

MSc Motorsport Engineering

Advanced Motorsport Dynamics

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Nomenclature

a distance from car's CoG to the front axle

b distance from car's CoG to the rear axle

c_f front spring damping

c_r rear spring damping

c_s suspension damping coefficient

c_{tf} front tire damping coefficient

c_{tr} rear tire damping coefficient

d track width

F_{arb} Vertical force applied to each wheel by the anti-roll bar

K_{arb} Anti-roll bar stiffness (Nm/rad)

k_f front spring stiffness

k_r rear spring stiffness

k_s suspension spring stiffness

k_t tire stiffness

m mass

M_{arb} Torque from anti-roll bar (N/m)

R_e effective radius

V Lineal velocity

V_y Lateral velocity

V_x Longitudinal velocity

$\dot{x}$ velocity of that component

$\ddot{x}$ Acceleration of that component

x_r road input (irregularities in road surface)

x_s vertical displacement of the sprung mass

x_u vertical displacement of the unsprung mass

y_f vertical displacement of the front wheels

y_r vertical displacement of the rear wheels

z vertical displacement of sprung mass

α weighting coefficient

α_s slip angle

β weighting coefficient

θ pitch angle

θ_L Left suspension deflections (radians)

θ_R right suspension deflections

σ slip ratio

ϕ roll angle

ω angular velocity

Introduction

Vehicle dynamics concern the response of vehicles to whatever driving condition they are exposed to and, as such, constitute one of the major study areas in automotive engineering. Recognising the necessity for analysis in the field of vehicle dynamics, to be pursued in optimisations about performance, handling, and safety ushers in the application of mathematical models able to represent complicated interactions of a vehicle's motion with forces.

Typical is the 7 Degrees of Freedom-7-DOF-vehicle dynamics model, which has been very useful in representing real driving scenarios. Major motions included in this model are longitudinal and lateral motions, vertical motions, roll, pitch, and yaw rotations, including wheel dynamics. The 7-DOF model integrates principles of suspension mechanics, mass distribution, and tire characteristics, enabling the engineer to study vehicle stability, handling performance, and dynamic responses under various conditions such as acceleration, braking, cornering.

The work presented here is dedicated to developing a 7-DOF vehicle dynamics model using the Simulink package, widely used within the automotive context for the study of vehicle behaviour under different conditions, key performance metrics assessment, and sensitivity analyses that are crucial for determination of the influence of critical parameters. An accuracy check of the model against theoretical expectations or experimental data will also be performed to ensure that it will be reliable for prediction in real-world responses. The proposal, again, will go further into the development and analysis of this model in insight into advanced vehicle dynamics, emphasising accuracy in modelling for motorsport engineering. It highlights the effectiveness of Simulink as a tool in vehicle performance simulation.

Developing a model

4 DOF

The 4-DOF Spring-Mass-Damper Model analyses the vertical dynamics of a vehicle by simulating ride comfort and road-holding ability. It considers the sprung mass (vehicle body) and unsprung masses (wheels), incorporating suspension and tire stiffness. The model demonstrates how a vehicle responds to road disturbances, helping in suspension tuning for optimal vibration isolation and stability. It analyses the vertical dynamics of a vehicle by simulating ride road-holding ability comfort and. It considers the sprung mass (vehicle body) and unsprung masses (wheels), incorporating suspension and tire stiffness. The model demonstrates how a vehicle responds to road disturbances, helping in suspension tuning for optimal vibration isolation and stability.

$$m_s \ddot{x}_s = -k_s(x_s - x_u) - c_s(\dot{x}_s - \dot{x}_u)$$

Each unsprung mass is influenced by the force from the suspension (acting upwards), and the force from the tire stiffness (acting downwards).

$$m_u \ddot{x}_u = -k_s(x_s - x_u) + c_s(\dot{x}_s - \dot{x}_u) - k_t(x_u - x_r)$$

Methodology

The basic model-7-degree-of-freedom vehicle model-describes the dynamics of a vehicle in heave, pitch, and roll, with additional DOF for the vertical displacement of each wheel. The model is normally used in ride and handling studies where suspension behaviour, load transfer, and body motions are analysed for various driving conditions. Below is the structured approach toward the development of a 7-DOF spring-mass-damper model, derived from basic dynamic equations.

Vertical motion of front sprung mass

$$-m_{f1}\ddot{z}_{f1} = k_{tf1}(z_{f1} - y_{f1}) - k_{f1}(z - z_{f1} - a\theta - \beta\phi) - c_{f1}(\dot{z} - \dot{z}_{f1} - a\dot{\theta} - \beta\dot{\phi}) + c_{tf1}(\dot{z}_{f1} - \dot{y}_{f1})$$

$$-m_{f2}\ddot{z}_{f2} = k_{tf2}(z_{f2} - y_{f2}) - k_{f2}(z - z_{f2} - a\theta - \alpha\phi) - c_{f2}(\dot{z} - \dot{z}_{f2} - a\dot{\theta} - \alpha\dot{\phi}) + c_{tf2}(\dot{z}_{f2} - \dot{y}_{f2})$$

Vertical motion of rear sprung mass

$$-m_{r1}\ddot{z}_{r1} = k_{tr1}(z_{r1} - y_{r1}) - k_{r1}(z - z_{r1} + b\theta - \beta\phi) - c_{r1}(\dot{z} - \dot{z}_{r1} + b\dot{\theta} - \beta\dot{\phi}) + c_{tr1}(\dot{z}_{r1} - \dot{y}_{r1})$$

$$-m_{r2}\ddot{z}_{r2} = k_{tr2}(z_{r2} - y_{r2}) - k_{r2}(z - z_{r2} + b\theta + \alpha\phi) - c_{r2}(\dot{z} - \dot{z}_{r2} + b\dot{\theta} - \alpha\dot{\phi}) + c_{tr2}(\dot{z}_{r2} - \dot{y}_{r2})$$

Heave

$$\begin{aligned} -m\ddot{z} = & k_{f1}(z - z_{f1} - a\theta - \beta\phi) + c_{f1}(\dot{z} - \dot{z}_{f1} - a\dot{\theta} - \beta\dot{\phi}) \\ & + k_{f2}(z - z_{f2} - a\theta - \alpha\phi) + c_{f2}(\dot{z} - \dot{z}_{f2} - a\dot{\theta} - \alpha\dot{\phi}) \\ & + k_{r1}(z - z_{r1} + b\theta - \beta\phi) + c_{r1}(\dot{z} - \dot{z}_{r1} + b\dot{\theta} - \beta\dot{\phi}) \\ & + k_{r2}(z - z_{r2} + b\theta + \alpha\phi) + c_{r2}(\dot{z} - \dot{z}_{r2} + b\dot{\theta} - \alpha\dot{\phi}) \end{aligned}$$

Roll Moment of Inertia

$$\begin{aligned} -I_{xx}\ddot{\theta} = & bk_{r1}(z - z_{r1} + b\theta - \beta\phi) + bc_{r1}(\dot{z} - \dot{z}_{r1} + b\dot{\theta} - \beta\dot{\phi}) \\ & + bk_{r2}(z - z_{r2} + b\theta + \alpha\phi) + bc_{r2}(\dot{z} - \dot{z}_{r2} + b\dot{\theta} - \alpha\dot{\phi}) \\ & - ak_{f1}(z - z_{f1} - a\theta - \beta\phi) - ac_{f1}(\dot{z} - \dot{z}_{f1} - a\dot{\theta} - \beta\dot{\phi}) \\ & - ak_{f2}(z - z_{f2} - a\theta - \alpha\phi) - ac_{f2}(\dot{z} - \dot{z}_{f2} - a\dot{\theta} - \alpha\dot{\phi}) \end{aligned}$$

Pitch Moment of Inertia

$$\begin{aligned} -I_{yy}\ddot{\phi} = & \alpha k_{f2}(z - z_{f2} - a\theta - \alpha\phi) + \alpha c_{f2}(\dot{z} - \dot{z}_{f2} - a\dot{\theta} - \alpha\dot{\phi}) \\ & + \alpha k_{r2}(z - z_{r2} + b\theta + \alpha\phi) + \alpha c_{r2}(\dot{z} - \dot{z}_{r2} + b\dot{\theta} - \alpha\dot{\phi}) \\ & - \beta k_{f1}(z - z_{f1} - a\theta - \beta\phi) - \beta c_{f1}(\dot{z} - \dot{z}_{f1} - a\dot{\theta} - \beta\dot{\phi}) \\ & - \beta k_{r1}(z - z_{r1} + b\theta - \beta\phi) - \beta c_{r1}(\dot{z} - \dot{z}_{r1} + b\dot{\theta} - \beta\dot{\phi}) \end{aligned}$$

Theoretical estimation using mass distribution

Calculations assume the car as a rigid body; chassis flex and suspension compliance are neglected for simplicity. Symmetry about the longitudinal axis is assumed for the pitch moment of inertia (I_{yy}) and about the lateral axis for the roll moment of inertia (I_{xx}). The mass distribution is approximated by treating components as simplified shapes and assuming left-right balance, front left and front right are symmetric.

Roll moment of inertia

$$I_{xx} = I_{xx}^{sprung} + I_{xx}^{unsprung}$$

$$I_{xx}^{sprung} = ms \left(\frac{T_f^2 + T_r^2}{12} + h_{CoG}^2 \right)$$

$$I_{xx}^{unsprung} = 2m_{usf} \left(\frac{T_f^2}{4} \right) + 2m_{usr} \left(\frac{T_r^2}{4} \right)$$

Pitch moment of inertia

$$I_{yy} = I_{yy}^{sprung} + I_{yy}^{unsprung}$$

$$I_{yy}^{sprung} = ms \left(\frac{w^2}{12} + h_{CoG}^2 \right)$$

$$I_{yy}^{unsprung} = 2m_{usf} \left(\frac{w_f^2}{4} \right) + 2m_{usr} \left(\frac{w_r^2}{4} \right)$$

$$w_f = 0.45w \text{ (front CG distance)}$$

$$w_r = 0.55w \text{ (rear CG distance)}$$

Using these calculations, with standard figures for the model a roll moment of inertia was calculated at $269.34 \text{ kg} \cdot \text{m}^2$ and a pitch moment of inertia (I_{yy}) of $277.71 \text{ kg} \cdot \text{m}^2$. Which were used in the Simulink model.

Anti-roll bar

Despite the common use of anti-roll bars in many Formula Ford chassis designs, there are instances where teams may omit or disconnect the bar entirely for testing or specific setup reasons. An anti-roll bar to isolate other suspension variables or establish a baseline vehicle setup. By eliminating its influence, engineers can more clearly observe the effects of spring rates, damper settings, and tires on overall ride and handling characteristics (Milliken & Milliken, 1995; Wong, 2008). In this case, the simulator Formula Ford was run without an anti-roll bar, so the corresponding 7-DOF Simulink model was built to reflect the same hardware configuration. By excluding the anti-roll bar from the model, it aligns with the actual test vehicle's suspension system, allowing for direct correlation between simulated and real-world data. While this choice inevitably affects the car's roll stiffness and handling characteristics—especially through corners—the objective in this scenario is to capture the tested setup as faithfully as possible. Any discrepancies in roll behaviour or lateral load transfer should be interpreted considering the absent anti-roll bar, highlighting the model's ability to replicate the specific dynamics of a car that is deliberately run without this common suspension component.

If the simulator test includes an anti-roll bar in the setup, it could be added to each wheel with the following equation in Simulink.

$$F_{arb} = \frac{K_{arb}}{d}(\theta_L - \theta_R)$$

$$M_{arb} = K_{arb}(\theta_L - \theta_R)$$

9 DOF and higher

The 7 DOF model can be expanded upon to increase accuracy as it can be used to analyse suspension deflection or unsprung mass behaviour. The equations for unsprung mass heave for front and rear wheels are added to expand on the 7 DOF model in the 9 DOF model.

9 DOF unsprung mass

$$m_{uf}\ddot{z}_{uf} = -k_{tf}(z_{uf} - z_f) - c_{tf}(\dot{z}_{uf} - \dot{z}_f)$$

$$m_{ur}\ddot{z}_{ur} = -k_{tr}(z_{ur} - z_r) - c_{tr}(\dot{z}_{ur} - \dot{z}_r)$$

The yaw moment of inertia (often denoted I_{zz}) represents how resistant a vehicle is to changes in its rotational speed around the vertical axis. In a Simulink model—or any dynamic vehicle simulation accurately defining I_{zz} is essential for capturing how quickly and faithfully the vehicle responds to steering inputs, external disturbances, or changes in tire forces (Milliken & Milliken, 1995; Gillespie, 1992). This parameter directly affects transient dynamics (e.g., rapid lane changes, corner entry, and exit) and steady-state behaviour, influencing how the vehicle's understeer or oversteer characteristics develop over time (Rajamani, 2006). An incorrect yaw moment of inertia will cause the simulated yaw rates, slip angles, and lateral load transfers to deviate from real-world data, leading to misleading conclusions about the vehicle's handling performance and overall stability (Blundell & Harty, 2004). Therefore, for simulations aiming to match logged data (e.g., from a data acquisition system like MoTeC), carefully measuring or estimating the yaw inertia is critical to achieving high-fidelity results (Jazar, 2017; Reimpell & Stoll, 2001).

10 DOF Yaw moment of inertia

To expand on and make the model more accurate, yaw can be calculated and used with the following formula.

$$\begin{aligned} I_{zz}\ddot{\psi} = & \alpha k_f 2(z - z_f 2 - a\theta - \alpha\phi) + \alpha c_f 2(\dot{z} - \dot{z}_f 2 - a\dot{\theta} - \alpha\dot{\phi}) \\ & + \alpha k_r 2(z - z_r 2 + b\theta + \alpha\phi) + \alpha c_r 2(\dot{z} - \dot{z}_r 2 + b\dot{\theta} + \alpha\dot{\phi}) \\ & - \beta k_f 1(z - z_f 1 - a\theta - \beta\phi) - \beta c_f 1(\dot{z} - \dot{z}_f 1 - a\dot{\theta} - \beta\dot{\phi}) \\ & - \beta k_r 1(z - z_r 1 + b\theta - \beta\phi) - \beta c_r 1(\dot{z} - \dot{z}_r 1 + b\dot{\theta} - \beta\dot{\phi}) \\ & + b k_r 1(z - z_r 1 + b\theta - \beta\phi) + b c_r 1(\dot{z} - \dot{z}_r 1 + b\dot{\theta} - \beta\dot{\phi}) \\ & + b k_r 2(z - z_r 2 + b\theta + \alpha\phi) + b c_r 2(\dot{z} - \dot{z}_r 2 + b\dot{\theta} + \alpha\dot{\phi}) \\ & - a k_f 1(z - z_f 1 - a\theta - \beta\phi) - a c_f 1(\dot{z} - \dot{z}_f 1 - a\dot{\theta} - \beta\dot{\phi}) \\ & - a k_f 2(z - z_f 2 - a\theta - \alpha\phi) - a c_f 2(\dot{z} - \dot{z}_f 2 - a\dot{\theta} - \alpha\dot{\phi}) \end{aligned}$$

Simulink model

This 7-DOF Simulink model is used for the creation and simulation of vehicle dynamics data, modelling key parameters such as lateral acceleration (Lateral G), vertical displacement (z), pitch (θ), roll (Φ), damper velocities, and tire forces (y_f , Z_{f1} , Z_{f2} , Z_{r1} , Z_{r2}). The model captures heave, pitch, and roll motions along with vertical wheel displacements, enabling the analysis of suspension behaviour, stability, and ride dynamics. It solves system equations to virtually simulate real-life responses due to road inputs and driving manoeuvres; it provides necessary data for performance evaluation and optimisation.

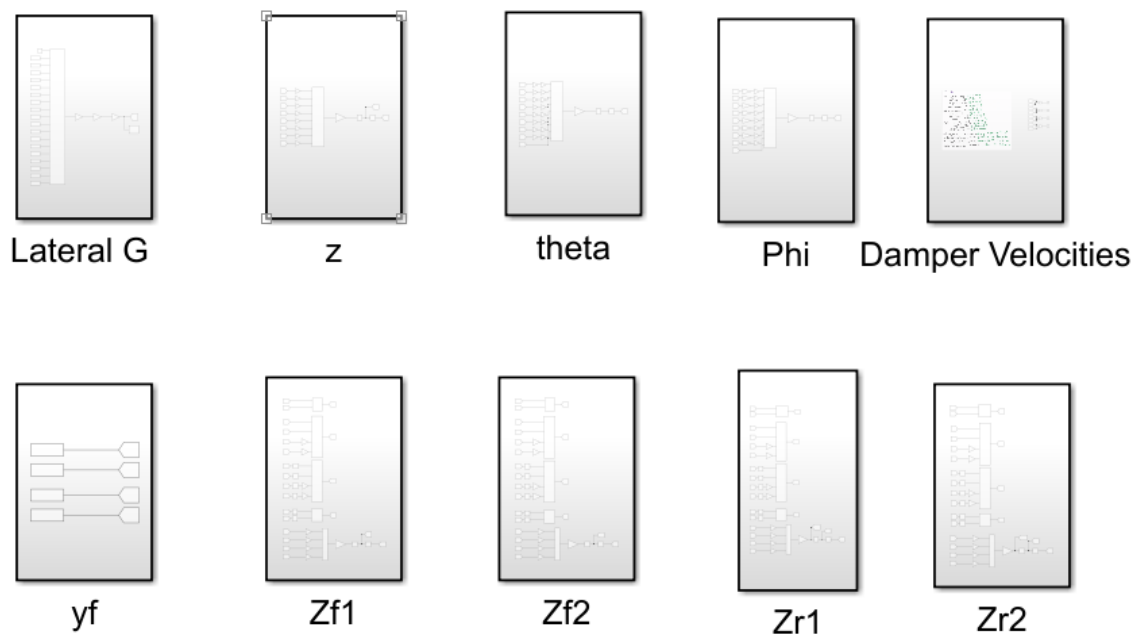


Figure 1 - 7 DOF Simulink model

The figures used in the model are from the below table.

Vehicle mass (kg)	M_T	514
Front corner Unsprung mass (kg)	M_{usf}	22
Rear corner Unsprung mass (kg)	M_{usr}	22
Vehicle sprung mass	M_s	426
Centre of Gravity Height (m)	CoG	0.277
Weight split F: R (%)	-	45:55
Wheelbase (m)	W	2.443
Front track (m)	T_f	1.62
Rear track (m)	T_r	1.57
Front tyre rolling radius (m)	r_f	0.2667
Rear tyre rolling radius (m)	r_r	0.2794
Front motion ratio	MR_f	1
Rear motion ratio	MR_r	1
Front Default Spring rate (N/mm)	K_{sf}	18
Rear Default Spring rate (N/mm)	K_{sr}	16
Front roll centre height (mm)	h_f	0.9
Rear roll centre height (mm)	h_r	59
Front slow bump	N/m/s	1660
Front fast bump	N/m/s	511
Front slow rebound	N/m/s	1915
Front fast rebound	N/m/s	511
Rear slow bump	N/m/s	1477
Rear fast bump	N/m/s	537
Rear slow rebound	N/m/s	1612
Rear fast rebound	N/m/s	537

Limitations of the Simulink model

A 7-Degree of Freedom (7-DOF) vehicle model typically includes the vehicle's heave (vertical displacement), roll, pitch, and yaw motions, along with the rotational degrees of freedom of the four wheels. In many cases, the vehicle body is assumed to be a rigid mass, and the suspension system is represented by linear (or mildly non-linear) spring-damper elements. Additionally, tire dynamics are frequently simplified by using linear stiffness and ignoring tire damping, while road irregularities are disregarded by assuming a flat, even surface (Blundell & Harty, 2004). These simplifications make the model more tractable and computationally efficient, providing a useful tool for preliminary studies or real-time simulations involving ride and handling.

Despite its practical advantages, the 7-DOF approach entails key limitations. When the suspension components and tire-road interactions are treated as linear, important non-linear characteristics such as combined lateral and longitudinal slip, transient relaxation, and hysteresis are not fully captured (Milliken & Milliken, 1995). This may eventually result in deviations between the outputs of the model and the response of the vehicle in real life, especially at extreme manoeuvres or on uneven terrains. Besides, the assumption of a single lumped unsprung mass at each wheel corner might be sufficient in moderate manoeuvres but fails to provide adequate or even actual dynamic responses when real-world suspension geometry—for example, bump steer and camber gain—and compliance become important, such as during high-speed lane changes (Wong, 2008).

By contrast, the Pacejka “Magic Formula” tire model offers a more comprehensive, empirically driven framework. It incorporates parameter sets that closely match real tire data across various slip angles, loads, and friction conditions (Pacejka, 2012). This makes it more accurate for simulating high-performance manoeuvres or predicting transient forces where lateral, longitudinal, and camber influences overlap (Rajamani, 2006). Consequently, while a 7-DOF model can be instructive in demonstrating basic vehicle dynamics and can be implemented comparatively easily, it cannot capture complex tire force-and-moment behaviour under combined slip conditions, nor can it represent detailed thermal and hysteresis effects. Further refinements, which may include the addition of Pacejka-based tire equations or more sophisticated multi-body suspension modelling, are generally needed to provide realistic high-fidelity simulations

Simulator work

Modelling differences

In vehicle dynamics, the 7-Degree of Freedom (7-DOF) Spring-Mass-Damper Model and the Pacejka Tire Model (Magic Formula) both play important roles, yet they monitor different aspects of a vehicle's motion. The 7-DOF model is good for ride comfort and handling characteristics, as it represents vertical, pitch, and roll movements of the vehicle body, added to the vertical displacements of each wheel (Blundell & Harty, 2004). It helps in thorough analyses of the dynamics of the suspension and the transfer of load under different conditions. For example, Subari et al. 2016 developed and tested a 7-DOF model for armoured vehicle lateral behaviours, indicating how complex the responses of the vehicle can be simulated.

Despite its utility, the 7-DOF model often assumes linear spring–damper elements and a rigid body, potentially overlooking real-world non-linearities such as bushing compliance, suspension kinematics, and road irregularities (Milliken & Milliken, 1995; Wong, 2008). It may also rely on simplified or linearised tire models, which can limit accuracy in combined slip or limit-handling situations. Nevertheless, the relative computational efficiency of the 7-DOF framework makes it suitable for real-time applications and preliminary feasibility studies (Gillespie, 1992).

By contrast, the Pacejka Tire Model focuses on tire–road interaction, providing empirically derived formulas that accurately predict tire forces and moments across various slip angles, slip ratios, and loads (Pacejka, 2012). This high-fidelity representation is critical for limit-handling manoeuvres—such as aggressive cornering or emergency braking—where tire slip and friction transitions dominate. Its empirical nature, however, demands extensive parameter identification for each tire type and operating condition (Rajamani, 2006). Despite these requirements, comparative studies confirm the accuracy of Pacejka's Magic Formula in replicating tire forces under different slip conditions (Fojtík & Bačík, 2021).

Integrating the Pacejka Model into a 7-DOF framework substantially enhances simulation fidelity, particularly in demanding manoeuvres like sudden lane changes or evasive actions, where both vehicle body dynamics and tire forces play crucial roles (A Preliminary Study for the Comparison of Different Pacejka Formulations Towards Vehicle Dynamics Behaviour, 2020). This synergy addresses the limitations of simpler tire representations in the 7-DOF model and the omission of suspension/body dynamics in pure Pacejka approaches. Nonetheless, the combined model may be computationally intensive, especially if advanced suspension geometry and high-frequency tire effects (e.g., transient relaxation) are included. Hence, balancing fidelity and computational cost is essential when applying these models to real-time systems, driver-in-the-loop simulators, or extensive parametric studies.

A multibody simulation (MBS) approach provides a more comprehensive and flexible framework compared to simplified lumped-parameter models like the 7-DOF or standalone tire models. In MBS, the vehicle and its subsystems (chassis, suspension links, steering components, etc.) are represented as

multiple interconnected rigid or flexible bodies, enabling detailed analysis of complex kinematic relationships, bushing compliance, and nonlinear geometry throughout the full range of motion (Blundell & Harty, 2004; Shabana, 2020). This level of fidelity allows engineers to study phenomena such as camber gain, bump steer, and dynamic toe changes, which are often oversimplified or omitted in more basic vehicle models. Consequently, MBS can yield highly accurate predictions of handling, ride comfort, and load transfer under diverse operating conditions. However, the added computational cost and modelling complexity require careful parameterisation, extensive testing data, and more robust numerical methods factors that can pose practical challenges for real-time or large-scale optimisation scenarios. Nonetheless, for in-depth design and analysis of modern vehicles where suspension geometry, compliance characteristics, and tire interactions play pivotal roles multibody simulation has become an indispensable tool (Rajamani, 2006).

The Pacejka Tire Model, often called the Magic Formula, provides a highly adaptable, semi-empirical approach to describing the non-linear behaviour of tires under different slip angles, slip ratios, and normal loads (Pacejka, 2012). By fitting a series of parameters—commonly referred to as stiffness, shape, and peak factors—to measured tire data, the model can accurately represent real-world tire forces and moments for a wide range of driving conditions (Bakker et al., 1987). Consequently, it is well suited to both steady-state handling analyses (e.g., cornering balance) and transient manoeuvres (e.g., sudden acceleration or braking), making it a standard in vehicle dynamics simulations. Its effectiveness extends to advanced multibody frameworks, where capturing nuanced tire-road interactions is pivotal for predicting limit behaviour and optimising chassis performance (Milliken & Milliken, 1995; Guiggiani, 2018). However, the empirical nature of the model requires parameter identification for each type of tire and operating condition, which in turn requires sound experimental data for accurate, high-fidelity simulations (Fojtik & Baík, 2021; Rajamani, 2006).

Pacejka Model

The Pacejka Model, also known as the Magic Formula, is an empirical tire model designed to predict the nonlinear forces and moments generated by tires under varying slip angles, slip ratios, and normal loads (Pacejka, 2012). It achieves this by fitting experimental data to a characteristic “magic” equation whose parameters often referred to as stiffness, shape, and peak factors—accurately replicate the real-world behaviour of tires at low slip and near their grip limits (Bakker, Nyborg, & Pacejka, 1987). Unlike simpler linear tire models, the Pacejka Model captures key effects such as combined slip (lateral and longitudinal), camber thrust, and force saturation, making it indispensable for vehicle dynamics simulations that demand realism in high-performance manoeuvres (Milliken & Milliken, 1995). Its flexibility and accuracy in reflecting measured tire data make it a standard in motorsport and automotive engineering, where understanding and optimising tire characteristics is crucial to improving cornering, braking, and overall handling performance (Rajamani, 2006). Pacejka formula is a function $y = f(x)$ which is used to predict and simulate forces developed by a tire, given an input x and a list of coefficients, which can be seen in the formula below.

$$Y(X) = D \cdot \sin(C \cdot \arctan(B \cdot X - E \cdot (B \cdot X - \arctan(B \cdot X))))$$

Where:

Y (X) Output force or moment (lateral, longitudinal force or aligning torque)

X input variable (slip angle or slip ratio)

B Stiffness factor (shape of the curve at small slips)

C Shape factor (general shape of the curve)

D Peak factor (scales the maximum force)

E Curvature Factor (affects the ‘fall-off’ of forces at high slip values)

Slip ratio / slip angle respectively

$$\sigma = \frac{\omega R_e - V}{|V|}$$

$$\alpha_s = \tan^{-1} \left(\frac{V_y}{V_x} \right)$$

Results

Simulator

Below is the first run, with the vehicle parameters that were listed. The slalom takes place within the 0-~550 index. The slight fluctuation in longitudinal G-force is to do with the pit limiter control strategy, both ignition and fuel could produce oscillations as seen in figures 2 and 3.

All the plots together tell a coherent story of a repeated side-to-side slalom manoeuvre. The lateral G-force data show prominent sine-wave swings, indicating consistent transitions between left and right cornering with peaks around ± 0.5 g. In contrast, the longitudinal G-force remains near zero most of the time, aside from a mild burst of positive acceleration and a short negative-G dip (likely braking).

These lateral oscillations manifest clearly in the tire load plots, where the outside tires carry more load during each turn while the inside tires' loads dip accordingly. Finally, the scatter plot of roll angle versus lateral G strongly underscores how closely body roll tracks lateral acceleration: as G-force increases in either direction, the roll angle grows proportionally. Overall, the data depict a cleanly executed slalom with predictable lateral weight transfers and minimal longitudinal effects.

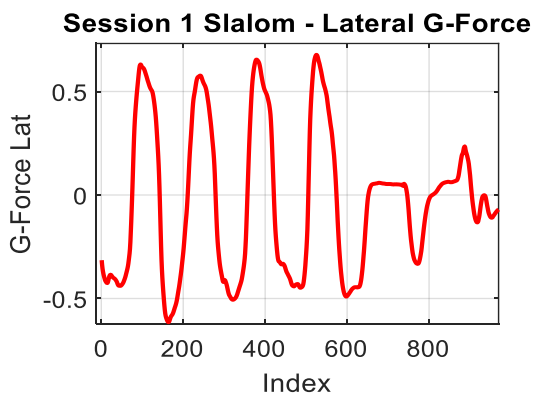


Figure 2 - Session 1 Slalom Lateral G-Force

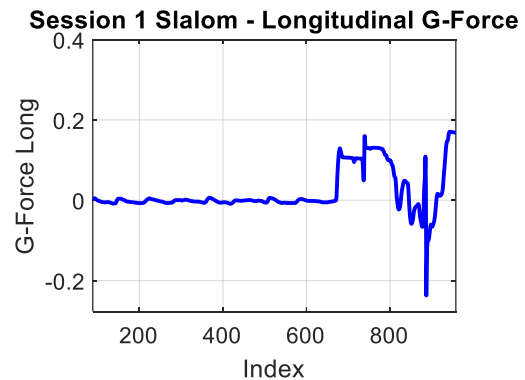


Figure 3 - Session 1 Slalom Longitudinal G-Force

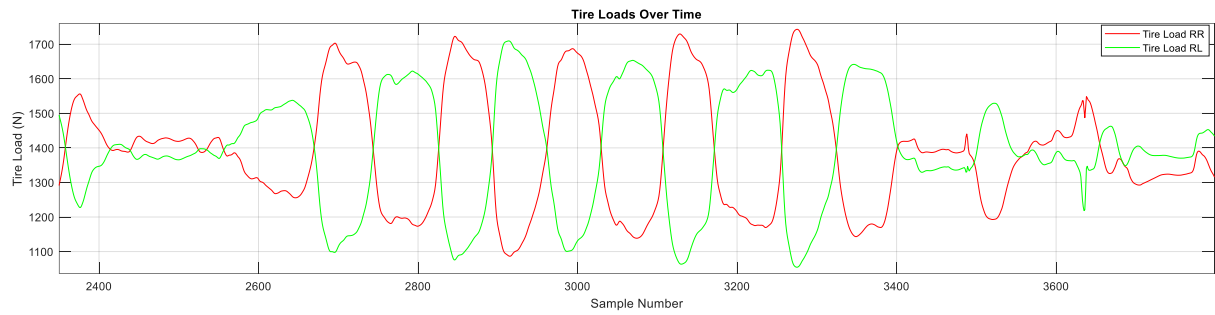


Figure 4 - Session 1 Rear tire loads

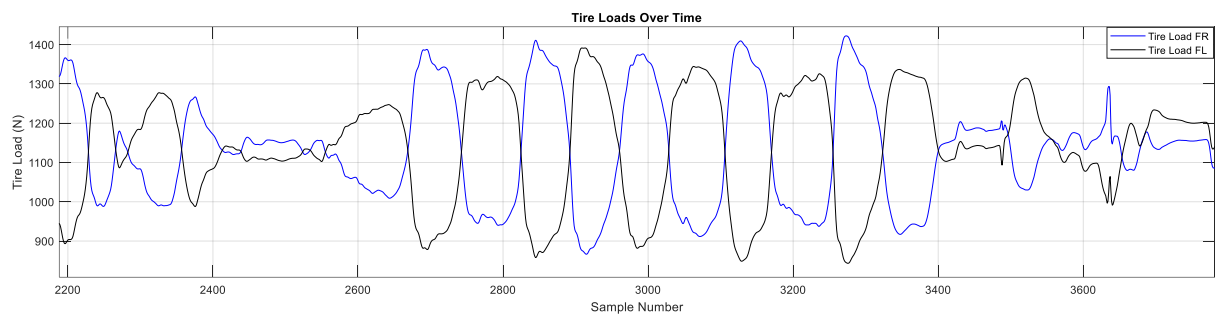


Figure 5 - Session 1 Front tire loads during slalom

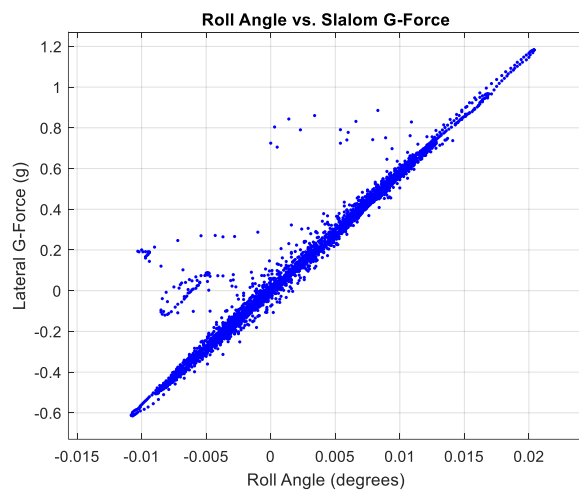


Figure 6 - Session 1 Roll Angle vs Slalom G-Force

For the stiffer than standard springs run, the Simulink model underestimates the lateral G-force compared to the MoTeC data. The underestimation of lateral G-forces in a 7-DOF Simulink model compared to MoTeC simulation data likely stems from simplified tire modelling, lack of aerodynamics, oversimplified suspension behaviour, and idealised assumptions about weight transfer and road-tire interaction. Enhancing the model with Pacejka's tire equations, aerodynamics, and nonlinear suspension geometry would help bridge the gap between Simulink's predictions and the simulator results. A similar underestimation can be seen with the tire loads from the slalom test due to the same reasons.

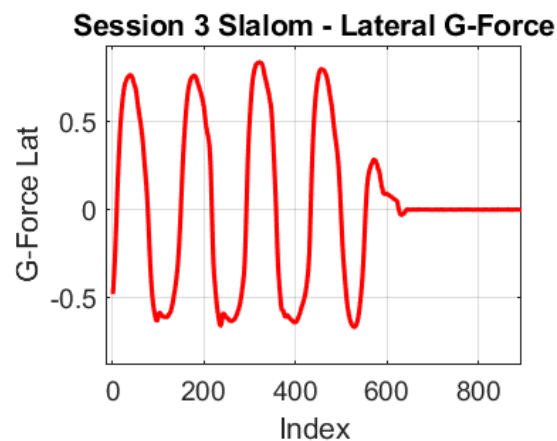


Figure 7 - Session 3 slalom later G-Force

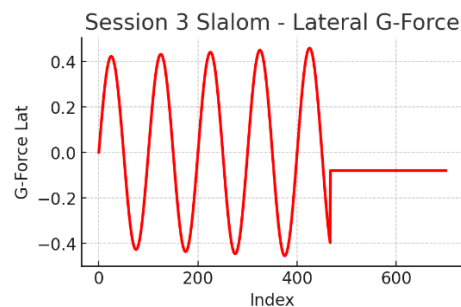


Figure 8 - Session 3 slalom lateral G-Force simulink

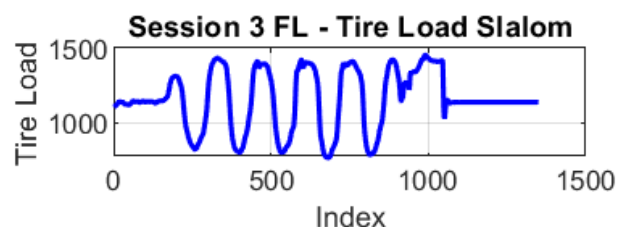


Figure 9 - Season 3 Slalom tire load

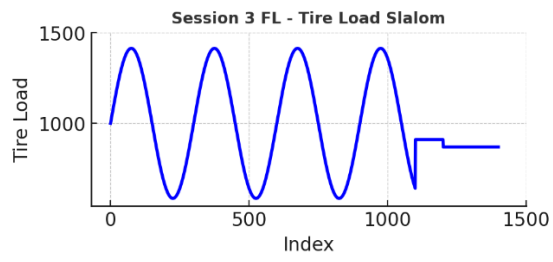


Figure 10 - Session 3 tire load slalom Simulink

Conclusion

The development and simulation of the 7-DOF vehicle dynamics model provide a valuable framework for understanding the complex behaviour of vehicles under various driving conditions. While simpler models, such as the 4-DOF model, are useful for basic vertical motion analysis, they lack the necessary detail to accurately represent real-world vehicle handling. In contrast, the 7-DOF model effectively captures additional roll and pitch dynamics, making it more suitable for advanced vehicle performance analysis.

A key challenge in vehicle modelling is the accurate representation of tire-road interactions. To enhance simulation accuracy, integrating the Pacejka Tire Model (Magic Formula) significantly improves predictions of tire forces, particularly in high-performance driving scenarios. The combination of the 7-DOF framework with the Pacejka Model enables a more holistic understanding of vehicle behaviour, bridging the gap between theoretical models and real-world performance.

The Simulink-based simulation successfully demonstrated the importance of key vehicle parameters, including suspension stiffness, roll moment of inertia, and tire characteristics. However, discrepancies between the Simulink model and MoTeC simulator data highlight the limitations of simplified assumptions in tire modelling and aerodynamics. Further refinement, such as incorporating nonlinear suspension characteristics and aerodynamic effects, could improve the accuracy of future simulations.

In conclusion, this study underscores the importance of advanced modelling techniques in motorsport engineering. By refining and validating vehicle dynamics models, engineers can enhance vehicle performance, improve safety, and optimise handling characteristics. Future work should focus on incorporating more complex tire models, real-time data integration, and multi-body simulations to achieve even greater accuracy in vehicle dynamics research.

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