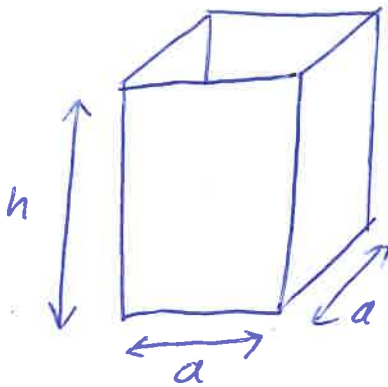


OPTIMISATION - A MATHEMATICAL EXAMPLE

①



WHAT IS h & a SUCH THAT

- VOLUME, V IS A MAX : OBJECTIVE
- AREA, $A = 2m^2$: CONSTRAINT
- a & h ARE : VARIABLES

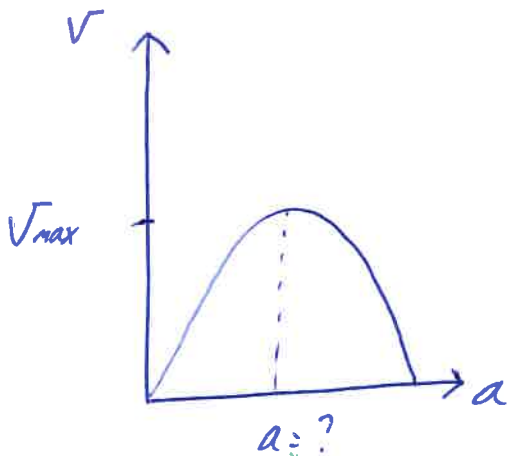
NOW $V = a^2 h$ ①

$$A = a^2 + 4ah$$

$$\therefore h = \frac{A - a^2}{4a}$$
 ②

PUT ② IN ①

$$\begin{aligned}\therefore V &= \frac{a^2(A - a^2)}{4a} \\ &= \frac{a(A - a^2)}{4}\end{aligned}$$



LET $x = a$: OUR VARIABLE

$$V = \frac{x(A - x^2)}{4} \quad (1) = \frac{xA}{4} - \frac{x^3}{4}$$

FOR MAX

$$\frac{dV}{dx} = 0 = \frac{A}{4} - \frac{3x^2}{4}$$

$$0 = \frac{A - 3x^2}{4}$$

$$3x^2 = A$$

$$x = \pm \sqrt{\frac{A}{3}}$$

~~$$\text{IF } \frac{dV}{dx} = \frac{A}{4} - \frac{3x^2}{4}$$~~

~~$$\frac{dV}{dx} = -3$$~~

NOW $A = 2$

$$x = \pm \sqrt{\frac{2}{3}} = \pm 0.816$$

OBTAINING THIS MUST
BE +ve.

$$x = 0.816 \text{ m}$$

FROM ①

$$V = \frac{x(A - x^2)}{4}$$

$$V = \frac{0.816(2 - 0.816^2)}{4} = \underline{\underline{0.272 \text{ m}^3}}$$

FROM ①

$$h = \frac{V}{a^2} = \frac{0.272}{0.816^2} = \underline{\underline{0.409 \text{ m}}}$$



British beans have five per cent more tomato in their sauce than the German product Photo: PAUL GROVER

By Fiona Govan

4:27PM BST 16 Sep 2009

European fans of the product, which for decades has been sold with the slogan "Beanz Meanz Heinz", were outraged after the British variety was declared superior to its European counterpart.

Tins in Germany and Austria were also found to have a more watery and less tasty sauce than the British equivalent.

An Austrian newspaper conducted a comparative study of the British beans with those sold locally after receiving complaints from its readers following the launch earlier this year of Heinz Gebackene Bohnen, a version of the British original destined for the German speaking world.

"The readers were correct," said David Rogers, editor of the *Austrian Times*. "We did a poll which found that many people felt the German Heinz baked beans were substandard and when we tested them we had to agree."

CAN OPTIMISATION EXAMPLE

①

MINIMISE MATERIAL USAGE FOR SAME VOLUME OF EXISTING CAN WHICH IS 80mm $\varnothing$ & 100mm TALL:



CONSTRAINT: EXISTING VOLUME, $V = \pi r^2 h$

$$= \pi \times (40 \times 10^{-3})^2 \times 100 \times 10^{-3}$$
$$= 5.03 \times 10^{-4} \text{ m}^3$$
$$= \underline{\underline{50.3 \text{ cL}}}$$

OBJECTIVE : MINIMISE
SURFACE AREA

VARIABLES ; r & h

• AREA

$$A = 2\pi r^2 + \text{SIDES}$$

$$= 2\pi r^2 + 2\pi r h$$

$$= 2\pi(r^2 + rh)$$

$$\underline{\underline{A = 2\pi r^2 + 2\pi r h}} \quad (1)$$

• VOLUME

$$V = \pi r^2 h$$

$$5.03 \times 10^{-4} = \pi r^2 h$$

$$\therefore h = \frac{5.03 \times 10^{-4}}{\pi r^2} \quad (2)$$

POT (2) IN (1)

$$A = 2\pi r^2 + 2\pi r \left(\frac{5.03 \times 10^{-4}}{\pi r^2} \right)$$

$$\underline{\underline{A = 2\pi r^2 + 10.06 \times 10^{-4} r^{-1}}}$$

FOR MINIMUM

$$\frac{dA}{dr} = 0$$

$$\therefore 0 = 2\pi \times 2r + 10.06 \times 10^{-4} \cdot -r^{-2}$$

$$0 = 4\pi r - 10.06 \times 10^{-4} r^{-2}$$

$$0 = 4\pi r - \frac{10.06 \times 10^{-4}}{r^2}$$

$$0 = 4\pi r^3 - 10.06 \times 10^{-4}$$

$$10.06 \times 10^{-4} = 4\pi r^3$$

3

$$\therefore r = 0.043m$$

$$= \underline{\underline{43mm}}$$

$$h = \frac{5.03 \times 10^{-4}}{\pi r^2} = \frac{5.03 \times 10^{-4}}{\pi \times (0.043)^2} = 0.086m$$

$$= \underline{\underline{86mm}}$$

CHANGE ϕ FROM 80mm TO 86mm
 h FROM 100mm TO 86mm } SAME AS EXPECTED!

$$AREA = 2\pi r^2 + 2\pi r h$$

$$A_{old} = 2\pi (40 \times 10^{-3})^2 + 2\pi (40 \times 10^{-3})(100 \times 10^{-3})$$

$$= \underline{\underline{0.0352m^2}}$$

$$A_{new} = 2\pi (43 \times 10^{-3})^2 + 2\pi (43 \times 10^{-3})(86 \times 10^{-3})$$

$$= \underline{\underline{0.0349m^2}}$$

$$SAVING \underline{\underline{3 \times 10^{-4} m^2}}$$

HEINZ SELL 1.5 MILLION CANS / DAY

$$= 3 \times 10^{-4} \times 1.5 \times 10^6$$

$$= \underline{\underline{450m^2}}$$