

1) Obtain the unit vector in the direction from the origin towards the point $P(3, -3, -2)$

Soln.

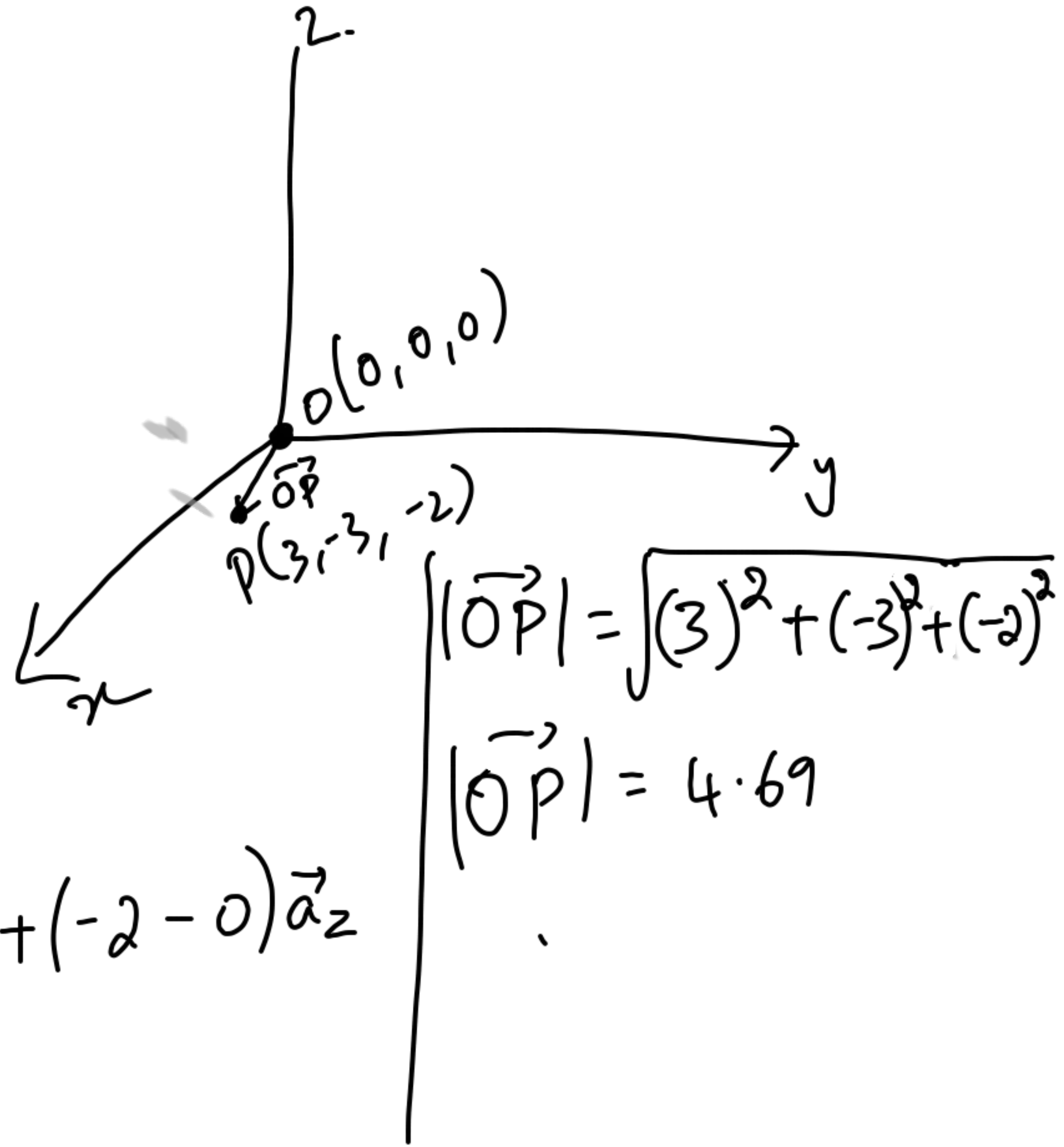
$$O \overset{x_1}{(0, \overset{y_1}{0}, \overset{z_1}{0})}$$

$$P \overset{x_2}{(3, \overset{y_2}{-3}, \overset{z_2}{-2})}$$

$$\vec{OP} = (x_2 - x_1)\vec{a}_x + (y_2 - y_1)\vec{a}_y + (z_2 - z_1)\vec{a}_z$$

$$\vec{OP} = (3 - 0)\vec{a}_x + (-3 - 0)\vec{a}_y + (-2 - 0)\vec{a}_z$$

$$\vec{OP} = 3\vec{a}_x - 3\vec{a}_y - 2\vec{a}_z$$



$$|\vec{OP}| = \sqrt{(3)^2 + (-3)^2 + (-2)^2}$$

$$|\vec{OP}| = 4.69$$

$$\vec{a}_{op} = \frac{\vec{OP}}{|\vec{OP}|}$$

$$\vec{a}_{op} = \frac{3\vec{a}_x - 3\vec{a}_y - 2\vec{a}_z}{4.69}$$

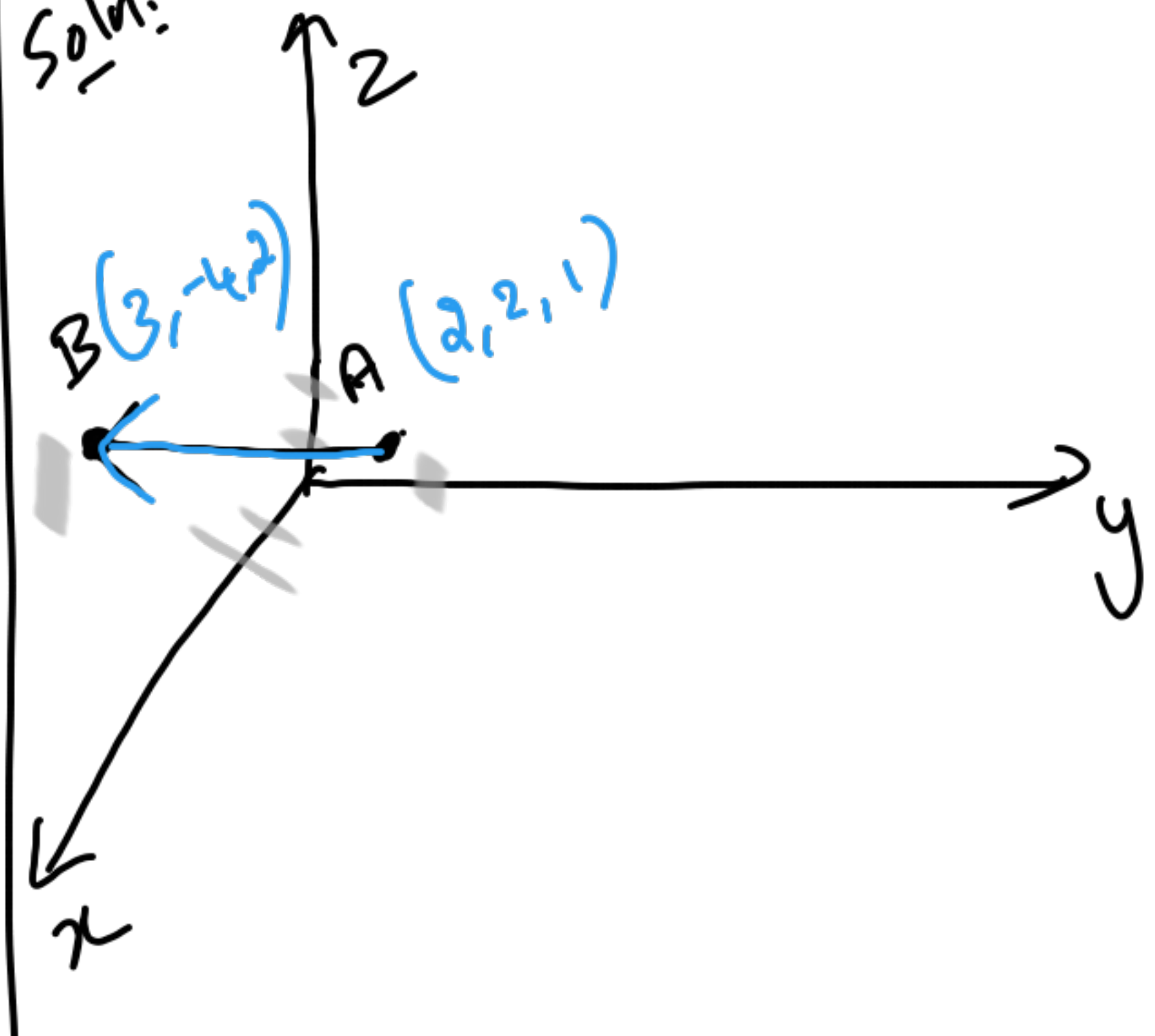
$$\vec{a}_{op} = 0.639\vec{a}_x - 0.639\vec{a}_y - 0.426\vec{a}_z$$

$$|\vec{a}_{op}| = \sqrt{(0.639)^2 + (-0.639)^2 + (-0.426)^2}$$

$$|\vec{a}_{op}| = 1$$

2) Two points $A(2, 2, 1)$ and $B(3, -4, 2)$ are given in the Cartesian System. Obtain the vector from A to B and a unit vector from A to B.

Soln:



$$\vec{A} = 2\vec{a}_x + 2\vec{a}_y + \vec{a}_z$$

$$\vec{B} = 3\vec{a}_x - 4\vec{a}_y + 2\vec{a}_z$$

$$\vec{AB} = \vec{B} - \vec{A}$$

$$= (3-2)\vec{a}_x + (-4-2)\vec{a}_y + (2-1)\vec{a}_z$$

$$\vec{AB} = \vec{a}_x - 6\vec{a}_y + \vec{a}_z$$

$$|\vec{AB}| = \sqrt{(1)^2 + (-6)^2 + (1)^2}$$

$$|\vec{AB}| = 6.16$$

Unit vector

$$\vec{a}_{AB} = \frac{\vec{AB}}{|\vec{AB}|}$$

$$= \frac{\vec{a}_x - 6\vec{a}_y + \vec{a}_z}{6.16}$$

$$\vec{a}_{AB} = 0.16\vec{a}_x - 0.97\vec{a}_y + 0.16\vec{a}_z$$

$$|\vec{a}_{AB}| = \sqrt{(0.16)^2 + (-0.97)^2 + (0.16)^2}$$

$$|\vec{a}_{AB}| = 1$$

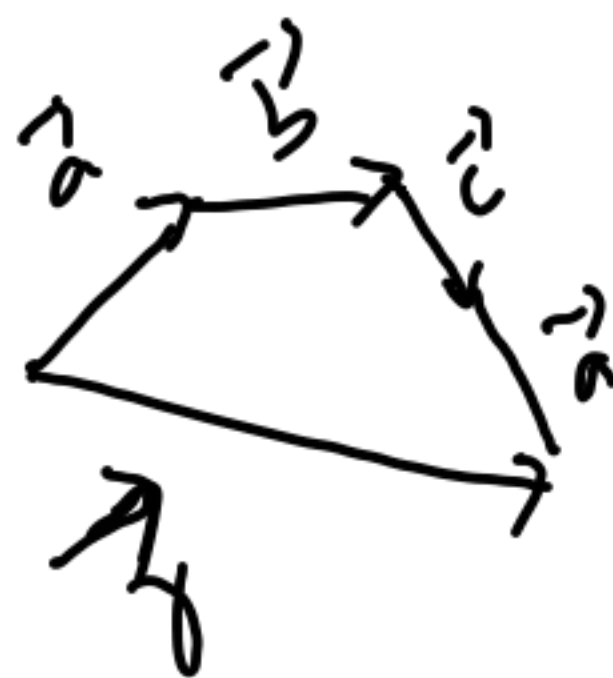
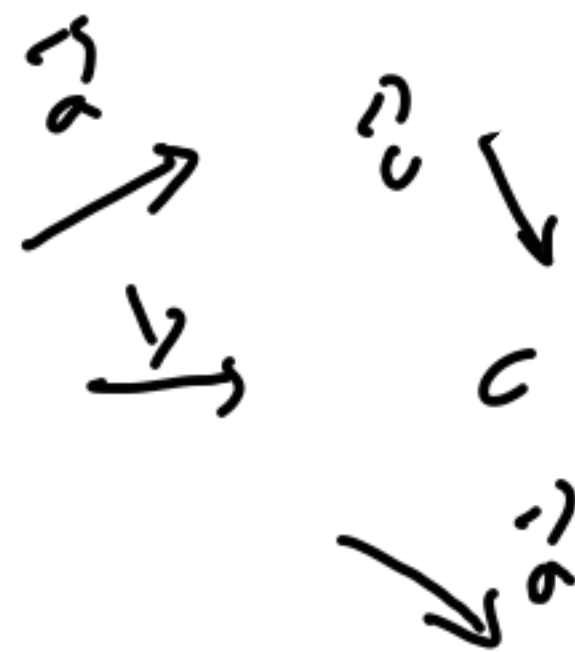
Vectors - magnitude & direction.

Scalars $\rightarrow$ magnt..



$$\vec{A} - \vec{B} = 0$$

(identical.)



$$|\vec{a}| = 1$$

$$\vec{a}_{OA} = \frac{\vec{OA}}{|\vec{OA}|}$$

$$P \rightarrow Q$$

$$\vec{PQ} = \vec{Q} - \vec{P}$$

$$|\vec{PQ}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

Cartesian. (x, y, z)

$$d\vec{l} = dx \vec{a}_x + dy \vec{a}_y + dz \vec{a}_z$$

Cylindrical. (r, ϕ, z)

$$d\vec{l} = dr \vec{a}_r + r d\phi \vec{a}_\phi + dz \vec{a}_z.$$

Spherical. (r, θ, ϕ)

$$d\vec{l} = dr \vec{a}_r + r d\phi \vec{a}_\phi + r \sin\theta d\theta \vec{a}_\theta$$

$$\vec{A} \cdot \vec{B} = |\vec{A}| |\vec{B}| \cos \theta_{AB}$$

$$\theta_{AB} = \cos^{-1} \left(\frac{\vec{A} \cdot \vec{B}}{|\vec{A}| |\vec{B}|} \right)$$

$$\vec{A} \times \vec{B} = |\vec{A}| |\vec{B}| \sin \theta_{AB} \vec{a}_{AB}$$

$$\vec{A} \times \vec{B} = \begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

Cylinder:

$$\vec{A} \times \vec{B} = \begin{vmatrix} \vec{a}_r & \vec{a}_\phi & \vec{a}_z \\ A_r & A_\phi & A_z \\ B_r & B_\phi & B_z \end{vmatrix}$$

Sphere:

$$\vec{A} \times \vec{B} = \begin{vmatrix} \vec{a}_r & \vec{a}_\theta & \vec{a}_\phi \\ A_r & A_\theta & A_\phi \\ B_r & B_\theta & B_\phi \end{vmatrix}$$

3). Given the vector $\vec{A} = 2\vec{a}_x - 5\vec{a}_y - 4\vec{a}_z$ and $\vec{B} = 3\vec{a}_x + 5\vec{a}_y + 2\vec{a}_z$. Find the dot product and angle between both the vectors.

Soln.

$$\vec{A} = 2\overset{A_x}{\vec{a}_x} - 5\overset{A_y}{\vec{a}_y} - 4\overset{A_z}{\vec{a}_z}.$$

$$\vec{B} = 3\overset{B_x}{\vec{a}_x} + 5\overset{B_y}{\vec{a}_y} + 2\overset{B_z}{\vec{a}_z}.$$

Find the dot product.

$$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$$

$$\vec{A} \cdot \vec{B} = (2 \times 3) + (-5 \times 5) + (-4 \times 2)$$

$$\vec{A} \cdot \vec{B} = -27.$$

$$|\vec{A}| = \sqrt{2^2 + (-5)^2 + (-4)^2} = 6.71$$

$$|\vec{B}| = \sqrt{(3)^2 + (5)^2 + (2)^2} = 6.16$$

$$\vec{A} \cdot \vec{B} = |\vec{A}| |\vec{B}| \cos \theta_{AB}$$

$$\theta_{AB} = \cos^{-1} \left(\frac{\vec{A} \cdot \vec{B}}{|\vec{A}| |\vec{B}|} \right)$$

$$= \cos^{-1} \left(\frac{-27}{6.71 \times 6.16} \right) \quad \theta_{AB} = 130.76^\circ$$

2) Show that $\vec{A} = 4\vec{a}_x - 2\vec{a}_y - \vec{a}_z$ and $\vec{B} = \vec{a}_x + 4\vec{a}_y - 4\vec{a}_z$ are perpendicular to each other.

Soln.

$$\vec{A} = 4\vec{a}_x - 2\vec{a}_y - \vec{a}_z$$
$$\vec{B} = \vec{a}_x + 4\vec{a}_y - 4\vec{a}_z.$$

If $\vec{A} \cdot \vec{B} = 0$, then perpendicular to each other

$$\vec{A} \cdot \vec{B} = (4 \times 1) + (-2 \times 4) + (-1 \times -4)$$

$$\vec{A} \cdot \vec{B} = 0$$

3) Given two Vectors. $\vec{A} = 3\vec{a}_x + 2\vec{a}_y - \vec{a}_z$ and $\vec{B} = -\vec{a}_x + 4\vec{a}_y + 2\vec{a}_z$.

Find the Vector that is perpendicular to both the vector and verify the resulting vector is perpendicular to $\vec{A}$ and $\vec{B}$

Soln.

To find the perpendicular Vector take cross product

$$\vec{A} \times \vec{B} = \begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$$\vec{A} \times \vec{B} = \begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ 3 & 2 & -1 \\ -1 & 4 & 2 \end{vmatrix}$$

$$= \vec{a}_x \begin{vmatrix} 2 & -1 \\ 4 & 2 \end{vmatrix} - \vec{a}_y \begin{vmatrix} 3 & -1 \\ -1 & 2 \end{vmatrix} + \vec{a}_z \begin{vmatrix} 3 & 2 \\ -1 & 4 \end{vmatrix}$$

$$= \vec{a}_x [(2 \times 2) - (-1 \times 4)] - \vec{a}_y [(3 \times 2) - (-1 \times (-1))] + \vec{a}_z [(3 \times 4) - (2 \times (-1))]$$

$$= \vec{a}_x [4 - (-4)] - \vec{a}_y [6 - 1] + \vec{a}_z [12 - (-2)]$$

$$= \vec{a}_x [4 + 4] - \vec{a}_y [5] + \vec{a}_z [12 + 2]$$

$$= 8\vec{a}_x - 5\vec{a}_y + 14\vec{a}_z$$

$$\vec{A} \times \vec{B} = 8\vec{a}_x - 5\vec{a}_y + 14\vec{a}_z$$

$$\vec{A} \times \vec{B} = \vec{R} = 8\vec{a}_x - 5\vec{a}_y + 14\vec{a}_z$$

To Verify if $\vec{R}$ is

Perpendicular to $\vec{A}$ and $\vec{B}$

To verify $\vec{A}$ and $\vec{R}$ are perpendicular to each other.

$$\vec{A} = 3\vec{a}_x + 2\vec{a}_y - \vec{a}_z$$

$$\vec{R} = 8\vec{a}_x - 5\vec{a}_y + 14\vec{a}_z.$$

$$\vec{A} \cdot \vec{R} = (3 \times 8) + (2 \times (-5)) + ((-1) \times 14)$$

$$= 24 - 10 - 14$$

$$\vec{A} \cdot \vec{R} = 0$$

To verify $\vec{B}$ and $\vec{R}$ are perpendicular to each other.

$$\vec{B} = -\vec{a}_x + 4\vec{a}_y + 2\vec{a}_z$$

$$\vec{R} = 8\vec{a}_x - 5\vec{a}_y + 14\vec{a}_z.$$

$$\vec{B} \cdot \vec{R} = ((-1) \times 8) + (4 \times (-5)) + (2 \times 14)$$

$$\vec{B} \cdot \vec{R} = -8 - 20 + 28$$

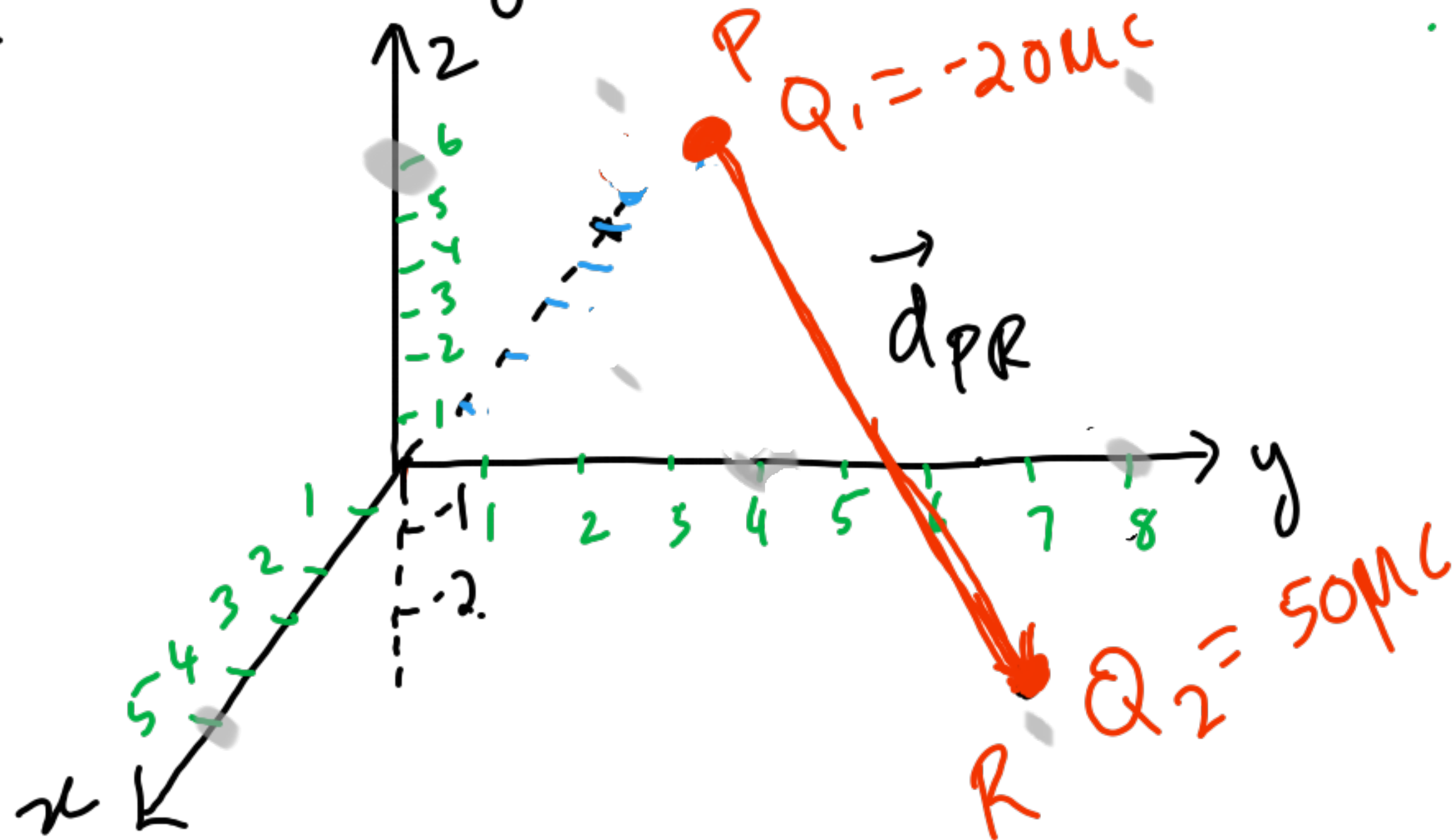
$$\vec{B} \cdot \vec{R} = 0$$

As the dot product is zero, Both the vectors are perpendicular.

4) A charge $Q_1 = -20 \mu\text{C}$ is located at a point $P(-6, 4, 6)$ and a charge $Q_2 = 50 \mu\text{C}$ is located at $R(5, 8, -2)$ in a free space. Find the force exerted on Q_2 by Q_1 .

The distances given are in meters.

Soln:



$$F = \frac{Q_1 Q_2}{4\pi\epsilon_0 d^2} \cdot \vec{a}$$

$$Q_1 = -20 \mu\text{C}$$

$$Q_2 = 50 \mu\text{C}$$

$$P(-6, 4, 6)$$

$$R(5, 8, -2)$$

$$\vec{P} = -6\vec{a}_x + 4\vec{a}_y + 6\vec{a}_z$$

$$\vec{R} = 5\vec{a}_x + 8\vec{a}_y - 2\vec{a}_z$$

$$\vec{d}_{PR} = \vec{R} - \vec{P}$$

$$\vec{d}_{PR} = (5 - (-6))\vec{a}_x + (8 - 4)\vec{a}_y + (-2 - 6)\vec{a}_z$$

$$\vec{d}_{PR} = 11\vec{a}_x + 4\vec{a}_y - 8\vec{a}_z$$

$$|\vec{d}_{PR}| = \sqrt{(11)^2 + (4)^2 + (-8)^2}$$

$$|\vec{d}_{PR}| = 14.18$$

$$\vec{a}_{PR} = \frac{\vec{d}_{PR}}{|\vec{d}_{PR}|} = \frac{11\vec{a}_x + 4\vec{a}_y - 8\vec{a}_z}{14.18}$$

$$\vec{a}_{PR} = 0.78\vec{a}_x + 0.28\vec{a}_y - 0.56\vec{a}_z$$

$$F = \frac{-20 \times 10^{-6} \times 50 \times 10^{-6}}{4\pi \times 8.854 \times 10^{-12} \times (14.18)^2} \vec{a}_{PR}$$

$$\vec{F} = -0.0447 (0.78\vec{a}_x + 0.28\vec{a}_y - 0.56\vec{a}_z)$$

$$\vec{F} = -0.034\vec{a}_x - 0.0126\vec{a}_y + 0.025\vec{a}_z$$

$$|\vec{F}| = 44.7 \text{ mN}$$

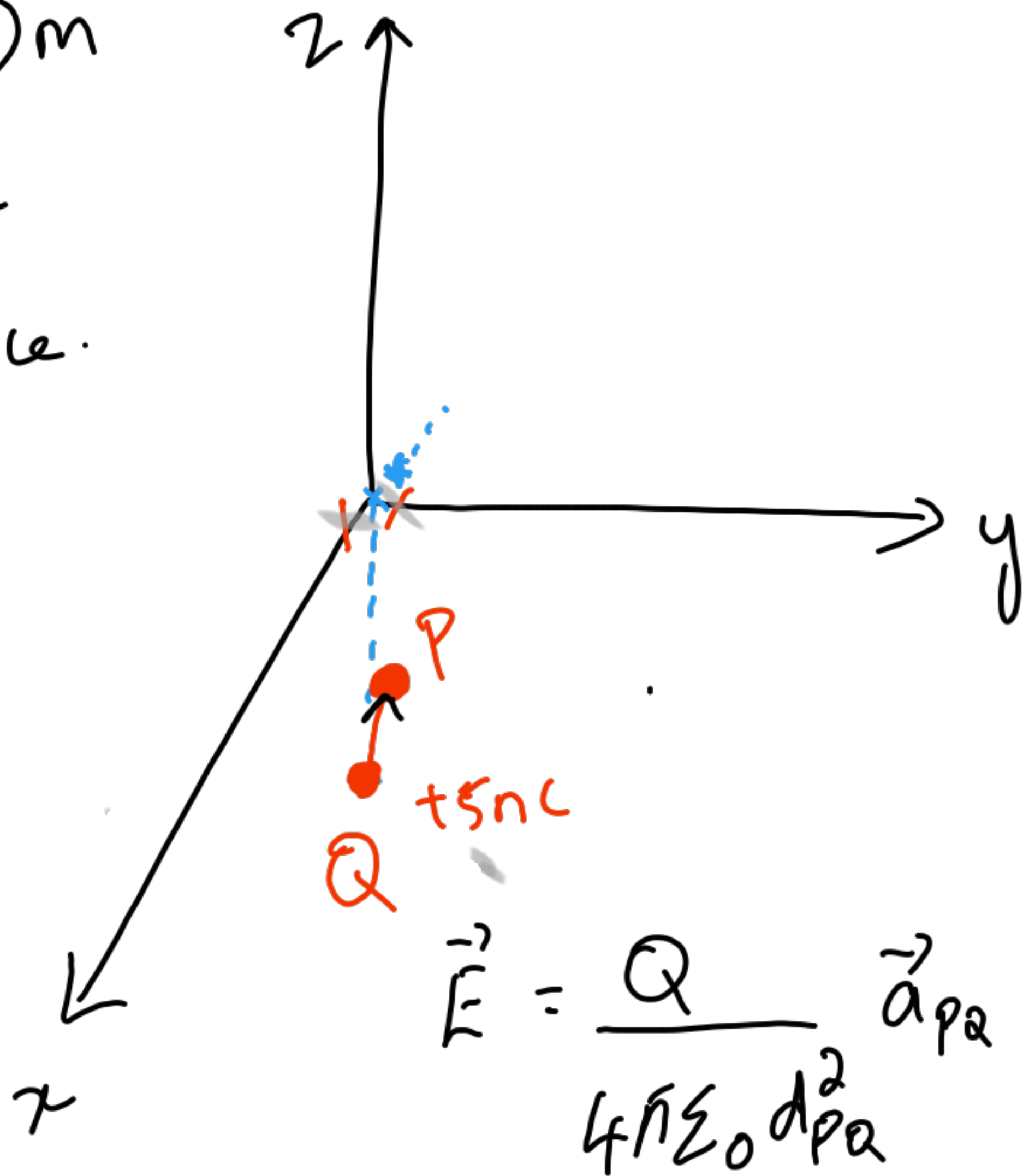
5) Determine the electric field intensity vector at $P(-0.2, 0, -2.3)\text{m}$ due to a point charge at $Q(0.2, 0.1, -2.5)\text{m}$ in space. $+5\text{nC}$

Soln.

$$P(-0.2, 0, -2.3)$$

$$Q(0.2, 0.1, -2.5)$$

$$\text{Point charge} = +5\text{nC}$$



$$Q = 5 \text{ nC}$$

$$= 5 \times 10^{-9} \text{ C}$$

$$\vec{P} = -0.2 \vec{a}_x - 2.3 \vec{a}_z$$

$$Q = 0.2 \vec{a}_x + 0.1 \vec{a}_y - 2.5 \vec{a}_z$$

$$\vec{d}_{PQ} = \vec{P} - \vec{Q}$$

$$= (-0.2 - 0.2) \vec{a}_x + (0 - 0.1) \vec{a}_y + (-2.3 - (-2.5)) \vec{a}_z$$

$$\vec{d}_{PQ} = -0.4 \vec{a}_x - 0.1 \vec{a}_y + 0.2 \vec{a}_z$$

$$|\vec{d}_{PQ}| = \sqrt{(-0.4)^2 + (-0.1)^2 + (0.2)^2} = 0.46$$

$$\vec{a}_{PQ} = \frac{\vec{d}_{PQ}}{|\vec{d}_{PQ}|}$$

$$= \frac{-0.4 \vec{a}_x - 0.1 \vec{a}_y + 0.2 \vec{a}_z}{0.46}$$

$$\vec{a}_{PQ} = -0.88 \vec{a}_x - 0.22 \vec{a}_y + 0.43 \vec{a}_z$$

$$\vec{E} = \frac{Q_1}{4\pi\epsilon_0 d_{PQ}^2} \vec{a}_{PQ}$$

$$= \frac{5 \times 10^{-9}}{4\pi \times 8.854 \times 10^{-12} \times (0.46)^2} \left(-0.88 \vec{a}_x - 0.22 \vec{a}_y + 0.43 \vec{a}_z \right)$$

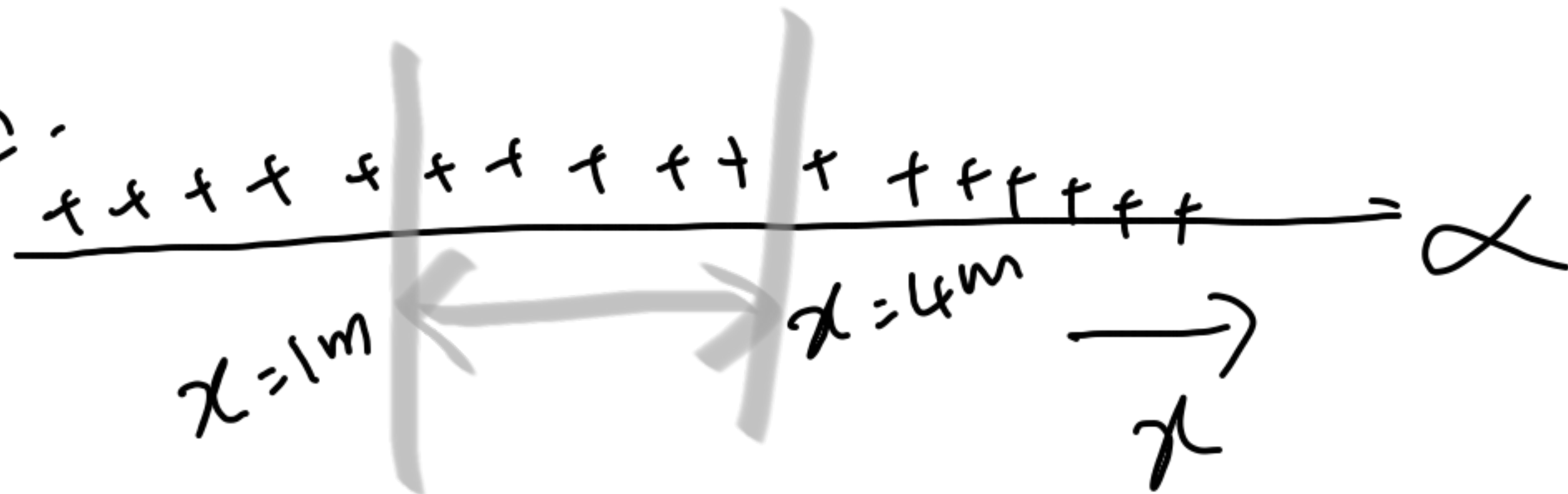
$$= 212.38 \left(-0.88 \vec{a}_x - 0.22 \vec{a}_y + 0.43 \vec{a}_z \right)$$

$$\vec{E} = -184.67 \vec{a}_x - 46.17 \vec{a}_y + 92.34 \vec{a}_z.$$

$$|\vec{E}| = 212.38 \text{ N/C}$$

b) An infinitely long charge is distributed along the x -axis with a linear charge density $\rho_L = 2x^2 \text{ C/m}$. Calculate the total charge Q distributed along the segment of the line from $x = 1\text{m}$ to $x = 4\text{m}$.

Soln.



line charge density

$$\rho_L = 2x^2 \text{ C/m}$$

$$x = 1\text{m} \text{ to } x = 4\text{m}$$

$$Q = \int_L \rho_L \, dl$$

$$Q = \int_L 2x^2 \, dx$$

$$Q = \int_1^4 2x^2 \, dx$$

$$Q = 2 \int_1^4 x^2 \cdot dx.$$

$$Q = 2 \left[\frac{x^3}{3} \right]_1^4$$

$$Q = 2 \left[\frac{(4)^3}{3} - \frac{(1)^3}{3} \right]$$

4 x 4 x 4

$$Q = 2 \left[\frac{64}{3} - \frac{1}{3} \right]$$

$$= 2 \left[\frac{64-1}{3} \right] = 2 \times \frac{63}{3} = \frac{126}{3} = 42 C$$

$$\boxed{Q = 42 C}$$

$$\left[\int x^n dx = \frac{x^{n+1}}{n+1} \right]$$

$$\int x^2 dx = \frac{x^{2+1}}{2+1} = \frac{x^3}{3}$$

1) A volume charge density is expressed as $\rho_v = 10z^2 x (\sin \pi y)$.

Find the total charge inside the volume $(-1 \leq x \leq 2)$

$(0 \leq y \leq 1)$ $(3 \leq z \leq 3.6)$

Soln

$$\rho_v = 10z^2 x (\sin \pi y)$$

$$Q = \int_V \rho_v \, dv$$

$$Q = \int_{-1}^2 \int_0^1 \int_3^{3.6} 10z^2 x (\sin \pi y) \, dx \, dy \, dz$$

$$Q = 10 \int_{-1}^2 \int_0^1 \int_3^{3.6} z^2 x (\sin \pi y) \, dx \, dy \, dz$$

$$Q = 10 \int_{-1}^2 x \, dx \int_0^1 \sin \pi y \, dy \int_3^{3.6} z^2 \, dz$$

$$Q = 10 \int_{-1}^2 x \, dx \int_0^1 \sin \pi y \, dy \int_3^{3.6} z^2 \, dz$$

$$Q = 10 \left[\left[\frac{x^2}{2} \right]_{-1}^2 \left[-\frac{\cos \pi y}{\pi} \right]_0^1 \left[\frac{z^3}{3} \right]_3^{3 \cdot 6} \right]$$

$$\int x^n dx = \frac{x^{n+1}}{n+1}$$

$$Q = 10 \left[\left[\frac{2^2}{2} - \frac{(-1)^2}{2} \right] \left[\frac{-\cos \pi(1)}{\pi} - \left(\frac{-\cos 0}{\pi} \right) \right] \left[\frac{(3 \cdot 6)^3}{3} - \frac{3^3}{3} \right] \right]$$

$$\int \sin \pi y dy = -\frac{\cos \pi y}{\pi}$$

$$\int \sin y dy = -\cos y$$

$$Q = 10 \left[\frac{4}{2} - \frac{1}{2} \right] \left[-\frac{\cos \pi}{\pi} + \frac{\cos 0}{\pi} \right] \left[\frac{46 \cdot 6}{3} - \frac{27}{3} \right]$$

$$Q = 10 \left[\frac{3}{2} \right] \left[-\frac{(-1)}{\pi} + \frac{1}{\pi} \right] [6 \cdot 55]$$

$$Q = 10 \left[1.5 \times \left(\frac{2}{\pi} \right) 6 \cdot 55 \right] = 62.57 \text{ C}$$

$$Q = 62.57 \text{ C}$$

