

ELEMENTS OF ANGLE MEASURING, TRAVERSING AND CALCULATION OF CO-ORDINATES

It was seen that the use of a magnetic compass obviated many of the restrictions imposed by chain surveying when setting out the basic survey network. The obvious drawbacks however, are the methods relative inaccuracy and susceptibility to local attraction of the compass card or needle.

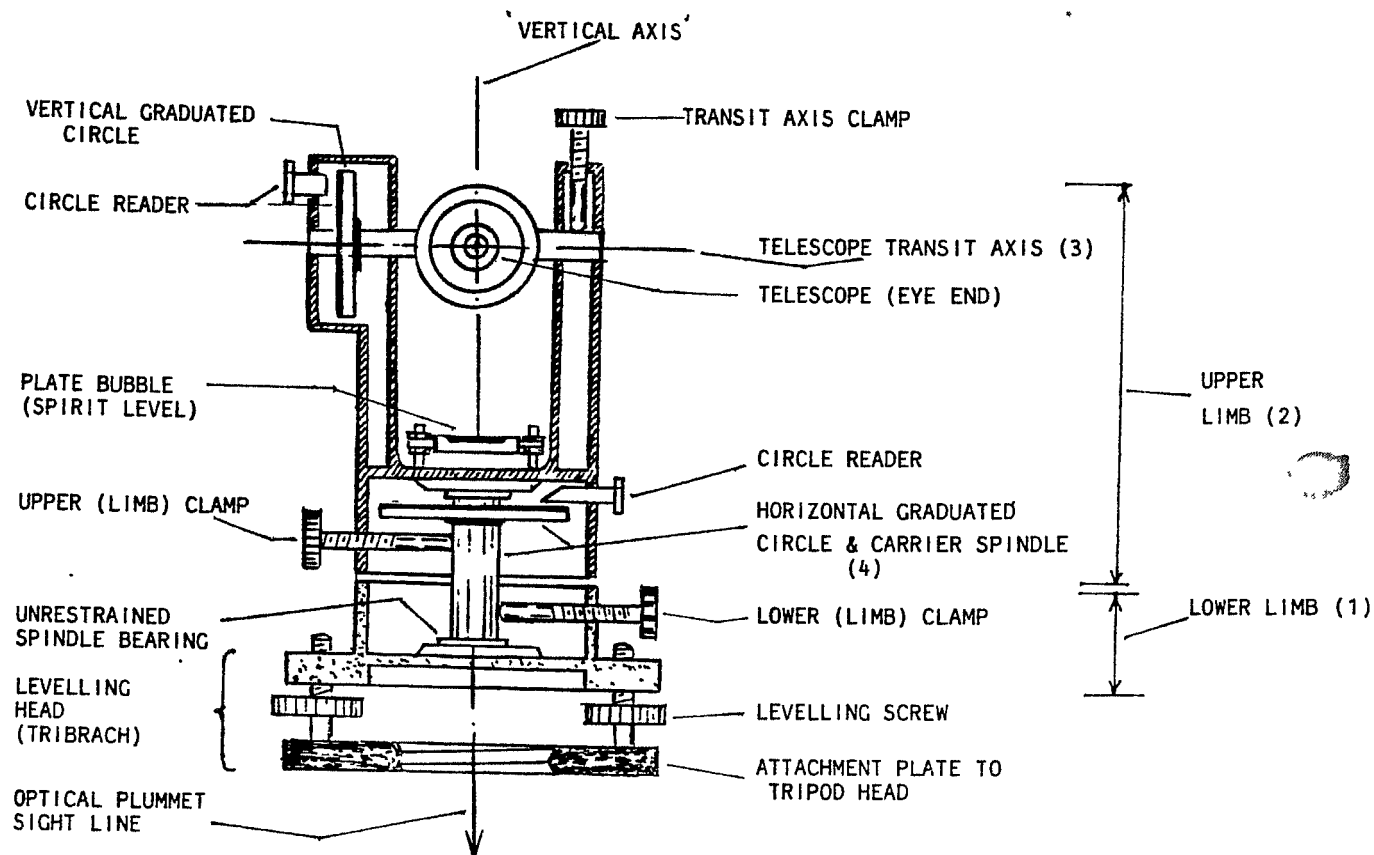
The third alternative is to measure the angles between survey lines. Depending on both the standard of angular and linear measurement very high degrees of accuracy are attainable, albeit that considerably more care is required to avoid cumulative error. As with compass surveying it has the advantage that as long as the lines comprise a closed figure it may be adequately verified without the use of check lines. In the event of particular subsidiary detail being required survey lines may be fixed within the main network in the most convenient positions for that purpose.

Angular Measurement

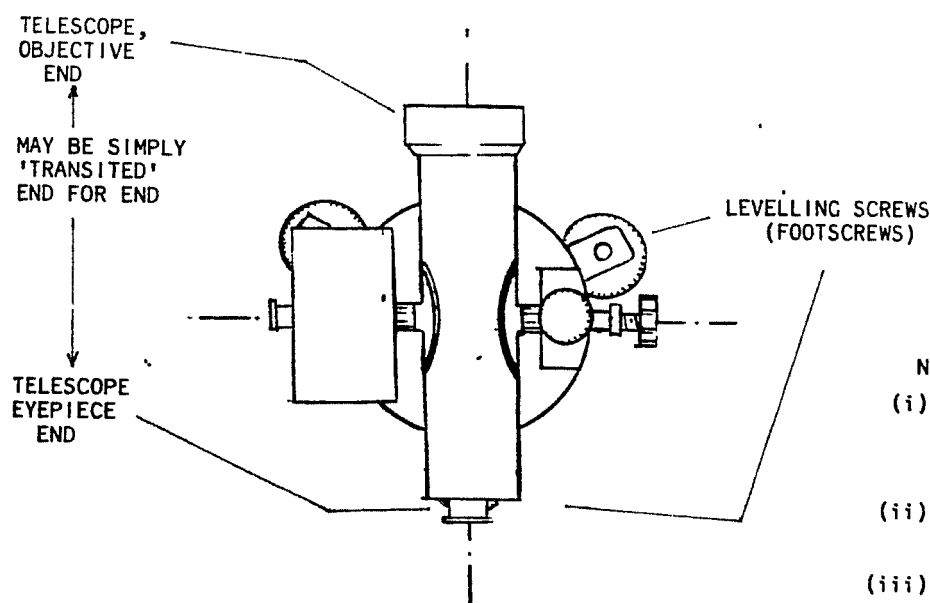
Stated simply, theodolites or optical transits are designed to measure accurate horizontal and vertical angles between distant points and the instrument itself. The design is such that if the vertical axis is truly vertical, (i.e. the plate bubble remains in the same position when rotated through 360°) by measuring angles on face left and face right the mean value will be virtually free of instrument errors. Fig.1 shows diagrammatically the basic features of a theodolite. No attempt is made to show usual refinements such as slow motion screws, optical micrometer systems, etc.

It will be seen that with the horizontal circle (lower) clamp off, the whole instrument may be rotated. In the clamped position only the upper portion (i.e. above horizontal circle) may be rotated. When the horizontal reference (upper) clamp is released. Such rotation may be read off against the horizontal circle. With both clamps applied the instrument is locked horizontally. Elevation or depression of the telescope when sighting is read against the vertical circle.

SIMPLIFIED FUNCTIONAL SKETCH OF 'THEODOLITE'/'OPTICAL TRANSIT' WITH DOUBLE CENTRES



SECTION THROUGH INSTRUMENT FROM OBSERVER'S POSITION ('FACE' OR 'CIRCLE' LEFT)



NOTES:

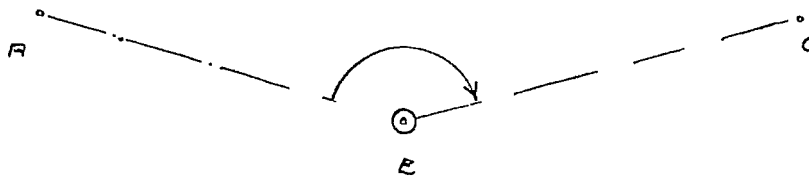
- (i) ASSEMBLIES MARKED (1), (2), (3) and (4), THOUGH COUPLED, MAY BE ROTATED INDEPENDENTLY.
- (ii) CLAMPS ARE PROVIDED WITH FINE (SLOW) ADJUSTMENT
- (iii) CIRCLE READERS ON ACTUAL INSTRUMENTS OPERATE VIA COMPLEX OPTICAL PATHWAYS TO DISPLAYS NEAR EYEPIECE.

PLAN VIEW OF INSTRUMENT

FIG. 1

Of the several methods of measuring angles two have been the most widely practiced. These are the repetition and reiteration methods respectively. The former, as implied, involves turning off the included angle several times so that its value accumulates. By dividing the final result by the number of times the angle was repeated, in theory at least, very accurate values are obtained. In practice however, it involves much manipulation of the lower clamp and slow motion screw and tends to conceal such inherent shortcomings as inadequate magnification for good station bisection. The method is now so unpopular that many modern instruments, by the omission of an orthodox lower clamp and slow motion screw, make its adoption impossible.

Reiteration may be executed by two procedures. In both the instrument is carefully centred and levelled over the station (B) and those to be observed (A & C) are marked clearly with targets.



(a) Closing the horizon.

- (i) With vertical circle "face left" and reference set to an approximately required reading on the horizontal circle, direct telescope to A, apply lower clamp and carefully bisect target. Take circle reading and book.
N.B. The lower clamp is not used in subsequent manipulation.
- (ii) Release upper clamp and "swing" right onto C clamp and bisect, take circle reading and book.
- (iii) Again release upper clamp and bisect A, read circle and book.
- (iv) Transit (turn end for end) telescope, release upper clamp and re-bisect and clamp on A, read circle and book. The instrument will now be in the "face right" position.
- (v) Release upper clamp swing telescope left onto C etc., and A etc. as before.

One round of angles has now been taken out of a minimum of two. The number of rounds will be increased with the importance or difficulty of the work. Other initial settings are chosen and the procedure repeated. If closure back onto A differs markedly from the original reading the round is rejected.

Angle ABC will be the mean of all satisfactory rounds.

(b) Double Foresight.

- (i) as in (a) (i)
- (ii) as in (a) (ii)
- (iii) Release upper clamp, transit telescope, swing it to left (being on face right) and re bisect C. Read circle.
- (iv) Release upper clamp, swing telescope left and bisect A, clamp and read horizontal circle.

The observing of vertical angles is similar in both procedures. Since the inclination is usually required to be that at which the linear measurements are taken it is usual to sight to targets on both faces at the same height as the instrument after completing horizontal angling. When a height target is bisected the altitude bubble adjusting screw is used to bring the bubble to the centre of its run and the inclination is read off the vertical circle at the reference point.

Examples of booking.

OBS = OBSERVED
Inst = Instrument

Closing Horizon.

Inst. Stn.	OBS. Stn.	Horizontal F.L.	Readings F.R.	Mean	OBS. Angle	ADJ.	Final Angle
B	A	1°01'20"	181°01'40"	1°01'30"	161°02'20"	+10"	161°02'30"
	C	162°03'40"	342°04'00"	162°03'50"	198°57'20"	+10"	198°57'30"
	A	1°01'00"	181°01'20"	1°01'10"	359°59'40"		360°00'00"

(360 + 1°01'00) - 181°01'20 = 198°57'20" ERROR
First round values, take at least one further round.

Double Foresight

Inst. Stn.	OBS. Stn.	Horizontal F.L.	Readings F.R.	Mean	Observed Angle
B	A	1°01'20"	181°01'40"	1°01'30"	161°02'20"
	C	162°03'40"	342°04'00"	162°03'50"	

360° - 359°59'40" = 20"

RO = Reference
objection

I used
this

First round values, take at least one further round.

Closing horizon is usually used with high order instruments for
surveying



Application of angular measurements

It is usual for the work to be carried out systematically, moving progressively from station to station and forming a traverse. From Fig.2 it may be seen that the angles of the simple figures may be measured whilst traversing in a clockwise or anti-clockwise direction, viz.

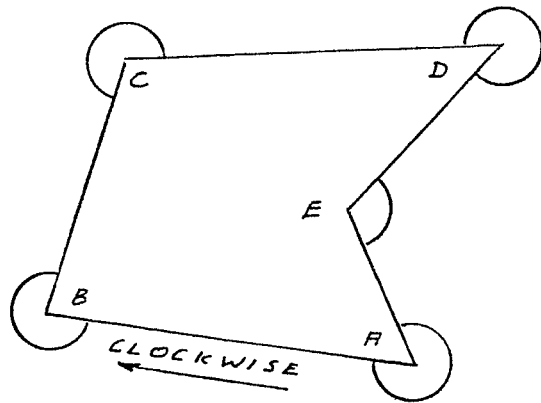
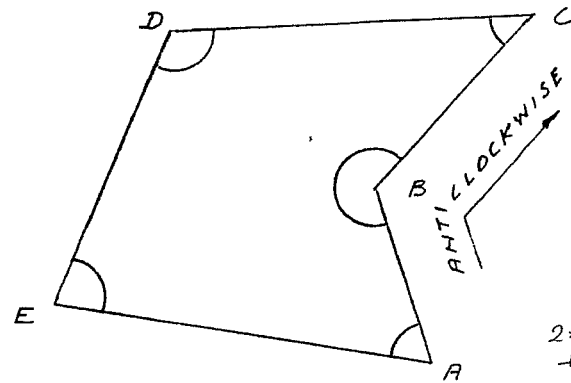


Fig.2

(a)



(b)

$$2 \times 90^\circ \times \text{Nos of Side} + 4 \times 90^\circ$$

$$540^\circ + 360^\circ$$

The convention, as in levelling, that a backsight is first taken followed by a foresight in the direction of the survey implies that either exterior or interior angles are measured depending on whether traversing clockwise or anti-clockwise.

In the former case the sum of the exterior angles should be equal to twice the number of right angles as the figure has sides, plus four right angles.

i.e. Fig.2 (a) $(2 \times 90^\circ \times 5) + (4 \times 90^\circ) = 1260^\circ 00' 00''$.

In the latter case the sum of the interior angles should be equal to twice the number of right angles as the figure has sides less four right angles.

i.e. Fig.2 (b) $(2 \times 90^\circ \times 5) - (4 \times 90^\circ) = 540^\circ 00' 00''$.

(The rules may be verified by considering a rectangle)

The first operation, having completed a traverse, is thus to sum up the mean observed angles and compare the results with the appropriate theoretical value.

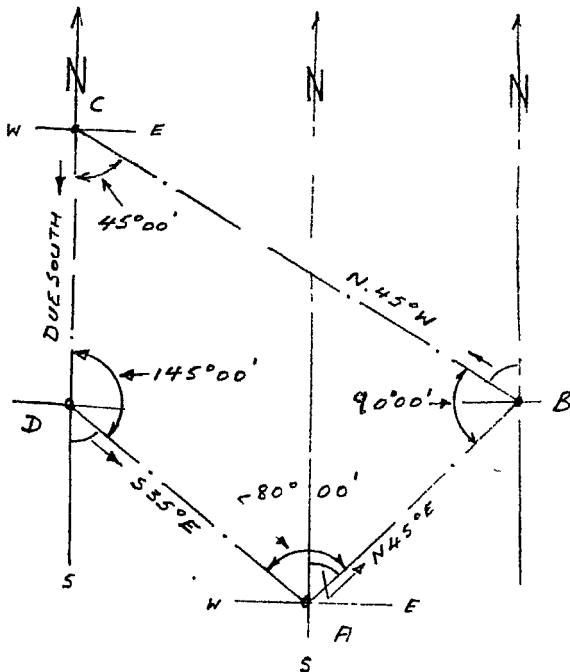
$$(2n \pm 4) 90$$

if Anti-clockwise interior angles
 if clockwise exterior angles

720

Bearings of Lines

The angles of the survey having been adjusted it is necessary for plotting and calculation to determine azimuth and quadrant bearings of survey lines.



Bearing A-B	45°00'	N45°00'E
Le B	90°00	
	135°00	
add	180°00	
Bearing B-C	315°00	N45°00'W
Le C	45°00	
	360°00	
subtract	180 00	
Bearing C-D	180°00'	Due S
Le D	145°00	
	325°00	
subtract	180°00	
Bearing D-A	145°00	S35°00'E
Le A	80°00	
	225°00	
subtract	180°00	
Bearing A-B	45°00'	N45°00'E

Having added the angle to the previous azimuth bearing the rules for finding the resulting bearing may be stated as follows:-

- (i) If sum is less than 180°, add 180°
- (ii) If sum is more than 180° subtract 180°
- (iii) If remainder is more than 360°, subtract 360°

Horizontal Length of Lines

Before plotting the traverse it is necessary to know the horizontal equivalent of the measured lengths of the survey lines.

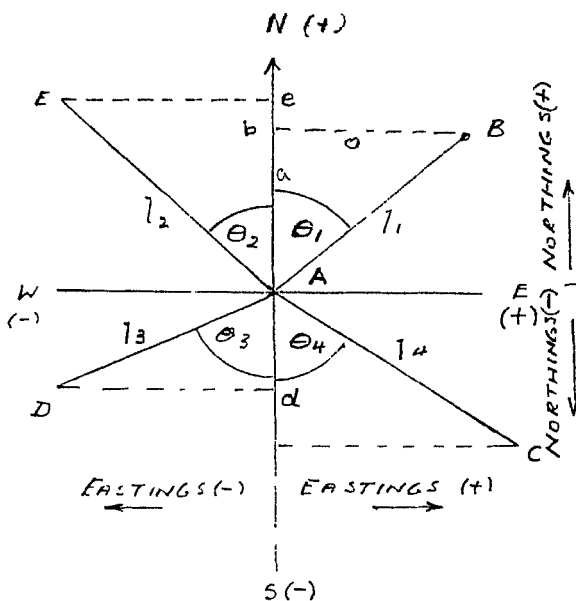
These can be found by the methods set out in the notes on chain surveying. The most convenient in traversing being horizontal length = Inclined length x Cos inclination.

40
80 S A
T C
270

Plotting the Traverse

Although sufficient data is now available to plot the survey as described in Magnetic Compass Traversing (and this is frequently done as a rough check) in order to obtain the full benefit of the precision of the work rectangular co-ordinates are determined for each survey station.

Rectangular Co-ordinates



From the sketch it will be seen that $\theta_1, \theta_2, \theta_3$ & θ_4 are the quadrant bearings of lines radiating from A to B, C, D & E respectively.

Also that the "horizontal" ordinates b-B, c-C, d-D & e-E may be calculated by multiplying the appropriate length by the sine of the particular value of θ involved

$$\begin{aligned} \text{i.e. } b-B &= l_1 \sin \theta_1 (+) \\ d-D &= l_3 \sin \theta_3 (-) \end{aligned}$$

Similarly the "vertical" ordinates may be found by multiplying the appropriate length by the cosine of the particular value of θ involved,

$$\begin{aligned} \text{i.e. } A-b &= l_1 \cos \theta_1 (+) \\ A-d &= l_3 \cos \theta_3 (-) \end{aligned}$$

The "horizontal" ordinates are known as Eastings (formerly departures) and the "vertical" ordinates as Northings (formerly Latitudes).

Eastings when measured in an easterly direction are regarded as positive and if westerly-negative. Northings when measured in a northly direction are regarded as positive and if southerly as negative.

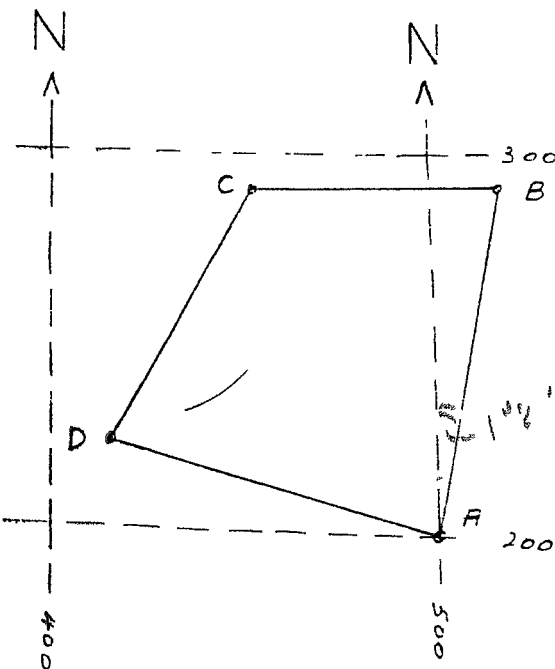
By calculating individual or partial values of Eastings and Northings for successive survey stations and adding these progressively and algebraically the relativity of any station in the survey to other stations may be determined.

Further, if the 'Total' co-ordinate value of a surveyed station relative to the origin of a particular co-ordinate grid is known, all other surveyed stations may be related to that grid and any other surveys orientated thereto.

The calculation of the co-ordinates of a simple traverse, shown, has been carried out using 5 figure logarithmic tables.

With the advent of moderately priced scientific calculators the cumbersome use of tables may be avoided. To obtain, say, sin values however, the calculator must handle degrees and decimals thereof only. Some are provided with a direct conversion facility. If not, it is a simple matter to change, say, $0^{\circ}22'15''$, i.e. $(22 \times 60) + 15''/3600 = 0^{\circ}.3708$. To convert back $0^{\circ}.3708$ multiply by 60 = $22'.248$, and, seconds = $0.248 \times 60 = 14.88''$ or $15''$. Total = $0^{\circ}22'15''$.

As in levelling since the traverse is closed the sum of eastings should = 0 similarly $\sum \Delta N = 0$. To adjust for same the Bowditch method, p.11 is applied. The following example illustrates the points touched upon.



Bearing A-B No. $1^{\circ}02'E$ Co-ords A E500.00
N 200.00 Mean Angles A $72^{\circ}09'$ B $88^{\circ}59'$
C $130^{\circ}53'$ D $68^{\circ}03' = 360^{\circ}04'$ Adjusted
Angles A $72^{\circ}08'$ B $88^{\circ}58'$ C $130^{\circ}52'$
D $68^{\circ}02' = 360^{\circ}00'$

Lengths & inclinations

A-B	80.76 m Level	
B-C	36.16 m dipping $5^{\circ}07'$	Horizontal 36.020
C-D	70.32 m Level	
D-E	85.23 m rising $2^{\circ}10'$	85.165
		85.165

SKETCH PLAN OF
SURVEY SUPERIMPOSED
ON GRID

A-B	1°02'	N 1°02'E	D-A	108°54'
L ^e B	<u>88°58</u>		L ^e A	<u>72°08'</u>
	90°00			181 02
+ 180 00			- 180 00	
B-C	270°00	Due W	A-B	1°02' N 1°02'E
L ^e C	<u>130 52</u>			
	400 52			
- 180 00				
C-D	220 52	S 40°52' W		
L ^e D	<u>68 02</u>			
	288 54			
- 180 00				
D-A	108 54	S71°06'E		

T	LINE	A Z BRG	Quad BRG	Horiz. Length m	Log Eastings	PARTIAL m		TOTAL m	
					Log Sin Brg. Log length Log Cos Brg. Log Northings	EASTINGS	NORTHINGS	EASTINGS	NORTHINGS
	AB	1°02'	N1°02'E	80.760	0.16329 2.25609 1.90720 1.9999 1.9071	+ 1.456 (-0.001)		500.00 (501.455) 501.456	200.00 (280.753) 280.747
	BC	270°00'	Due W	36.020	Not Required	(-)36.020 (- 0.000)		(465.435) 465.436	(280.775) 280.747
	C-D	220°52'	S40°52'W	70.320	1.66286 1.81578 1.84708 1.87866 1.72574	(-)46.010 (-0.001)		(419.424) 419.426	(227.582) 227.569
	D-A	108°54'	S71°06'E	85.165	1.90619 1.97593 1.93026 1.51047 1.44073	+80.578 (- 0.002 Rounded up to balance)	(+ 0.006) (-)27.588	(500.000) 500.004	(200.00) 199.981
				272.265	Totals	(+)0.004	(-) 0.019	To be adjusted	

BOWDITCH ADJUSTMENT FOR CLOSED SURVEYS

Summation errors in both eastings and northings are distributed to individual values in proportion to the length of the particular line concerned compared with the total horizontal length of traverse, i.e.

$$\text{adjustment} = \text{length of particular line} \times \frac{\text{summation error}}{\text{total length of traverse}}$$

All values may be conveniently held in a calculator without writing down details.

e.g. Take the error in the northing values on P8 which is (-) 0.019

The adjustment to the northings line AB = $+ 80.747 \times \frac{0.019}{272.265}$ or 6.98×10^{-5}

a constant (K)

$$AB = + 80.747 \times K = 0.006$$

$$BC = + 36.020 \times K = 0.002$$

$$CD = + 70.320 \times K = 0.005$$

$$DA = + 85.165 \times K = 0.006$$

As will be realised,
AB is accurate enough

$$\begin{array}{rcl} \text{Check} & + & 0.019 \checkmark \\ 81m \times K & = & 0.006 \end{array}$$

The error in the eastings is dealt with in the same way though being very small it will only 'show up' in the longer lines.

STANDARDS OF ACCURACY

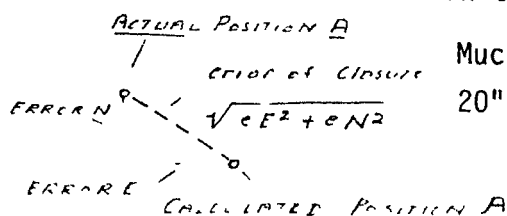
ANGULAR : In the example the error of summation is $0^{\circ}04'00''$.

Somewhat higher than would usually be expected. Using a 20" theodolite for normal work it should not exceed $\pm 20'' \sqrt{n}$ (n = number of stations). In this case the theodolite reads to a minute of arc hence summation error should not exceed $\pm 60'' \sqrt{4} = 02'00''$ ($\frac{1}{2}$ of that in example)

LINEAR

After applying angular correction this is calculated using the error of closure, given by 1 in $\frac{\text{Horizontal traverse length}}{\text{error } E^2 + \text{error } N^2}$

In the example 1 in $\frac{272.256}{\sqrt{0.004^2 + 0.019^2}} = 1 \text{ in } 14022$
Much better than normal. When using a 20" theodolite 1 in 10,000 would be adequate.



1000 ft closing error underground min work