

School of Engineering

Introduction to Vehicle Transient Dynamics Moment of Inertia

Rotation – Linear Parallels

Linear Motion

Rotational Motion

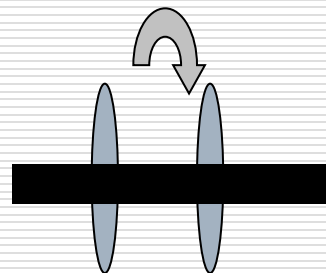
Position	x	θ	Angular position
Velocity	v	ω	Angular velocity
Acceleration	a	α	Angular acceleration
Motion equations	$x = \bar{v}t$	$\theta = \bar{\omega}t$	Motion equations
	$v = v_0 + at$	$\omega = \omega_0 + \alpha t$	
	$x = v_0t + \frac{1}{2}at^2$	$\theta = \omega_0t + \frac{1}{2}\alpha t^2$	
	$v^2 = v_0^2 + 2ax$	$\omega^2 = \omega_0^2 + 2\alpha\theta$	
Mass (linear inertia)	m	I	Moment of inertia
Newton's second law	$F = ma$	$\tau = I\alpha$	Newton's second law
Momentum	$p = mv$	$L = I\omega$	Angular momentum
Work	Fd	$\tau\theta$	Work
Kinetic energy	$\frac{1}{2}mv^2$	$\frac{1}{2}I\omega^2$	Kinetic energy
Power	Fv	$\tau\omega$	Power

Capri Accelerating

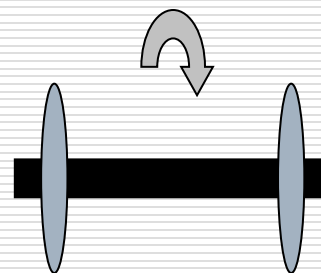


moment of inertia

- ❑ Force required to twist long dumbbell is higher than required for the short dumbbell
- ❑ Longer dumbbell has higher inertia
- ❑ Same force is required to lift both dumbbells
- ❑ Both dumbbells are the same mass
- ❑ Need to know mass and inertia properties to fully understand system behaviour



Short Dumbbell
Easy to Twist



Long Dumbbell
Hard to Twist

Spinning Skater

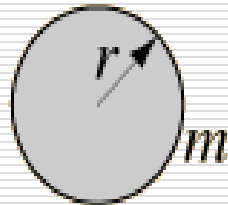


Moment of Inertia



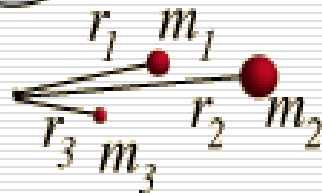
$$I = mr^2$$

For a point mass the moment of inertia is just the mass times the radius from the axis squared. For a collection of point masses (below) the moment of inertia is just the sum for the masses.



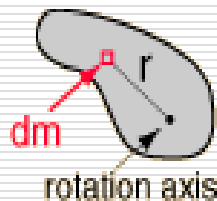
$$I = kmr^2$$

For an object with an axis of symmetry, the moment of inertia is some fraction of that which it would have if all the mass were at the radius r .



$$I = \sum_i m_i r_i^2 = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + \dots$$

Sum of the point mass moments of inertia.



$$I = \int_0^M r^2 dm$$

Continuous mass distributions require an infinite sum of all the point mass moments which make up the whole. This is accomplished by an integration over all the mass.

Linear $F = ma$

Newton's Second
Law

Angular $\tau = I\alpha$

Linear $p = mv$

Momentum

Angular $L = I\omega$

Moment
of Inertia
 I

Linear $KE = \frac{1}{2}mv^2$

Kinetic Energy

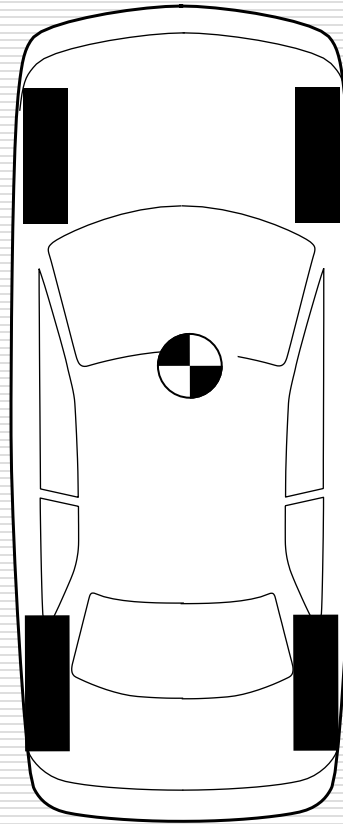
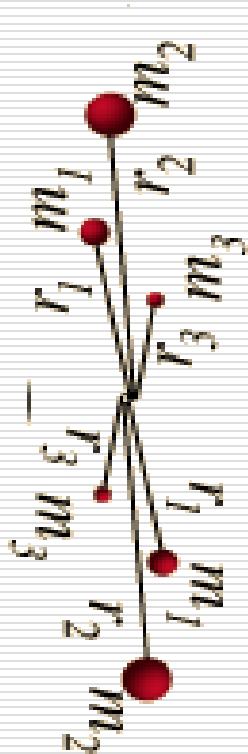
Angular $KE = \frac{1}{2}I\omega^2$

Linear $F_{net}d = \Delta \left(\frac{1}{2}mv^2 \right)$

Work-Energy

Angular $\tau_{net}\theta = \Delta \left(\frac{1}{2}I\omega^2 \right)$

Moment of Inertia



Formula Student

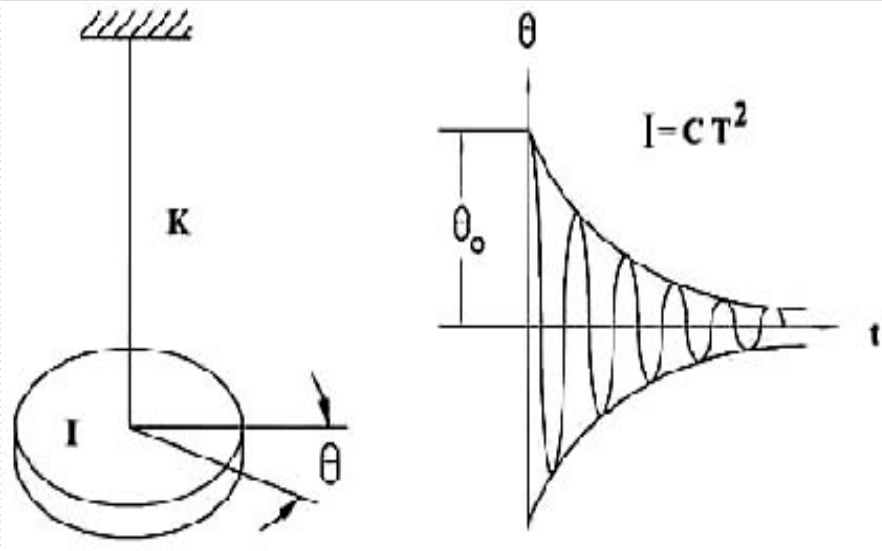
☐ Lumped mass distribution problem

Vehicle rotational inertia

- ☐ How can we ascertain vehicle rotational inertia
 - ☐ Put everything in CAD and ask the computer
 - ☐ Measure it
-

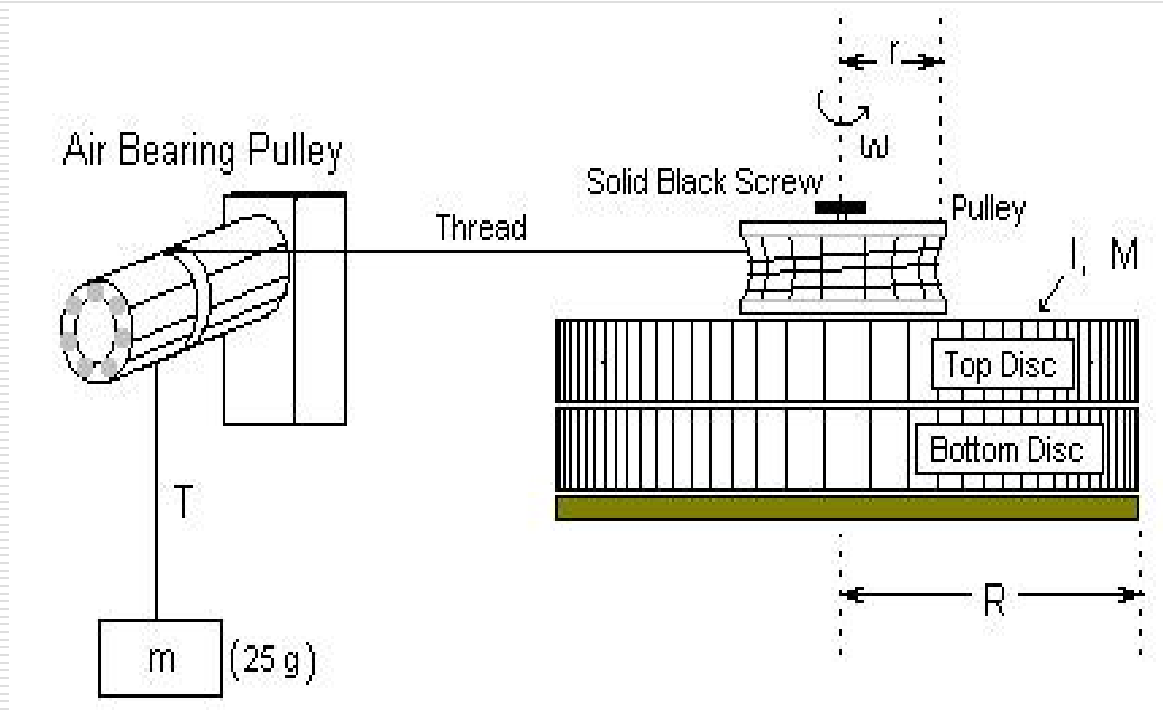
Inertia measurement

Torsional pendulum



Inertia measurement

Torsional force



Applied torque = Inertia x angular acceleration

Vehicle Inertial measurement



CIC's Moment of Inertia Facilities provide measurement of:

- 3-Axis, Moment and Product of Inertias
- Principal Inertias and their Direction Cosines
- Total Mass, Centre of Gravity Location and Mass Distribution

Mass range	From 1kg to 2,500kg
Dimension range	Nominally 6500 (L) x 2800 (W) x 2000 (H) mm max

Inertial Measurement Centre of Gravity And Inertial Measurement System

- The vehicle is clamped to the 'centre table' of the SPMM via multi-axis load cells. The centre table is then automatically driven through a sequence of manoeuvres to enable the Centre of Gravity and Moments of Inertia of the body to be determined. A test sequence typically lasts less than 15 minutes.



Mass Distribution Study

- Where to spend money

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Introduction to Vehicle Transient Dynamics Moment of Inertia

Vehicle cornering

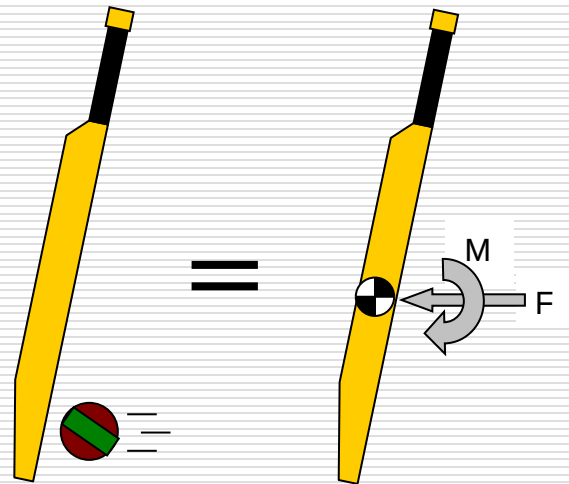
- ❑ So far we have looked at steady state cornering where the vehicle rotates very slowly as it goes around a corner
 - ❑ Vehicle mass will have little effect on this behaviour
 - ❑ What happens when we start to corner ?
-

Watch the spin



inertial conjugate

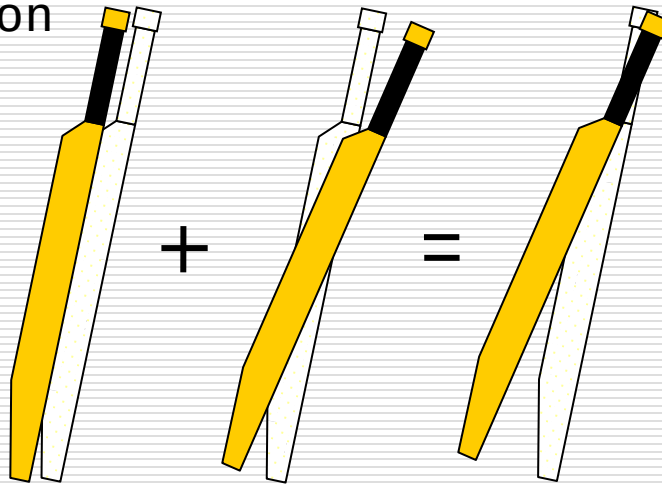
- ❑ Inertial conjugate is sometimes referred to as the 'centre of percussion'
- ❑ Bat is a solid entity and transfers energy to the ball
- ❑ Force and moment at CG result from ball / bat contact



Equivalent
Forces and
Moments of Ball
Striking Bat

inertial conjugate

- ❑ The result is a linear acceleration and a rotational acceleration of the bat
- ❑ At the CG there is no rotation only translation
- ❑ Below the CG the rotational acceleration adds to linear acceleration
- ❑ Above the CG the rotational acceleration subtracts from the linear acceleration



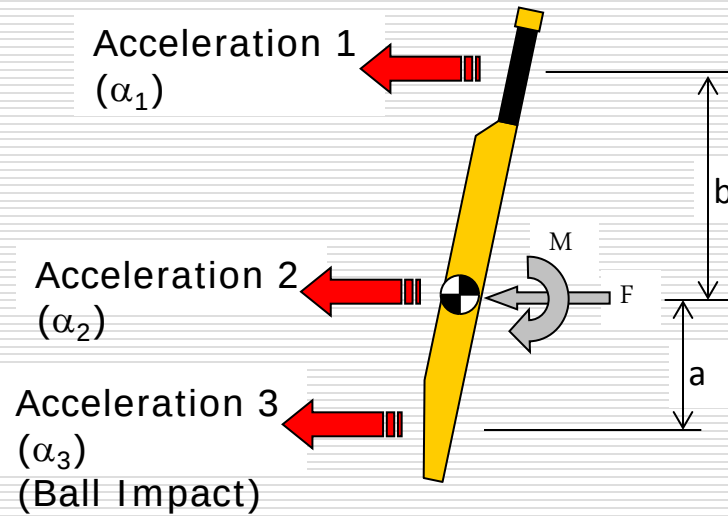
Combining Linear and
Rotational Acceleration

inertial conjugate

$$\alpha_1 = \frac{F}{m} - \frac{M.b}{I_{zz}}$$

$$\alpha_2 = \frac{F}{m}$$

$$\alpha_3 = \frac{F}{m} + \frac{M.a}{I_{zz}}$$



Accelerations at
Various Points on
Bat

- I_{zz} = Moment of Inertia
- m = Mass
- F = Force of Ball Strike
- a = Distance from Ball Strike to CG
- b = Distance from CG to Batsman's Hand
- M = Force of Ball (F) x Distance from Ball to CG
- (a)

Linear $F = ma$

Newton's Second
Law

Angular $\tau = I\alpha$

Linear $p = mv$

Momentum

Angular $L = I\omega$

Moment
of Inertia
 I

Linear $KE = \frac{1}{2}mv^2$

Kinetic Energy

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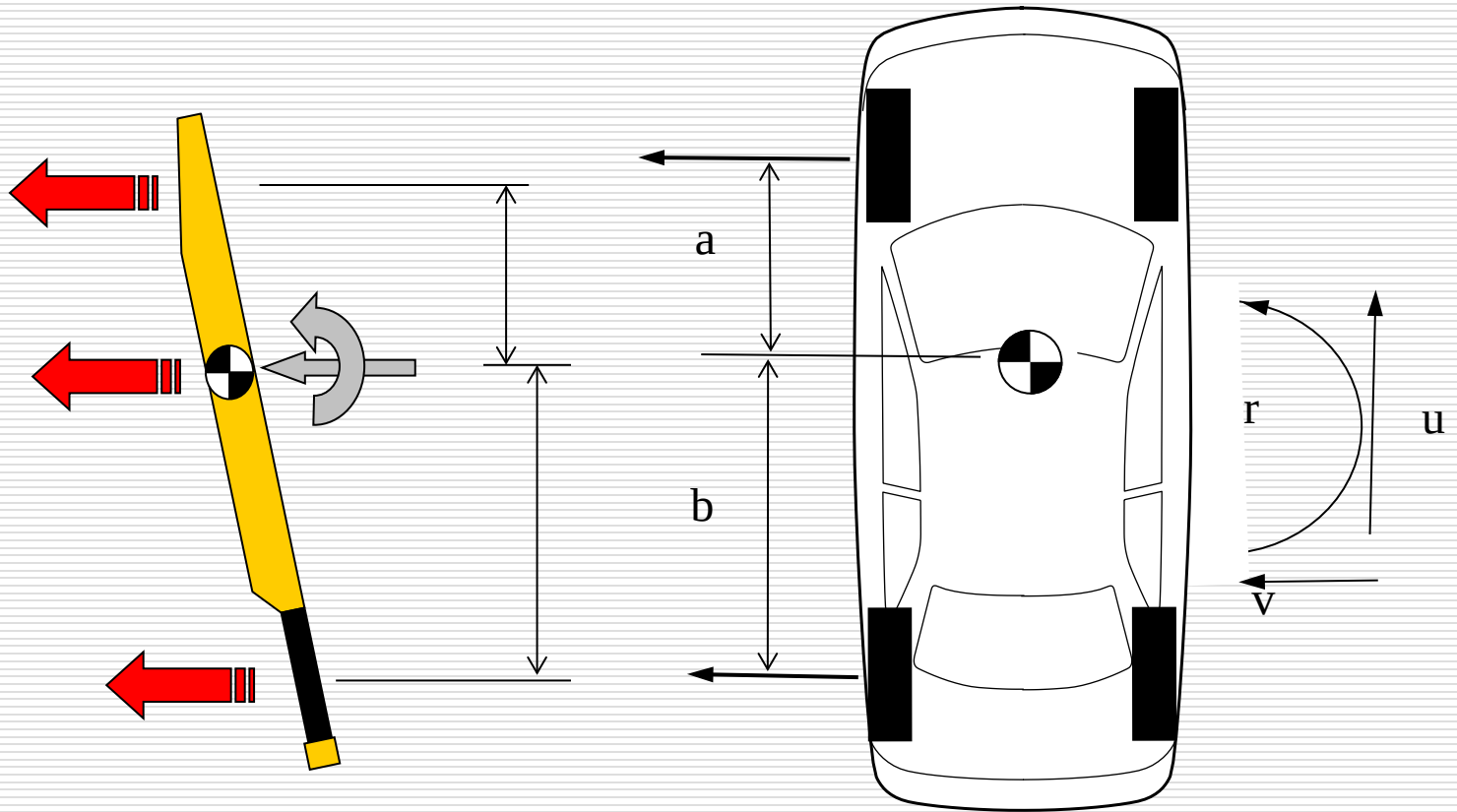
Work-Energy

Angular $\tau_{net}\theta = \Delta \left(\frac{1}{2}I\omega^2 \right)$

inertial conjugate

- There is a point on the other side of the CG to the ball contact where linear acceleration is zero
 - This is the inertial conjugate or 'centre of percussion'
 - The same theory is applied to a vehicle
 - The force application is at the front wheels
 - Resolves to a force and moment at the CG
 - The lateral acceleration and hence lateral force, direction and magnitude at the rear tyres is determined by mass, moment of inertia and distance from CG to front and rear axles
-

inertial conjugate



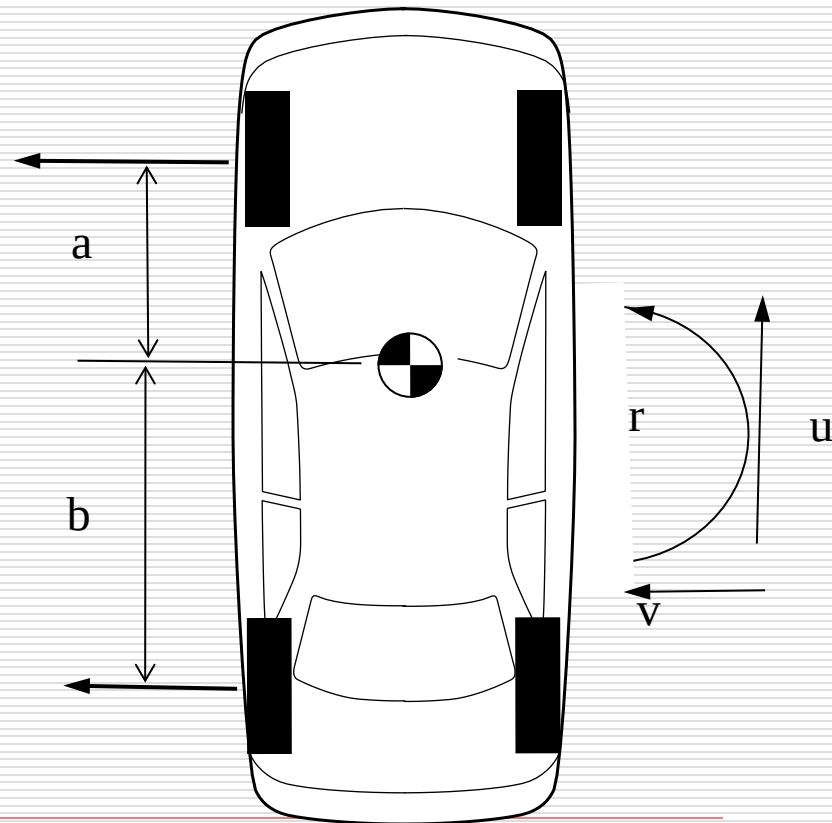
inertia match theory

- Consider a four-wheeled vehicle manoeuvring in the ground plane

$$F_{yf} \cdot a - F_{yr} \cdot b = 0$$

$$F_{yf} + F_{yr} = m \cdot r \cdot u \quad (= m \cdot a_y)$$

- Steady state, yaw acceleration is zero and the solution is trivial:



inertia match theory

- When entering a turn, a lateral acceleration and yaw acceleration exists
- At instant of turn in rear tyre slip angle is zero

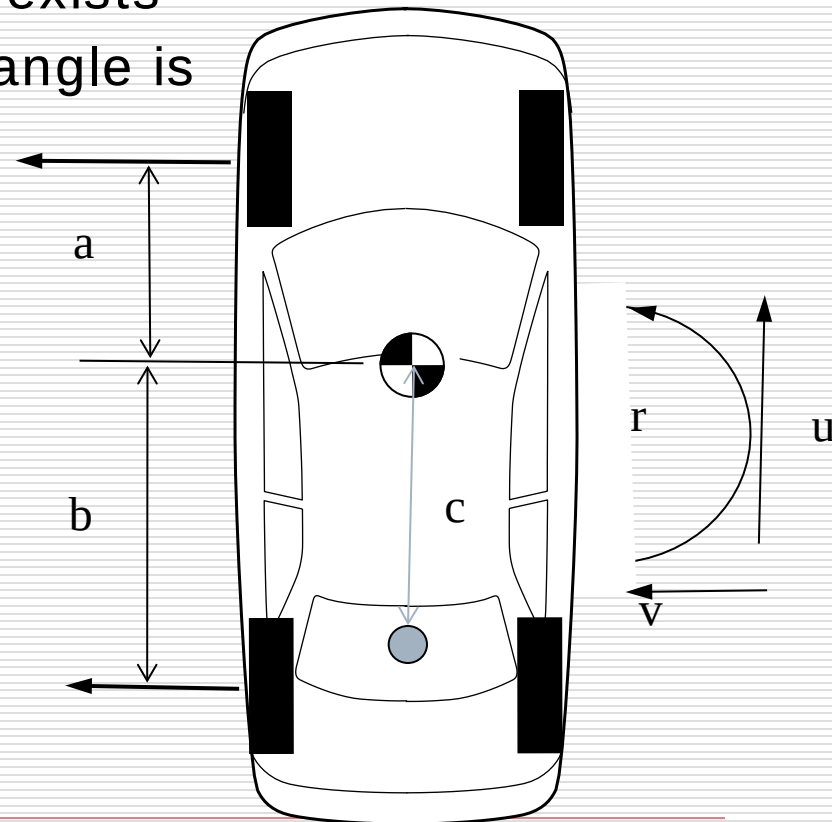
$$F_{yf} \cdot a - F_{yr} \cdot b = I_{zz} \dot{\delta}$$

$$F_{yf} + F_{yr} = m a_y$$

$$\frac{F_{yf}}{m} = \frac{F_{yf} \cdot a_y}{I_{zz}} \times c$$

$$I_{zz} = m \cdot k^2$$

$$\frac{F_{yf}}{m} = \frac{F_{yf} \cdot a_y}{m \cdot k^2} \times c$$

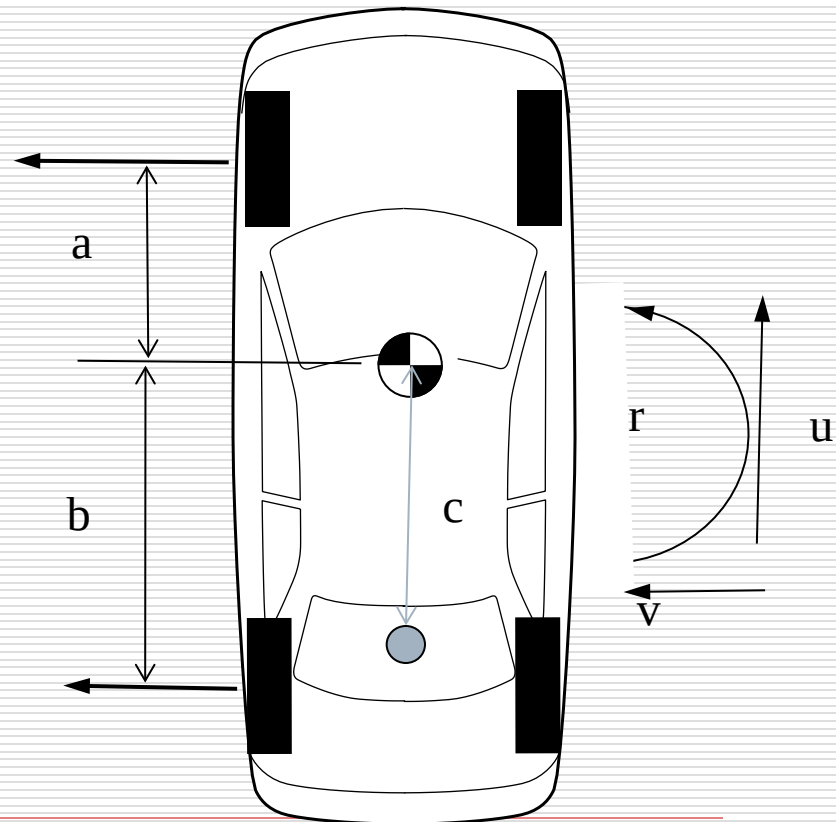


inertia match theory

$$\frac{F_{yf}}{m} = \frac{F_{yf} \cdot a_y}{m \cdot k^2} \times c$$

$$\frac{k^2}{a} = c$$

$$\frac{k^2}{a \cdot b} = 1 \quad (\text{when } c=b)$$

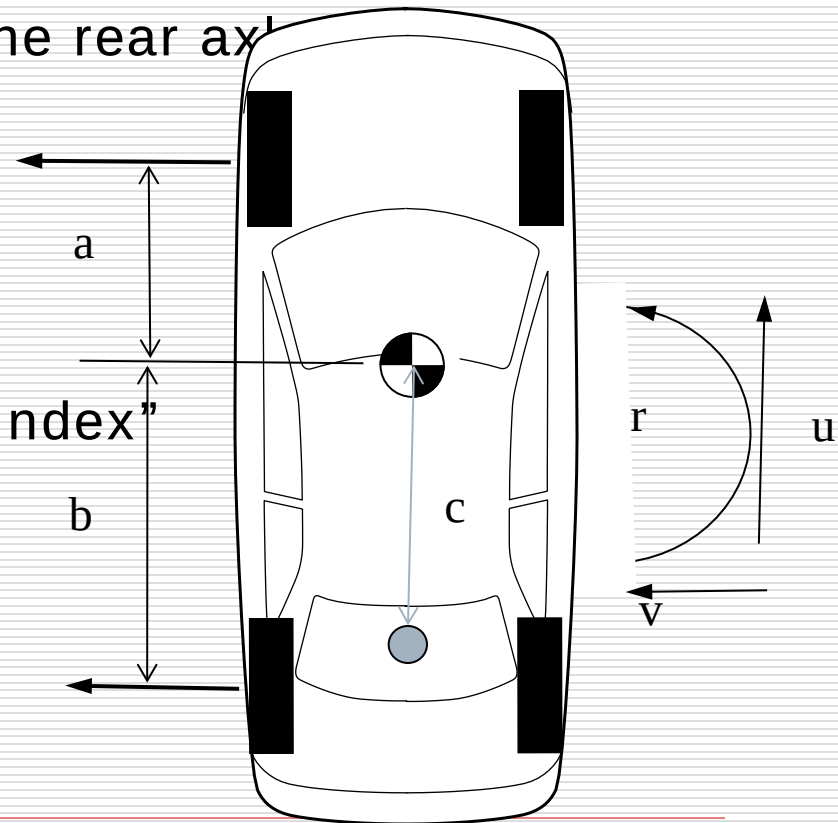


dynamic index

- $\frac{k^2}{a \cdot b}$ therefore describes the distance of the centre of rotation with respect to the rear axle

$$D.I = \frac{I_{zz}}{m \cdot a \cdot b}$$

- It is referred to as the “Dynamic Index”
- DI fraction is length ratio c / b
- $DI > 1$ implies $c > b$
- $DI < 1$ implies $c < b$

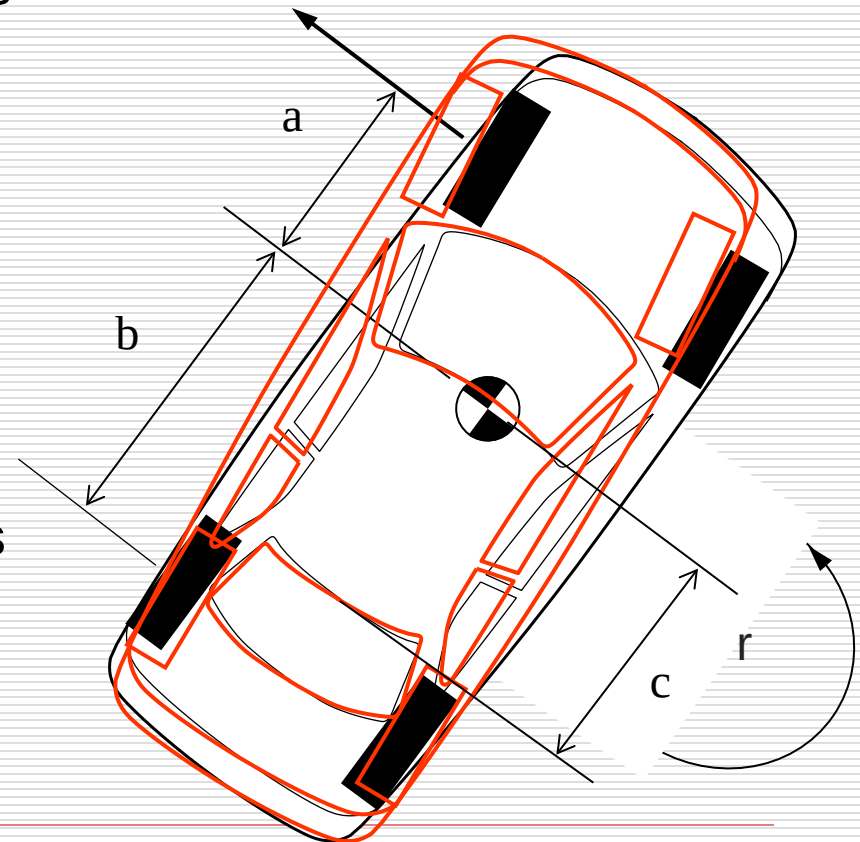


dynamic index

- The effect of the rotation centre not being at the rear axle gives a lateral velocity at the rear axle of

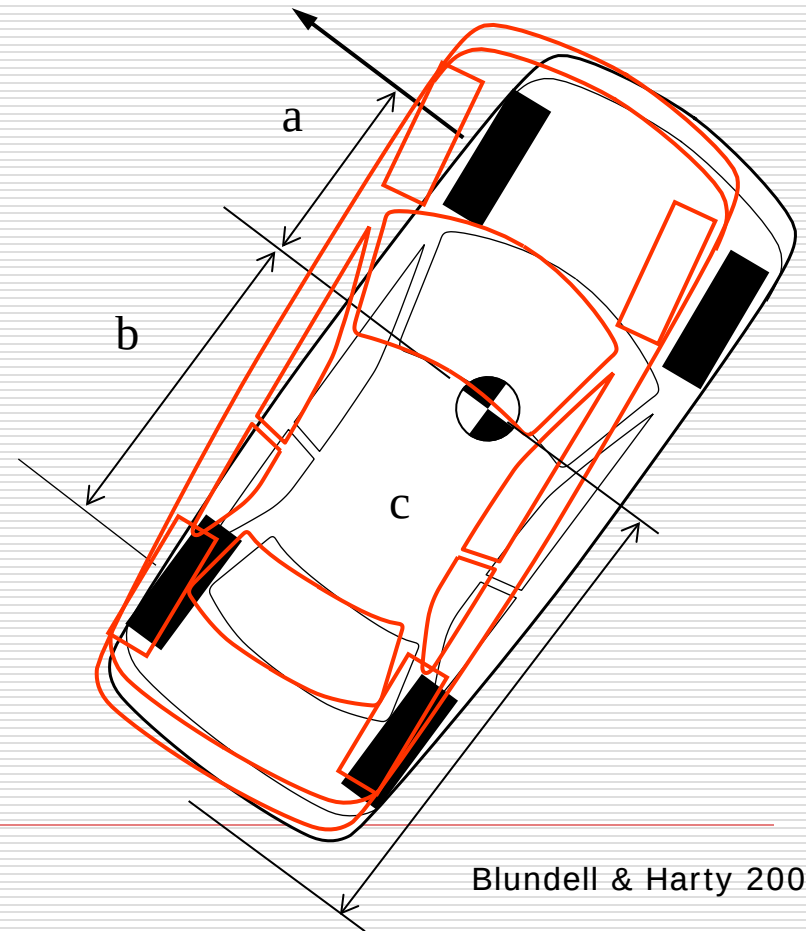
$$v_r = r \cdot (c - b)$$

- If $DI < 1$ ($c < b$ - inertial conjugate is ahead of the rear axle) then the rear of the car moves outward; inertial effects *add* to rear tyre slip angle



dynamic index

- If $DI > 1$ ($c > b$ - inertial conjugate is behind the rear axle) then the rear of the car moves inward; inertial effects *reduce* rear tyre slip angle
- Roll characteristics also affect rear tyre slip angle on turn in



dynamic index

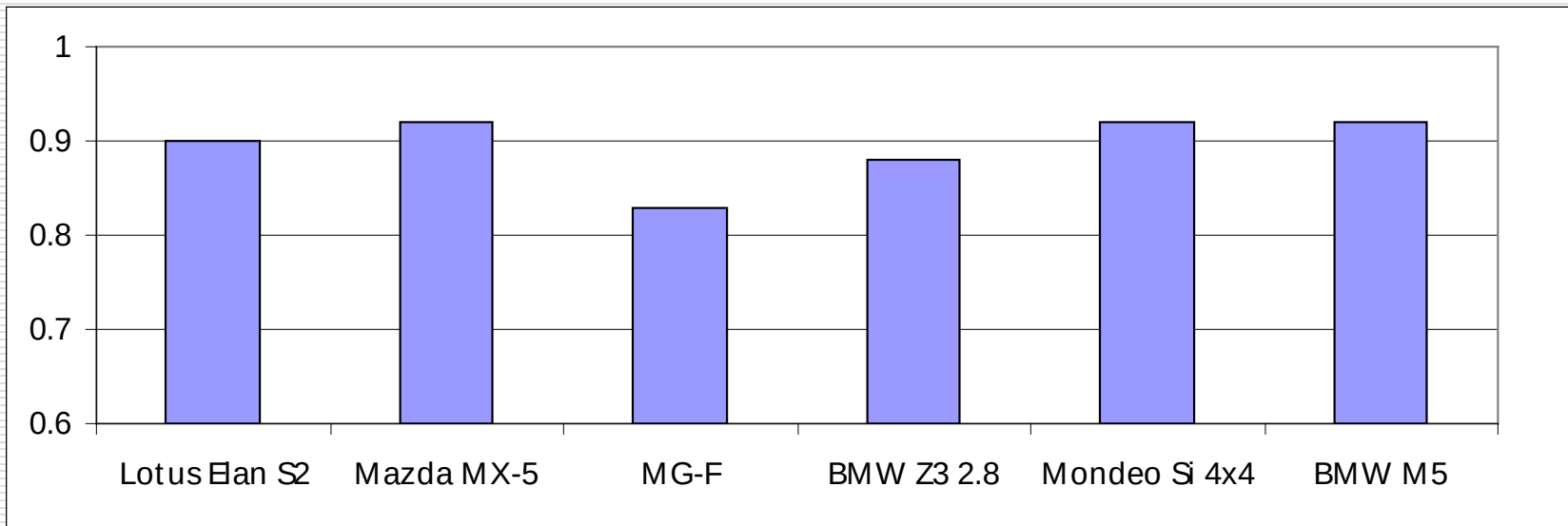
- DI is a significant subjective modifier to perceived agility
 - Lower DI generally implies a more agile, nimble, lively feel
 - However, an excessively low DI results in a twitchy, unmanageable vehicle
 - There are ample examples of very capable vehicles which have a DI between 0.9 and 0.92 which may be an indication to a good compromise between agility and control
-

Formula Student

- Now calculate the dynamic index for the formula student car.

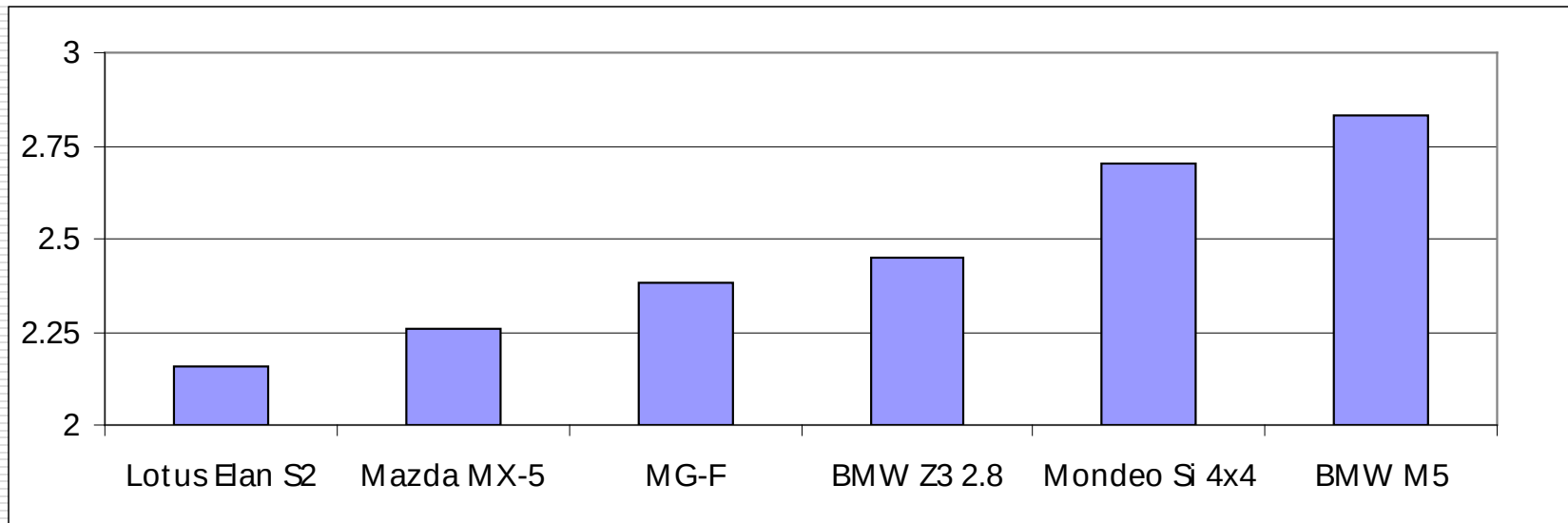
examples

□ Dynamic Index comparison:

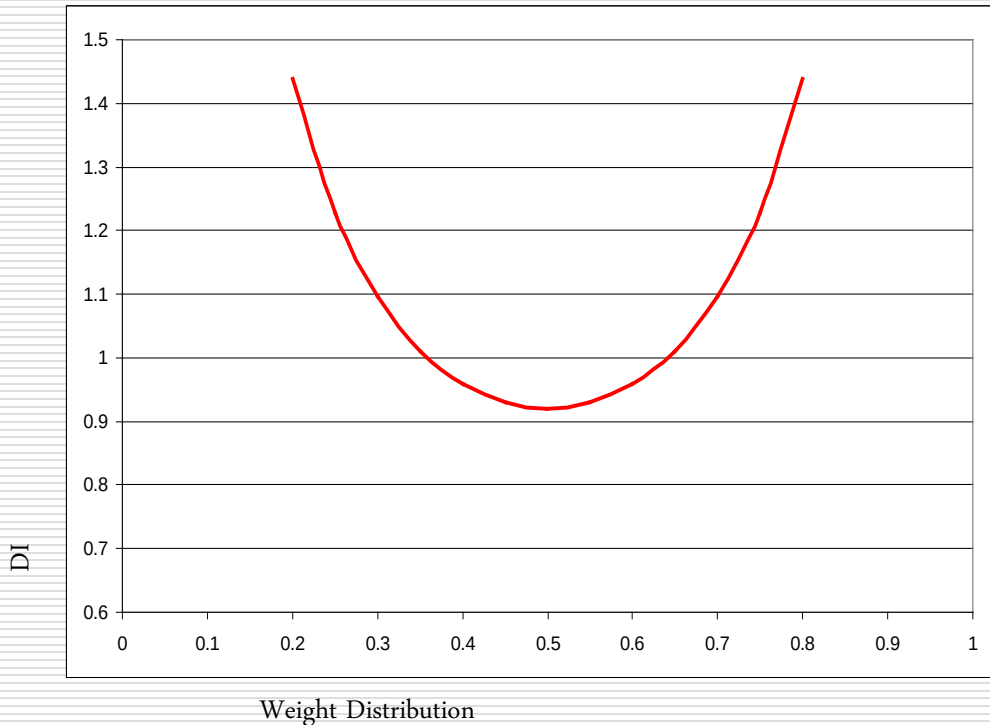


examples

□ Wheelbase comparison:



dynamic index



- ❑ Inertia match is the relationship between the CG position, wheelbase and yaw inertia.
- ❑ With “typical” vehicle layouts, yaw inertia (and hence DI) may be higher than is desirable
- ❑ 50/50 weight distribution gives lowest DI *for given yaw inertia*

inertia match summary

- ❑ The DI influences the way the rear tyres respond during a transient manoeuvre
 - ❑ It influences initial turn in, vehicle agility and driver workload for post limit control
 - ❑ If the inertia, wheelbase and mass are correctly matched downstream tuning becomes easier
 - ❑ A poor inertia match forces downstream tuning in specific directions to compensate for fundamental handling characteristics
-

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Introduction to Vehicle Transient Dynamics

Mark Sandford