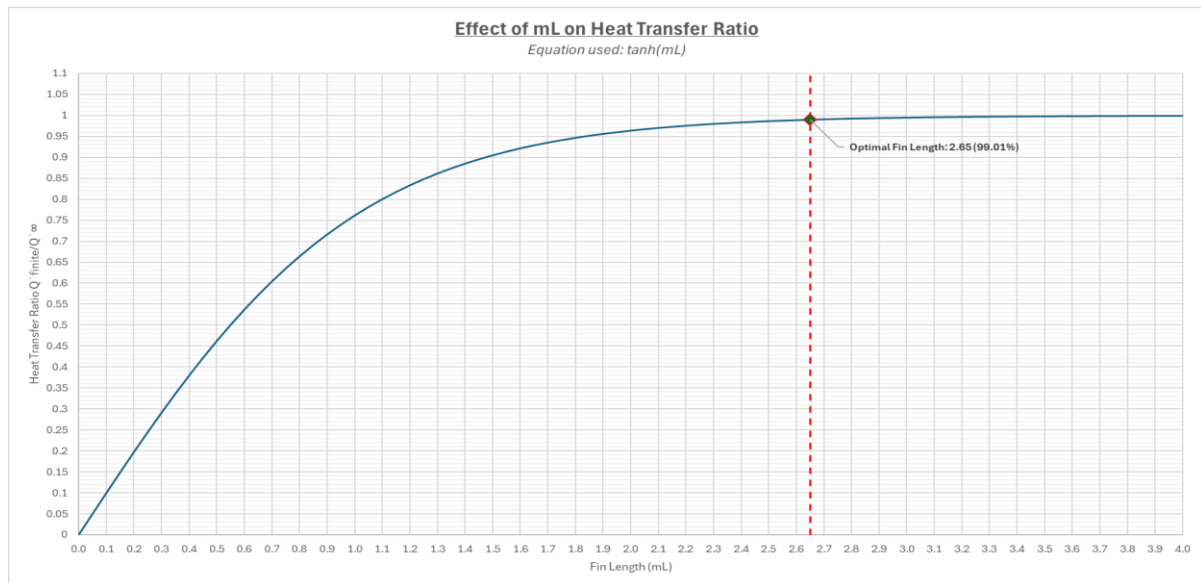


Figure 3: Effect of mL on Heat Transfer Ratio.

4 Thermal Analysis and Heat Loss Comparison

The thermal analysis and heat loss comparison of a very long rod of diameter (7.5mm diameter) attached to a heat surface maintained at 110°C and exposed to ambient air at 18°C has been carried out for four different materials. The properties of these materials are summarised in Table 1.

Material Name	Heat Transfer Coefficient (W/m ² .K)	Thermal Conductivity (W/m.K)
2024 Aluminium Alloy	30	186
Brass (70 Cu/30 Zn)	30	99
Copper	15	398
304 Stainless Steel	7.5	16

Table 1: Included materials in analysis and properties.

4.1 Temperature Distribution Along the Rod Materials

The temperature distribution along each of the four rod materials was evaluated using the theory of a uniform cross-section fin with an adiabatic tip boundary condition. The rods were analysed over a length of 0.4 m in increments of 10 mm, the resulting temperature profiles were plotted on a common graph to allow direct comparison between the materials.

The temperature distribution was calculated using Equation (1), derived from the adiabatic tip fin theory:

$$T(x) = T_{\infty} + (T_b - T_{\infty}) \frac{\cosh[m(L - x)]}{\cosh(mL)} \quad (Eq. 1)$$

Where:

$T(x)$: Temperature at position "x" on the rod.

T_b : Base temperature of the rod.

T_{∞} : Ambient temperature.

m: parameter dependant on material thermal conductivity, convection coefficient and geometry of the rod.

L: Length of the rod.

cosh: Hyperbolic cosine function.

Microsoft excel was used to implement Equation (1) and calculate the temperature distribution for each material. To ensure the reliability of the results, the Excel formula was validated through a manual calculation to confirm the accuracy of the code. The validation process, including the step by step calculations and a comparison is presented in Appendix A. The manual calculation returned the results consistent with the excel model.

Figure 4 plots the temperatures profiles of all four materials, this graph clearly highlight how the choice of fin material impacts the effectiveness of heat dissipation in thermal management systems. Materials with higher thermal conductivity, such as Copper retains more heat along the length of the rod. This results in a more gradual temperature drop, whereas materials such as 304 Stainless Steel shows a steep temperature decline due to having lower thermal conductivity.

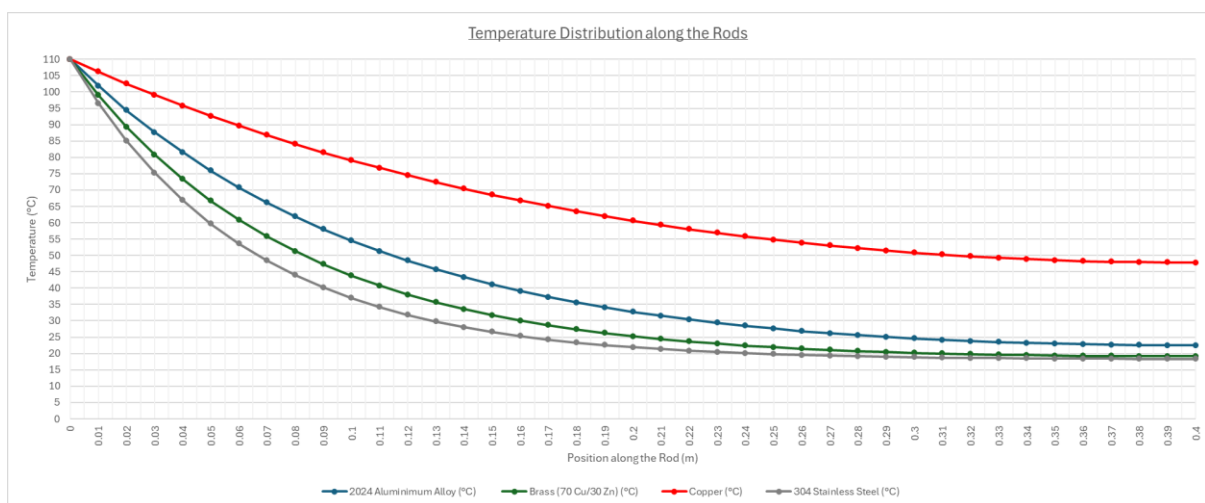


Figure 4: Temperature Distribution Along Rod Length of Four Different Materials.

4.2 Heat Loss Along the Rod Materials

The corresponding heat loss values for each of the four rod materials were determined. The heat loss was calculated using the derived Equation (2):

$$\dot{Q} = \sqrt{hPkA_c} \cdot (T_b - T_\infty) \cdot \tanh(mL) \quad (Eq. 2)$$

Where:

h : Heat transfer coefficient ($W/m^2.K$)

P : Perimeter of the rod.

k : Thermal conductivity of the material ($W/m.K$)

A_c : Cross-sectional area of the rod.

T_b : Base temperature of the rod.

T_∞ : Ambient temperature.

m : parameter dependant on material thermal conductivity, convection coefficient and geometry of the rod.

L : Length of the rod.

Microsoft excel was used to implement Equation (2) and calculate the heat loss for each material. To ensure the reliability of the results, the Excel formula was validated through a manual calculation to confirm the accuracy of the code. The validation process, including the step by step calculations and a comparison is presented in Appendix B. The manual calculation returned the results consistent with the excel model. The results are presented in Table 2.

Material Name	Heat Loss (W)
2024 Aluminium Alloy	7.0032
Brass (70 Cu/30 Zn)	5.1150
Copper	6.8617
304 Stainless Steel	1.0282

Table 2: Comparison of Heat Loss for Different Rod Materials.

4.3 Optimum Length Fin

The optimum lengths for the four rod materials were determined based on the assumption outlined in Section 3, using Equation (3):

$$L_{opt} = \frac{2.65}{m} \quad (Eq. 3)$$

This equation ensures that the rod meets the minimum length requirement to behave as optimum extended surfaces. Table 3 presents the calculated optimum lengths for each

material, along with the difference from the original rod length of 0.4 m and the corresponding percentage increase.

Material Name	Optimum Length Fin (m)	Difference (m)	Difference (%)
2024 Aluminium Alloy	0.2857	- 0.1143	- 28.57
Brass (70 Cu/30 Zn)	0.2085	- 0.1915	- 47.89
Copper	0.5911	+ 4.4834	+ 1020.84
304 Stainless Steel	0.1676	- 0.2474	- 58.10

Table 3: Optimum rod lengths by material.

The results in Table 3 demonstrate that the optimum rod length varies considerably between materials, largely due to differences in thermal conductivity. For three out of the four materials, the optimum length was shorter than the original 0.4 m indicating potential savings in material without compromising thermal performance.

To assess the impact of these length changes, a comparison was made between the heat loss at the original length and the heat loss at the optimum length using Equation (2). The outcomes are presented in Figure 5, where the solid bars represent heat loss as the original 0.4 m length and the lined bars show the corresponding heat loss at the optimum length.

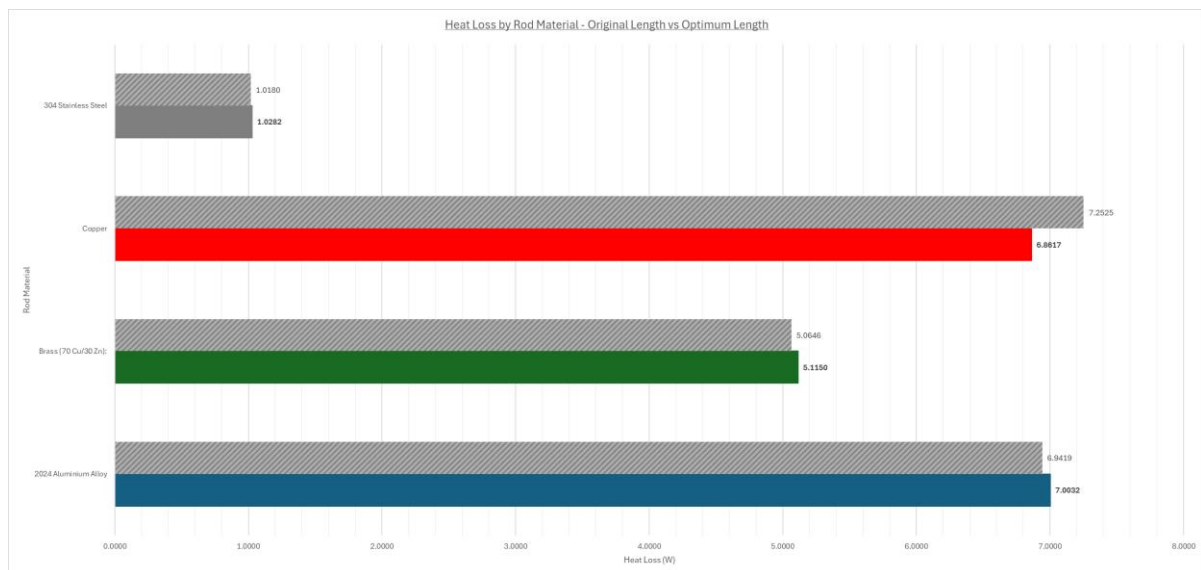


Figure 5: Heat Loss by Rod Material - Original vs Optimum Length.

The graph shows that for 2024 Aluminium Alloy and Brass, the optimum lengths are shorter than the original and only result in a slight reduction in heat loss. This suggests that the original rods were oversized and could be shorted without significantly impacting performance. Copper shows a substantial increase in heat loss due to the much longer optimum length indicating that the original 0.4 m was undersized.

4.4 Temperature Distribution Optimum Lengths

The temperature distribution along each four rod materials was recalculated using their optimum lengths applying Equation (1). This temperature profiles were generated over the full length of each rod, with calculated performed at 10 mm increments. The temperature profiles were plotted on a common graph as show in Figure 6.

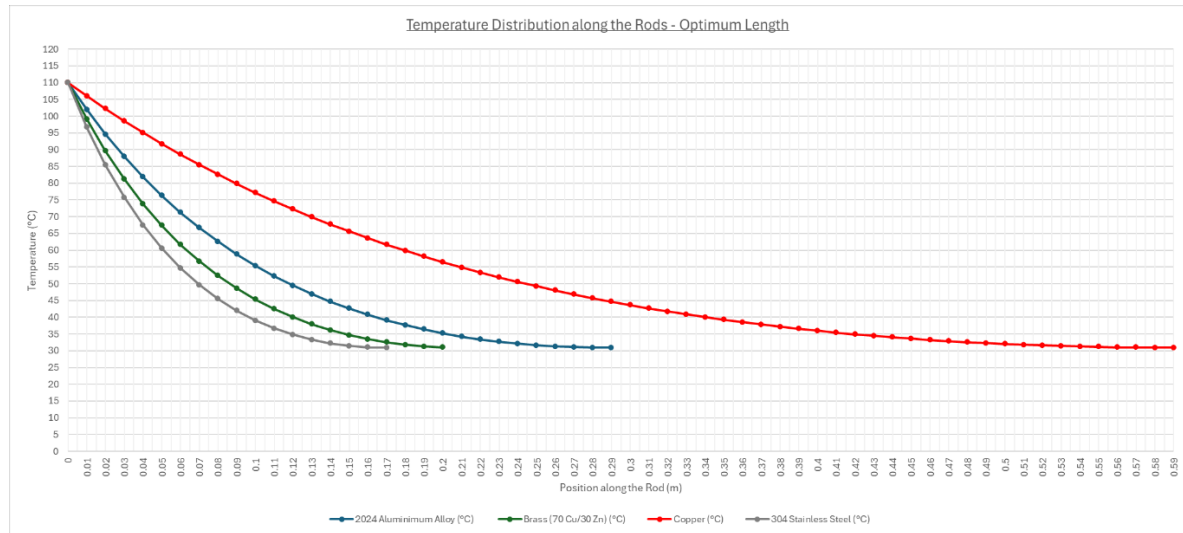


Figure 6: Temperature distribution along the Rods - Optimum Length.

4.5 Comparison of Infinite Fin Approximation

The condition for the infinite fin approximations was applied to each of the four rod materials to determine minimum required rod length. The infinite fin assumption requires the tip temperature to closely approach the ambient temperature. The minimum lengths necessary for each material were calculated based on their m values and the relationship:

$$e^{-mL} < 0.02$$

Therefore, Equation (4) was used to calculate the lengths stated in Table 4.

$$L = \frac{3.912}{m} \quad (Eq. 4)$$

Material Name	Infinite Fin Length (m)
2024 Aluminium Alloy	0.4218
Brass (70 Cu/30 Zn)	0.3077
Copper	0.8726
304 Stainless Steel	0.2474

Table 4: Infinite Fin Length.

The temperature distributions were recalculated for each rod using Equation (5) the infinite fin temperature equation:

$$T(x) = T_{\infty} + (T_b - T_{\infty})e^{-mx} \quad (Eq. 5)$$

Microsoft excel was used to implement Equation (5) and calculate the temperature distribution for each material. To ensure the reliability of the results, the Excel formula was validated through a manual calculation to confirm the accuracy of the code. The validation process, including the step by step calculations and a comparison is presented in Appendix C. The manual calculation returned the results consistent with the excel model.

4.5.1 2024 Aluminium Alloy Comparison

Figure 7 compares the temperature distribution along the 2024 Aluminium Alloy rod for the three different fin lengths. The original length 0.4 m, the optimum length 0.2857 m based on the adiabatic tip condition and the infinite fin length 0.4218 m based on the $mL > 3.912$ criteria. Graph shows that the optimum length is 29% shorter than the original offering efficient heat dissipation with reduced material usage.

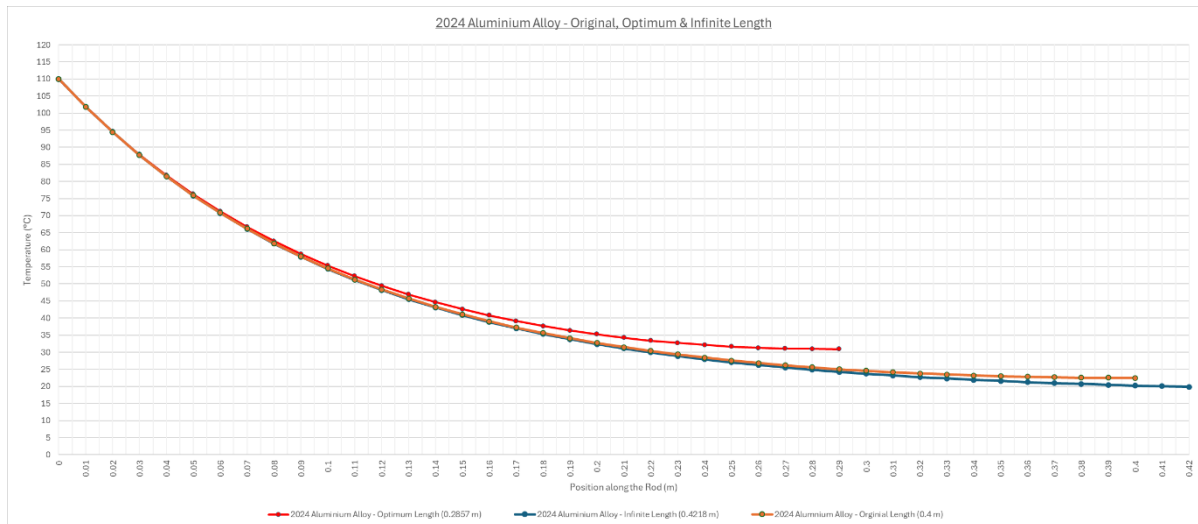


Figure 7: 2024 Aluminium Alloy - Original, Optimum and Infinite Temperature Distribution.

4.5.2 Brass (70 Cu/30 Zn) Comparison

Figure 8 compares the temperature distribution along the Brass (70 Cu/30 Zn) rod for the three different fin lengths. The original length 0.4 m, the optimum length 0.2085 m based on the adiabatic tip condition and the infinite fin length 0.3077 m based on the $mL > 3.912$ criteria. The optimum length is 48% shorter than the original rod, showing efficient heat dissipation. The infinite fin length is 28% shorter than the original with a more gradual temperature drop.

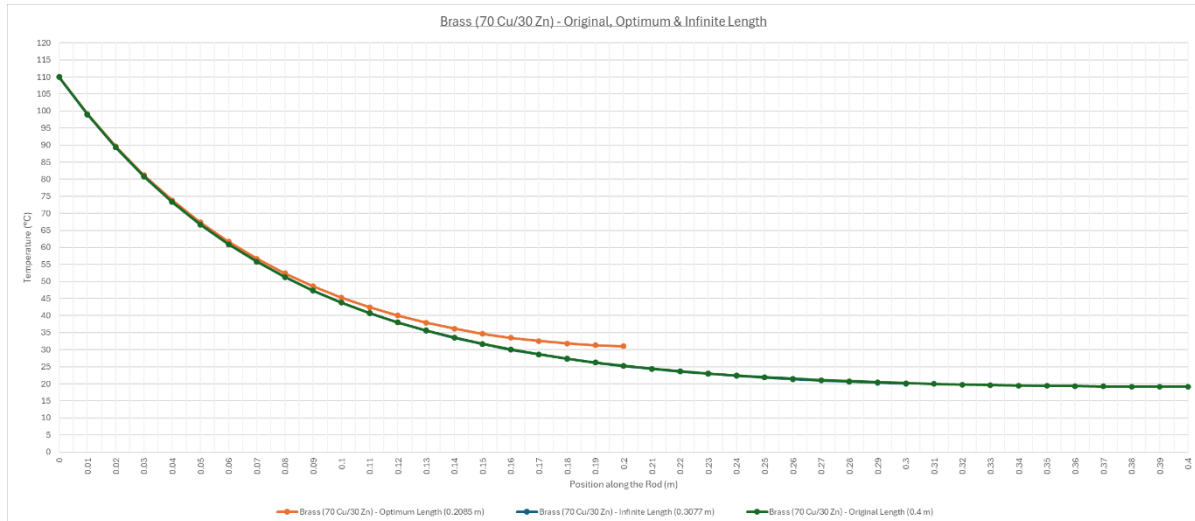


Figure 8: Brass (70 Cu/30 Zn) - Original, Optimum and Infinite Temperature Distribution.

4.5.3 Copper Comparison

Figure 9 compares the temperature distribution along the Copper rod for the three different fin lengths. The original length 0.4 m, the optimum length 0.5911 m based on the adiabatic tip condition and the infinite fin length 0.8726 m based on the $mL > 3.912$ criteria. The optimum length returned 48% longer than the original and provided a more accurate representation of heat dissipation. The infinite fin returned 118% longer than the original, indicating that the original rod was shorter than required.

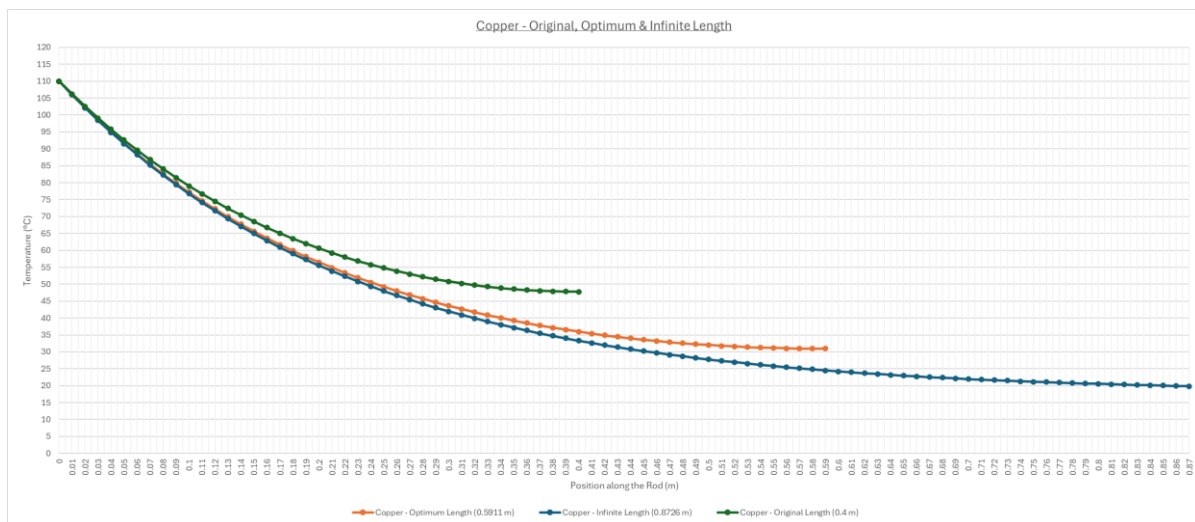


Figure 9: Copper - Original, Optimum and Infinite Temperature Distribution.

4.5.4 304 Stainless Steel Comparison

Figure 10 compares the temperature distribution along the 2024 Aluminium Alloy rod for the three different fin lengths. The original length 0.4 m, the optimum length 0.1676 m based on the adiabatic tip condition and the infinite fin length 0.2474 m based on the $mL > 3.912$

criteria. The temperature distribution for 304 Stainless Steel shows that shorter lengths are sufficient for this low-conductivity material to achieve effective heat dissipation. The optimum length of 0.1676 m is 58% shorter than the original 0.4 m rod, while the infinite fin length of 0.2474 m is still 38% shorter than the original. Both the optimum and infinite models show that most of the temperature drop occurs early along the rod, with diminishing returns beyond a certain length.



Figure 10: 304 Stainless Steel - Original, Optimum and Infinite Temperature Distribution.

5 Conclusion

In conclusion, this report has demonstrated how the performance of extended surfaces varies significantly with material properties, fin length and boundary conditions. The evaluation of both the temperature distribution and heat loss analysis of the four materials, found that the optimum fin length based on the adiabatic tip equation reduced material usage. The infinite fin equation, required longer rods than the optimum fin length in the majority of cases. The comparison between original, optimum and infinite fin lengths highlights that a one size fin length is not efficient. High conductivity materials such as copper benefit from extended lengths, whereas lower conductivity materials like the 304 Stainless Steel achieve effective performance in shorter fins.

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Appendices

Appendix A: Verification of MS Excel Temperature Distribution

Given Parameters	
Heat Transfer Coefficient ($W/m^2.K$)	186 $W/m^2.K$
Thermal Conductivity ($W/m.K$)	30 $W/m.K$
Base Temperature (T_b)	110 °C
Ambient Temperature (T_∞)	18 °C
Rod Diameter (d)	0.0075 m
Rod Length (L)	0.4 m
Position (x)	0.34 m

Calculate the cross sectional area (A_c)

$$A_c = \pi r^2$$

$$\pi(0.00375)^2$$

$$P = 4.418 \times 10^{-5} m^2$$

Calculate the perimeter (P)

$$P = \pi d$$

$$\pi \cdot 0.0075$$

$$P = 0.02356 m$$

Calculate m

$$m = \sqrt{\frac{hP}{kA_c}}$$

$$\sqrt{\frac{30 \cdot 0.02356}{186 \cdot 4.418 \times 10^{-5}}}$$

$$\sqrt{\frac{0.7068}{0.008218}}$$

$$m = 9.27$$

Use of the adiabatic tip equation

$$T(x) = T_\infty + (T_b - T_\infty) \frac{\cosh[m(L-x)]}{\cosh(mL)}$$

$$T(0.34) = 18 + (110 - 18) \frac{\cosh[9.27(0.4 - 0.34)]}{\cosh(9.27 \cdot 0.4)}$$

$$T(0.34) = 18 + 92 \cdot \frac{\cosh(0.556)}{\cosh(3.708)}$$

$$T = 23.22 ^\circ C$$

Excel Result @ 0.34 with the same parameters = **23.22 °C**

Appendix B: Verification of MS Excel Heat Loss

Given Parameters	
Heat Transfer Coefficient ($W/m^2.K$)	186 $W/m^2.K$
Thermal Conductivity ($W/m.K$)	30 $W/m.K$
Base Temperature (T_b)	110 $^{\circ}C$
Ambient Temperature (T_{∞})	18 $^{\circ}C$
Rod Diameter (d)	0.0075 m
Rod Length (L)	0.4 m
Position (x)	0.34 m
Perimeter (P) - From Temperature Distribution.	0.02356 m
Cross sectional area (A_c) - From temperature Distribution.	$4.418 \times 10^{-5} m^2$

Calculate m

$$m = \sqrt{\frac{hP}{kA_c}}$$

$$\sqrt{\frac{30 \cdot 0.02356}{186 \cdot 4.418 \times 10^{-5}}}$$

$$\sqrt{\frac{0.7068}{0.008218}}$$

$$m = 9.27$$

Use of heat loss equation

$$\dot{Q} = \sqrt{hPkA_c} \cdot (T_b - T_{\infty}) \cdot \tanh(mL)$$

$$\dot{Q} = \sqrt{30 \cdot 0.02356 \cdot 186 \cdot 4.418 \times 10^{-5}} \cdot (110 - 18) \cdot \tanh(9.27 \cdot 0.4)$$

$$\dot{Q} = \sqrt{0.00581} \cdot 92 \cdot \tanh(3.708)$$

$$\dot{Q} = 0.07625 \cdot 92 \cdot 0.9988$$

$$\dot{Q} = 7.00 W$$

Excel Result with the same parameters = **7.00 W**

Appendix C: Verification of MS Excel Infinite Fin Equation

Given Parameters	
Heat Transfer Coefficient ($W/m^2.K$)	186 $W/m^2.K$
Thermal Conductivity ($W/m.K$)	30 $W/m.K$
Base Temperature (T_b)	110 $^{\circ}C$
Ambient Temperature (T_{∞})	18 $^{\circ}C$
Rod Diameter (d)	0.0075 m
Rod Length (L)	0.4 m
Position (x)	0.34 m

Calculate the cross sectional area (A_c)

$$A_c = \pi r^2$$

$$\pi(0.00375)^2$$

$$P = 4.418 \times 10^{-5} m^2$$

Calculate the perimeter (P)

$$P = \pi d$$

$$\pi \cdot 0.0075$$

$$P = 0.02356 m$$

Calculate m

$$m = \sqrt{\frac{hP}{kA_c}}$$

$$\sqrt{\frac{30 \cdot 0.02356}{186 \cdot 4.418 \times 10^{-5}}}$$

$$\sqrt{\frac{0.7068}{0.008218}}$$

$$m = 9.27$$

Use of the infinite fin equation

$$T(x) = T_{\infty} + (T_b - T_{\infty})e^{-mx}$$

$$T(0.34) = 18 + (110 - 18)e^{-9.27 \cdot 0.34}$$

$$T = 21.92 ^{\circ}C$$

Excel Result @ 0.34 with the same parameters = **21.92 $^{\circ}C$**