

RH186462 Mechanical Assignment

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Applied Mechanics

Complex static and dynamic systems

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Summary

The assignment details the topics of complex static and dynamic systems where they are critically evaluated using a range of descriptive and calculated examples. Analytical work into the systems of non-coplanar balancing, epicyclic gearing, lifting devices, thick and thin walled cylinders and modes of torsion are presented, to demonstrate practical application of the discipline's theoretical concepts.

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1.0 Question 1

1.1 Task (a)

The process and application of balancing, is defined by Mehta (2016) who describes the technique as *“a method of correcting or eliminating unwanted inertia forces or moments in rotating or reciprocating masses”*.

Essentially the effective balancing of a system can be achieved through the addition or removal of mass in order to adjust a component's centre of gravity to match the subjected rotational axis. The fundamental concepts of balancing are based upon Newton's law of linear and angular motion. Due to the proportional relationship of force and speed as exhibited in Newton's law, the larger the mass or acceleration, the more significant the unbalance becomes. Wowk (1999) highlights this consequence writing *“if the speed is doubled, the force quadruples; if the speed is tripled the force increases by a factor of nine”*.

The effects of an unbalanced system has serious detrimental effects to both the lifespan of equipment and to the safety of its operators, ultimately resulting in the destruction of a component after a period of use through vibration and fatigue mechanisms.

The varying types of balancing can be grouped into static and dynamic systems. The former consists of force in relation to gravity, for instance as a mass is rotated its culmination of centrifugal force is equal to zero in any radial path. An example of static balancing can be demonstrated through the experiment of pivot balance (Sanders, 1947) where mass is moved along either side of a pivoted beam to the result of equilibrium in the moments of force against gravity.

The latter system of dynamic balancing is witnessed through the interaction of forces of inertia, Wowk (1999) describes this type of balance writing *“A rotating mass is said to be dynamically balanced when it does not vibrate in its running state”*.

Balancing can be achieved when the resulting moments involved, in the angular acceleration of dynamic components is equal to zero. An example of the importance of dynamic balancing can be seen in the rotation of a simple wheel, a dynamic imbalance would result in the ineffective capability of the wheel to spin, causing both vertical and lateral vibration. In high speed, high precision engineering such as a motor engine or aircraft propeller fabrication, the effects of dynamic imbalance result in significant detrimental consequence to the reliability and safe operation of equipment.



1.2 Task (b)

"An experimental balancing rig consists of four discs A, B, C and D mounted on a rotating shaft, the linear distances between each of the discs being 0.5m. On each disc provision is made for the attachment of unbalanced masses at different radii and angular displacement. If a mass of 0.6kg at radius 120mm is attached to B at 0° and a mass of 0.3kg at radius 120mm is attached at C at 120° find the magnitude and angular position relative to B of masses placed at A and D at radii 100mm and 90mm respectively to give the system complete static and dynamic balance."

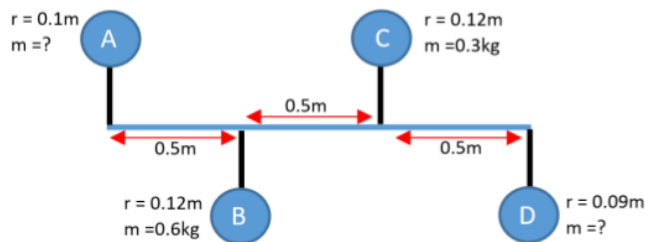


Figure 1: Co-planar balancing diagram

For complete balance Σ centrifugal forces = 0 and Σ moments = 0

Solving graphically an M R L table is constructed:

Plane	Mass (kg)	Radius (m)	M R (kgm)	Length(m)	M R L (kgm ²)	θ
A	M_A	0.1	$0.1M_A$	0	0	θ_A
B	0.6	0.12	0.072	0.5	0.036	0°
C	0.3	0.12	0.036	1	0.036	120°
D	M_D	0.09	$0.09M_D$	1.5	$0.135 M_D$	θ_D

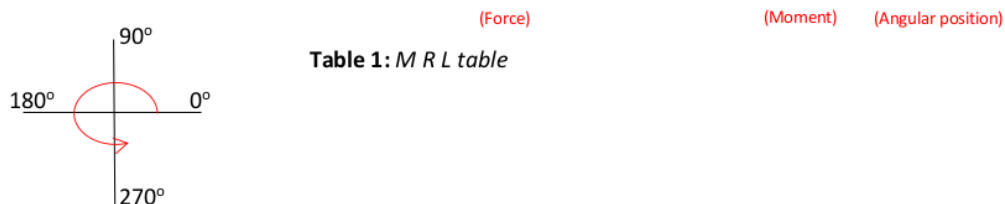


Table 1: M R L table

An MRL polygon (moment) is constructed; scale set 1cm = 0.01 Kg m²

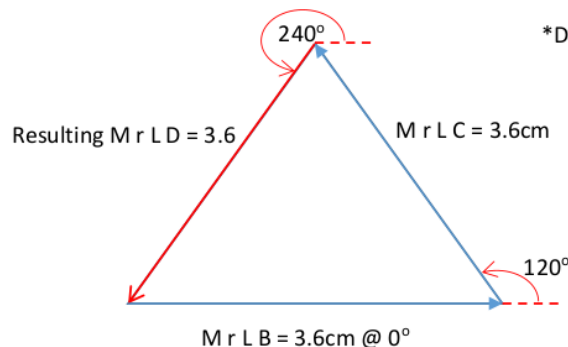


*Diagram for illustration only – not to scale

Mass of D = $0.135m_D =$

$$\frac{(3.6 \times 0.01)}{0.135} = 0.26$$

Therefore:

Mass of D required = 0.26 kg

Angular position D = 240°

An MR polygon (force) is constructed; scale set 1cm = 0.01 Kg^m²

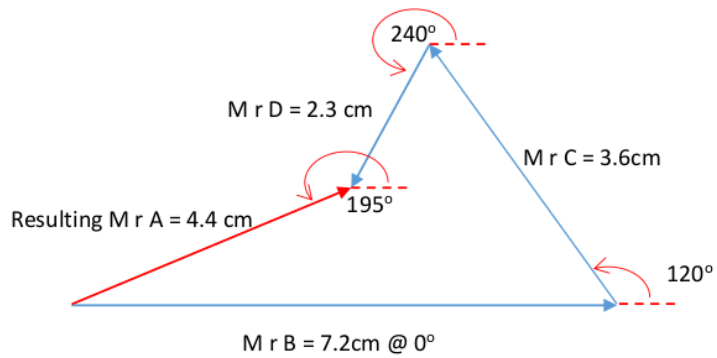


*Diagram for illustration only – not to scale

Mass of A = $0.1m_A =$

$$\frac{(4.4 \times 0.01)}{0.1} = 0.44$$

Therefore:

Mass of A required = 0.44 kg

Angular position D = 195°

The completed M R L table is formulated:

Plane	Mass (kg)	Radius (m)	M r (kgm)	Length(m)	M r L (kgm ²)	θ
A	0.44	0.1	0.044	0	0	195°
B	0.6	0.12	0.072	0.5	0.036	0°
C	0.3	0.12	0.036	1	0.036	120°
D	0.26	0.09	0.023	1.5	0.035	240°

Table 2: Completed M R L table



2.0 Question 2

2.1.1 Task 1 (a)

A typical epicyclic gearbox configuration;

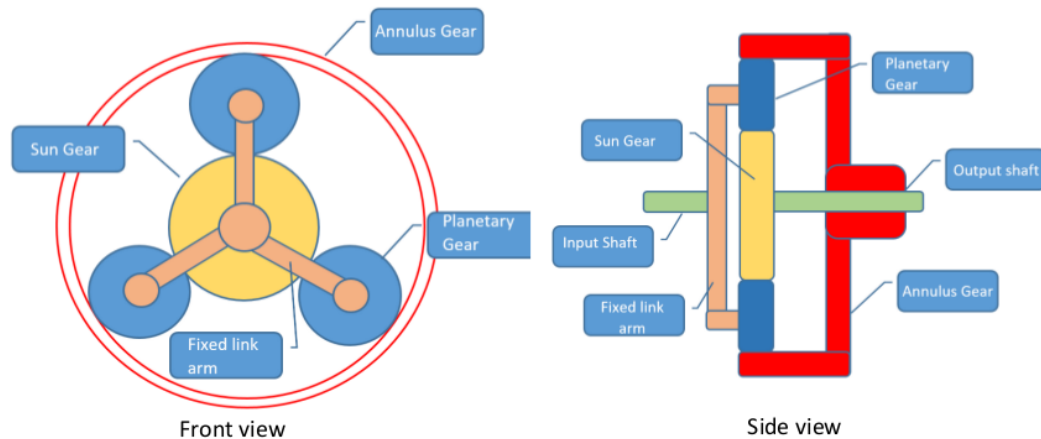


Figure 2: *Epicyclic gearbox configuration*

2.1.2 Task 1 (b)

Sun Gear – The sun gear is positioned centrally to the epicyclic gearbox and in a typical application is connected to a motor shaft via a mechanically connected clamping system, the outer surface of the sun gear is meshed with the planetary gears.

Planetary gear – The gear or gears are situated on an evenly spaced concentric orbit to the sun gear and are housed via connection with the fixing arm and mesh linkages between the sun and annulus gears. The number of planets housed within the gearbox has a direct effect on the distribution of load, altering the number of teeth between the various gears results in a range of torque transmission ratios that a system is capable of.

Annulus Gear – Often considered the housing of the gearbox the ring annulus is configured to have internal integrated teeth that secure the planetary gears in place. The annulus gear is constrained via a mechanical clamping system which can be fixed or driven to transfer the resulting torque transmission.

Fixed link arm – The fixed link arm constrains the planetary gears in concentric orbit around the sun gear.

Input/ output shaft – The component shaft mechanically connects the gears to the driving force and output receiver of the system conveying the resultant torque transmission energy for use.

2.1.3 Task 1 (c)

Expressions when the sun, planet and ring gears are fixed can be gained through the method of relative rotation:

2.1.3.1 (c) (i) The arm is fixed – Star arrangement

Assumptions:

- The sun (S) gear is the driver.
- The annulus (A) gear is driven.
- The arm (L) is fixed and therefore planetary gears (P) are idle.
- T_x = number of gear teeth, $T_A = T_S + 2(T_P)$

	S	L	P	A
Fix L and rotate sun + 1 rev	+1	0	$+1(\frac{T_S}{T_P})$	$+1(\frac{T_S}{T_A})$

Table 3: Expression table for star arrangement

Therefore the ratio (R) expression for a gearbox in star arrangement with a Sun gear driver =

$$\frac{R_A}{R_S} = -1(\frac{T_S}{T_A})$$

2.1.3.2 (c) (ii) The sun is fixed – Solar arrangement

Assumptions:

- The planetary arm (L) is the driver.
- The annulus (A) gear is driven.
- The sun (S) gear is fixed.
- T_x = number of gear teeth, $T_A = T_S + 2(T_P)$

	S	L	P	A
Rotate whole gear train +1 rev	+1	+1	+1	+1
Fix L and rotate fixed gear – 1 rev	-1	0	$+1(\frac{T_S}{T_P})$	$+1(\frac{T_S}{T_A})$
Add lines 1 and 2 together	0	1	$1 + 1(\frac{T_S}{T_P})$	$1 + 1(\frac{T_S}{T_A})$

Table 4: Expression table for solar arrangement

Therefore the ratio (R) expression for a gearbox in solar arrangement with a planetary arm

as a driver = $\frac{R_A}{R_L} = 1 + 1(\frac{T_S}{T_A})$

2.1.3.3 (c) (iii) The ring is fixed – Planetary arrangement

Assumptions:

- The planetary arm (L) is the driver.
- The sun (S) gear is driven.
- The annulus (A) gear is fixed.
- T_x = number of gear teeth, $T_A = T_S + 2(T_P)$

	S	L	P	A
Rotate whole gear train +1 rev	+1	+1	+1	+1
Fix L and rotate fixed gear – 1 rev	$+1 \left(\frac{T_A}{T_S} \right)$	0	$-1 \left(\frac{T_A}{T_P} \right)$	-1
Add lines 1 and 2 together	$1 + 1 \left(\frac{T_A}{T_S} \right)$	1	$1 - 1 \left(\frac{T_A}{T_P} \right)$	0

Table 5: Expression table for planetary arrangement

Therefore the ratio (R) expression for a gearbox in planetary arrangement with a planetary

arm as a driver = $\frac{R_S}{R_L} = 1 + 1 \left(\frac{T_A}{T_S} \right)$

2.1.4 Task 1 (d)

In an epicyclic gearbox where all gears are free to rotate expressions for each of the components can be constructed using an altered relative motion method as below:

	S	L	P	A
Rotate whole gear train +m revs	+m	+m	+m	+m
Fix L and rotate sun gear + n revs	+ n	0	$-n \left(\frac{T_S}{T_P} \right)$	$-n \left(\frac{T_S}{T_A} \right)$
Add lines 1 and 2 together	$m + n$	m	$m - n \left(\frac{T_S}{T_P} \right)$	$m - n \left(\frac{T_S}{T_A} \right)$

Table 6: Expression table for free rotation arrangement

2.1.5 Task 1 (e)

Typical applications of epicyclic gear boxes are best suited to situations that require fast responses and the requirement of varying torque/output ratio transmissions. Nunney (2016) writes of the design of epicyclic gearboxes commenting *“The system is so named because of the behaviour of its geared elements, which rotate about a common axis, similar to that found in our solar system”*.

A most prominent example of epicyclic gearing in industrial application is its deployment within automatic transmissions of the automotive industry. The different gearing combinations presented by fixing and driving different components of the system, allow for selection between high and low torque selection in a compact and highly efficient system. Other uses include cranes that rely on increased and efficient transmission of torque to provide the necessary lifting strength. Gearing of this type also extends to application within mechanical watches and clockwork toys, which take advantage of their compact and predictable configuration.

It is demonstrated in the examples above that epicyclic gearboxes have the both operating and size advantages over traditional standard gear trains Nunney (2016) highlights a further example commenting *“Changes from one ratio to another can be effected without interruption of the drive”*.

Nunney identifies that when compared to similar gearing ratios, epicyclic configurations are lighter in weight and more compact. Chen (2014) offers an alternative perspective offering that epicyclic gear trains may have relatively low efficiency compared to simple gear system counterparts. However the added advantages of load distribution through the addition of planetary gears around the sun gear, and the nature of system stability provided by the configuration of gearing arrangement presents the provision and ease of switching gearing ratios by the fixing of various components. The features demonstrated by this meshed configuration make the use of epicyclic gearboxes suitable to a wide and encompassing range of industry uses.

2.1.6 Task 1 (f)

When considering the design and function of complex and compound epicyclic gear trains McFadden (2016) highlights the prominent design feature commenting *“the possibility of higher power to weight ratio is possible, while keeping the input and output shaft coaxial”*.

In keeping the input and output shafts in-line, the compound epicyclic system is adaptable to situations where limited space and auxiliary component configuration is restricted. The functionality of this design allows for the acceleration or reduction in final output transmission and the ability to switch between speeds and ratios without the need for multiple gear trains, which in a simple gear train would need to be situated adjacent to each other. In addition the enhanced capability of increasing the amount of planetary gears, reduces the amount of load by distribution across a larger surface area, the resulting effect reduces gear and teeth wear and increases the systems suitability to wider applications.

The system however is not without disadvantage, Forsthoffer (2011) identifies that compounded and complex epicyclic gearing involve maintenance intensive components. This aspect becomes apparent when considering the arrangement of the components, and the configuration of multiple gears, bearings and lubricant housed in an enclosed entity. If problems with the gear box were to occur the complexity of the different housings and gear faces become difficult to replace, rectify and monitor when compared to simple gear trains.



2.2 Task 2

"A compound epicyclic gear box is as shown below. Gear C has 200 teeth, B has 50, D has 50 and E has 200. Calculate the ratio of the gear box. Assume that B is fixed and that C and D are rigidly connected."

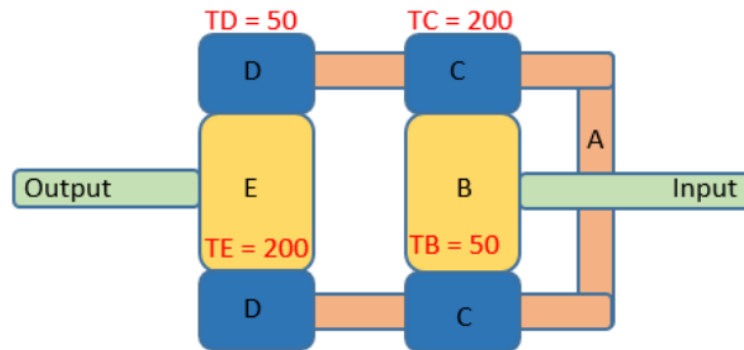


Figure 3: Epicyclic gearbox diagram

Assumptions:

- The sun gear (B) is the driver.
- Gears C and D are both planetary gears.
- The system's input is through B and output of E for the overall ratio.

Compound gear features calculated using the relationship: $\frac{\text{Driver}}{\text{Driven}}$ i.e. $\frac{B \times D}{C \times E}$

	A	B	C/D	E
Rotate whole gear train +1 rev	+1	+1	+1	+1
Fix Arm (A) and rotate fixed gear (B) – 1 rev	0	-1	$+1 \left(\frac{50}{200} \right)$	$-1 \left(\frac{50 \times 50}{200 \times 200} \right)$
Add lines 1 and 2 together	1	0	1.25	0.94

Table 7: Completed gear arrangement

Therefore the ratio for a gearbox can be stated as 0.94 : 1



3.0 Question 3

"The lifting device shown in figure Q3 consists of a motor M driving a drum D through a reduction gearbox. The load is lifted by a cable wrapped around the drum. Using the data provided calculate the vertical acceleration of the load neglecting the mass of the cable. Take the moment of inertia of the motor and gears on the motor shaft as 0.04 Kg m^2 and the moment of inertia of the drum and cables on the drum shaft as 25 Kg m^2 . The gear reduction is 30:1, the drum diameter is 800mm, the mass of the load is 500kg, the motor torque is 104Nm and the mechanical efficiency of the motor is 85%."

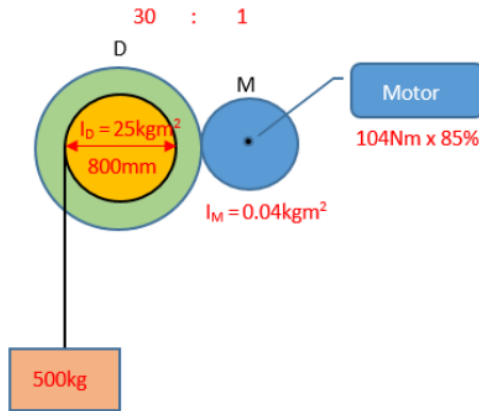


Figure 4: Lifting device diagram

Using the expressions for α (alpha) ω (omega) and neglecting frictional torque the total power required is considered:

$$T \cdot \omega = \begin{matrix} \text{Power needed to} \\ \text{accelerate motor} \\ \text{\& gearbox} \end{matrix} + \begin{matrix} \text{Power needed to} \\ \text{accelerate the} \\ \text{drum} \end{matrix} + \begin{matrix} \text{Power needed to lift} \\ \text{and accelerate the} \\ \text{load.} \end{matrix}$$

Therefore the governing equation is formed:

$$0.85 (T) (\omega_M) = (I_M \alpha_M) \omega_M + (I_D \alpha_D) \omega_D + m(g + a)r_D \cdot \omega_D$$

Substituting in the values of the question where:

$$T = 104 \text{ Nm}, \quad I_M = 0.04 \text{ kg m}^2, \quad I_D = 25 \text{ kg m}^2, \quad m = 500 \text{ kg}, \quad r_D = 0.4 \text{ m},$$

The relationship of velocity and acceleration at the motor can be considered through the ratio of the gearbox and the drum: $\omega_M = 30 \cdot \omega_D$ and $\alpha_M = 30 \cdot \alpha_D$

Therefore:

$$0.85 (104) (30 \times \omega_D) = (0.04 (30 \times \alpha_D))(30 \times \omega_D) + (25 \times \alpha_D) \omega_D + 500(9.81 + a)(0.4) \omega_D$$

Collecting the terms and removing the common term ω_D :

$$0.85 (104) (30 \times \cancel{\omega_D}) = (0.04 (30 \times \alpha_D)) (30 \times \cancel{\omega_D}) + (25 \times \alpha_D) \cancel{\omega_D} + 500(9.81 + a)(0.4) \cancel{\omega_D}$$

$$\therefore 2652 = 36 \alpha_D + 25 \alpha_D + 1962 + 200a$$

Using the relationship that: $\alpha_D = \frac{a}{r_D} \therefore \alpha_D = \frac{a}{0.4}$

$$\therefore 2652 = 36 \left(\frac{a}{0.4}\right) + 25 \left(\frac{a}{0.4}\right) + 1962 + 200a$$

$$\therefore 2652 - 1962 = 61 \left(\frac{a}{0.4}\right) + 200a$$

$$\therefore 690 = 61 \left(\frac{a}{\frac{4}{10}}\right) + 200a$$

$$\therefore 690 = \frac{610}{4} a + 200a$$

$$\therefore 690 = 152.5 a + 200a$$

$$\therefore 690 = 352.5a$$

$$\therefore \frac{690}{352.5} = a = 1.96$$

The vertical acceleration of the load is calculated to be 1.96 ms^{-2}



4.0 Question 4 – 4.1 Task (a)

The defining state of whether a cylinder is considered thick or thin, lies in the components relationship between thickness and diameter. Investigating the design and fabrication of a given cylinder, if when calculated the diameter over thickness is greater than a factor of 20 then the cylinder is considered thin. In cases such as these circumferential and longitudinal stress are considered constant across the cylinders thickness, and thus the resistant hoop stress is also considered constant which constitutes an essential factor when accounting for pressure calculations and safe limits of operation.

In circumstances where the diameter over thickness is less than 20, the cylinder is considered thick, due to increase in the variance of stresses and pressure gradient that enact on both the internal and external surfaces of a thick walled cylinder, the circumferential and longitudinal stress are again taken in account with the additional consideration of radial stress using Lamé's theory (Fisher Pub. Co, 1929). Conducting analysis of the principal stresses enacting on the thick walled cylinder, allow for safe working pressure envelopes to be established. An example industry application of a thick walled cylinder can be found in the use of pressure vessels within the system of a hydraulic press. The use of a thick walled cylinder in this instance, is to enable the hoop stress to be kept safely below the material yield stress, in order to withstand the action of significant pressure without rupture or fracture of the vessel.

Phalguna (2017) identifies the issues of stress distribution on thick walled cylinders writing that *"A pressurised thick cylinder may undergo failure in two modes i.e. elastic failure and plastic failure."*

This can be attributed to the deformation of the cylinder wall beyond the materials yield and ultimate strength capacity, resulting in failure. Stresses of this kind can also be significantly raised at areas of structural and geometrical discontinuity such as welds, change in section and locations of mechanical damage. Environmental and process factors also present an issue to thick walled cylinders, as higher tri-axial stresses are particularly prone to the mechanism of brittle fracture. The American petroleum institute (2013) provide definition of the failure mode writing *"the sudden rapid fracture under stress, where material exhibits little or no evidence of ductility or plastic deformation"*.

Successful mitigation of this mechanism can be completed at the design stage of the cylinder by implementing post fabrication heat treatment, in doing so residual stress created in the manufacturing process are prominently reduced.

4.2 Task (b)

"A steel pipe of internal diameter 30mm and external diameter 120mm is subject to an internal pressure of 12MPa and an external pressure of 5MPa."

(i) What are the radial and circumferential stresses at the inner and outer surfaces?

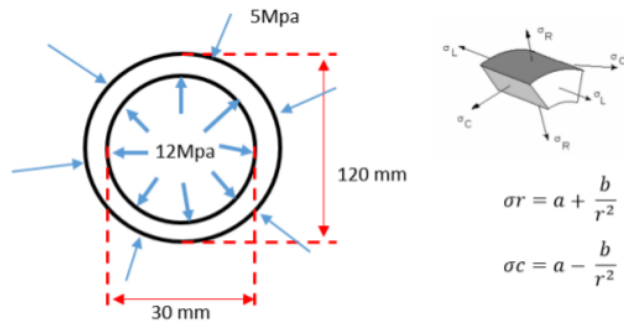


Figure 5: Radial and circumferential Parameter diagram

Firstly boundary conditions are established:

- (i) When the internal radius = 0.015m
The radial stress = 12MN/m²
∴ $r = 0.015\text{m}$ and $\sigma_r = -12 \times 10^6$
- (ii) When the external radius = 0.06m
The radial stress = 5MN/m²
∴ $r = 0.06\text{m}$ and $\sigma_r = -5 \times 10^6$

From:

$$\sigma_r = a + \frac{b}{r^2} \quad \therefore -12 \times 10^6 = a + \frac{b}{(0.015)^2} \quad \text{----- Equation 1}$$

$$\text{And} \quad -5 \times 10^6 = a + \frac{b}{(0.06)^2} \quad \text{----- Equation 2}$$

From equation 2:

$$-5 \times 10^6 = a + 277.7b$$

$$\therefore a = -5 \times 10^6 - 277.7b \quad \text{----- Sub back into equation 1}$$

$$\therefore -12 \times 10^6 = (-5 \times 10^6 - 277.7b) + 4444.4b$$

$$\therefore -12 \times 10^6 = -5 \times 10^6 + 4166.7b$$

$$\therefore -12 \times 10^6 + 5 \times 10^6 = 4166.7b$$

$$\therefore -7 \times 10^6 = 4166.7b$$

$$\therefore \frac{-7 \times 10^6}{4166.7} = b = -1680$$

Returning -1680(b) to find a:

$$a = -5 \times 10^6 - 277.7(-1680) \\ = -4.5 \times 10^6$$

With the entities of a and b found the circumferential stress can be calculated at both boundary parameters:

$$\sigma_c = a - \frac{b}{r^2} \quad \text{When } r = 0.015\text{m:} \quad -4.5 \times 10^6 - \frac{(-1680)}{(0.015)^2} = 2.9 \times 10^6$$

$$\sigma_c = a - \frac{b}{r^2} \quad \text{When } r = 0.06\text{m:} \quad -4.5 \times 10^6 - \frac{(-1680)}{(0.06)^2} = -4 \times 10^6$$

To summarise the results a table of boundary conditions is constructed:

Radius (m)	$\sigma_r \text{ (N/m}^2\text{)}$	$\sigma_c \text{ (N/m}^2\text{)}$
0.015	-12×10^6	2.9×10^6
0.06	-5×10^6	-4×10^6



Table 8: Stress boundary conditions

(ii) Sketch the variation of hoop stress across the material, clearly showing the maximum and minimum positions of hoop stress.

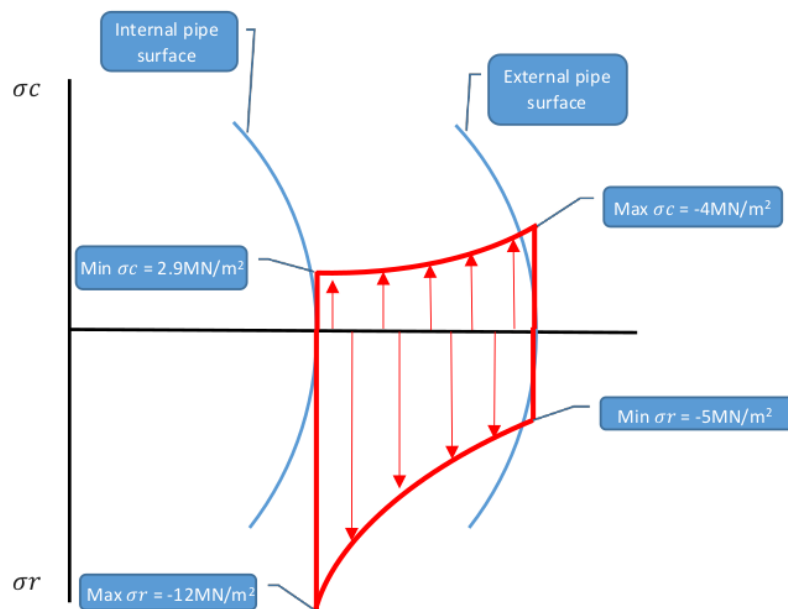


Figure 6: Hoop stress diagram

5.0 Question 5

"The shaft shown consists of a 30mm diameter solid section CD and a tube AC that has an outside diameter of 45mm and an inside diameter of 35mm. The shaft is subjected to concentrated torques at B, C, and D. The shear modulus of elasticity is $G = 75 \text{ GPa}$."

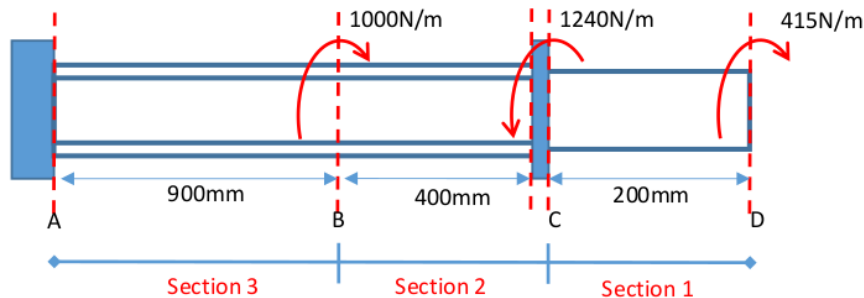


Figure 7: Shaft parameter diagram

(i) The maximum shear stress in the shaft.

Establish Torque values at the sections:



Clockwise direction = +ve $\therefore T_1 = -415 \text{ N/m}$

$$T_2 = -415 + 1240 = 825 \text{ N/m}$$

$$T_3 = -415 + 1240 - 1000 = -175 \text{ N/m}$$

If:

$$\frac{T}{J} = \frac{\tau}{r} = \frac{G \theta}{L} \quad \therefore \frac{T}{J} = \frac{\tau}{r} \Rightarrow \tau = \frac{T r}{J}$$

Establish J value for sections:

$$\text{Section AC: } J = \frac{\pi D^4}{32} = \frac{\pi d^4}{32} \quad \therefore J_{AC} = \frac{\pi 0.045^4}{32} - \frac{\pi 0.035^4}{32} = 2.55 \times 10^{-7} \text{ m}^4$$

$$\text{Section CD: } J = \frac{\pi D^4}{32} \quad \therefore J_{CD} = \frac{\pi 0.03^4}{32} = 7.95 \times 10^{-8} \text{ m}^4$$

Establish Shear stress for sections:

$$\text{Section 1: } \tau_1 = \frac{T_1 r_1}{J_{CD}} \quad \therefore \frac{-415 (0.015)}{7.95 \times 10^{-8}} = -78.3 \text{ Mn/m}^2$$

$$\text{Section 2: } \tau_2 = \frac{T_2 r_2}{J_{AC}} \quad \therefore \frac{825 (0.0225)}{2.55 \times 10^{-7}} = 72.8 \text{ Mn/m}^2$$

$$\text{Section 3: } \tau_3 = \frac{T_3 r_3}{J_{AC}} \quad \therefore \frac{-175 (0.0225)}{2.55 \times 10^{-7}} = -15.4 \text{ Mn/m}^2$$

Therefore max stress will be witnessed in **Section 1** at 78.3 Mn/m^2

ii) The angle of rotation of each section.

Using:

$$\frac{T}{J} = \frac{\tau}{r} = \frac{G \theta}{L} \quad \therefore \quad \frac{T}{J} = \frac{G \theta}{L} \quad \Rightarrow \quad \theta = \frac{TL}{JG}$$

Section 1: $\theta_1 = \frac{T_1 L_1}{J_{CD} G} \therefore \frac{-415 \times 0.2}{7.95 \times 10^{-8} \times 75 \times 10^9} = -0.014 \text{ Rads}$

Section 2: $\theta_2 = \frac{T_2 L_2}{J_{AC} G} \therefore \frac{825 \times 0.4}{2.55 \times 10^{-7} \times 75 \times 10^9} = 0.017 \text{ Rads}$

Section 3: $\theta_3 = \frac{T_3 L_3}{J_{AC} G} \therefore \frac{-175 \times 0.9}{2.55 \times 10^{-7} \times 75 \times 10^9} = -0.0082 \text{ Rads}$

iii) The angle of rotation of D relative to A.

$$\theta_{DA} = \theta_1 + \theta_2 + \theta_3$$

$\therefore$

$$\begin{aligned} \theta_{DA} &= -0.014 + 0.017 - 0.0082 \\ &= -0.0052 \text{ rads} \end{aligned}$$

Converted to degrees:

$$\frac{-0.0052}{2\pi} \times 360 = -0.3^\circ$$



6.0 Bibliography and references

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