

Electrical Engineering
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1. Abstract

This report will be covering the topic of power factor correction (P.F.C) using an R-L circuit then transitioning into a RL-C circuit when the P.F.C has been completed. The report will be covering relevant information needed prior to the theoretical approach such as a literature survey, following on to the method and results of P.F.C and a discussion of the results to ensure a complete understanding has been achieved.

2. Introduction

The circuit that was used for the P.F.C was an A.C motor circuit, this included supply characteristics, an overall impedance value to count for the cabling, transformer and switchgear prior to the motor, there was also an internal DC resistance of the windings and an inductance value of the windings. These values were taken into account when calculating the P.F.C. Initially the circuit was a resistor & inductor(RL) series circuit. In a circuit with resistance (R) and inductance (L), the load voltage is the sum of the resistor voltage (VR) and inductor voltage(VL), this will create a phase difference between the current and voltage, with a result of the current lagging the voltage. The degree of lag depends on the values of resistor and inductor, with the lag angle being between 0° and 90° . When dealing with a series circuit the current is shared by all components and, as a result, is considered the reference phasor (J.Bird, 2010). Furthermore this is why a P.F.C capacitor was added as this circuit was an inductive circuit, for example if a circuit was a pure capacitance circuit the voltage would lag the current since when an Ac voltage is supplied to the capacitor the voltage changes fast, this causes the capacitor to charge and discharge in response to the Ac sinusoidal waveform. The charge and discharge is not instantaneous and will have a time delay causing the voltage to lag the current this causes phase angle difference between them.

3. Literature Survey

3.1 Resonance in AC systems.

In AC systems resonance can be apparent from two means. One mean is parallel resonance and the other is series resonance. According to [2] Barry Kennedy/Melly Leap, parallel resonance in an AC system is when the capacitor is in parallel with an inductor and possibly a resistor in series. When the capacitance and inductance are adjusted, this would result in the current flow being maximized or minimized at a certain frequency. Furthermore series resonance is when a capacitor and inductor are in series, this would result in the inductance and capacitance needing to have a low impedance to ensure resonance with the current flow maximised was achieved.

3.2 Power Factor Correction in AC systems.

Power factor correction is used in AC systems typically on inductive loads. According to [2] Barry Kennedy/Melly Leap power factor is the ratio of the active/real power (used power that was intended for the application) to apparent power (the overall power that was delivered to the application). Power factor correction is used to mitigate the reactive power (power lost to influences not intended for the application) such as heat dissipation, voltage drop, lowers systems capacity, by mitigating these losses it will make the circuit more efficient. Power factor correction can use resistive compensation, one method which was used in this report was using a capacitor, synchronous machines and static Var systems can also be used for P.F.C. Some typical applications where P.F.C is normally applied is, induction motors, induction furnaces, power electronic devices etc.

3.3 Maximum Power Transfer in AC systems.

In AC circuits maximum power transfer can be achieved. Maximum power transfer is when the load impedance is equal to the internal resistance of the circuit. According to [1] John Bird, an example of this is when transformers are used, they can be used as a resistance matching device by implementing them into the circuit between the load and the source.

4. Method

The information that has been given prior, is to build the circuit and simulate it to show its performance before applying P.F.C, to then allow for analysis and calculations to be made to achieve a value for the new P.F.C.

The information that is known is as follows:

- Sinusoidal source – 565.7 Peak to peak (400Vrms)
- Frequency – 50 Hz
- Rsrc – 2.0 Ohms
- Load resistor value - 71 Ohms
- Load inductor value – 267 mH

These values can now be used to build a circuit which will also aid In visualising how the wave form is performing prior to any power factor correction which is illustrated below in figure (1).

Initial design of the circuit and analogue graph of the wave forms prior to P.F.C

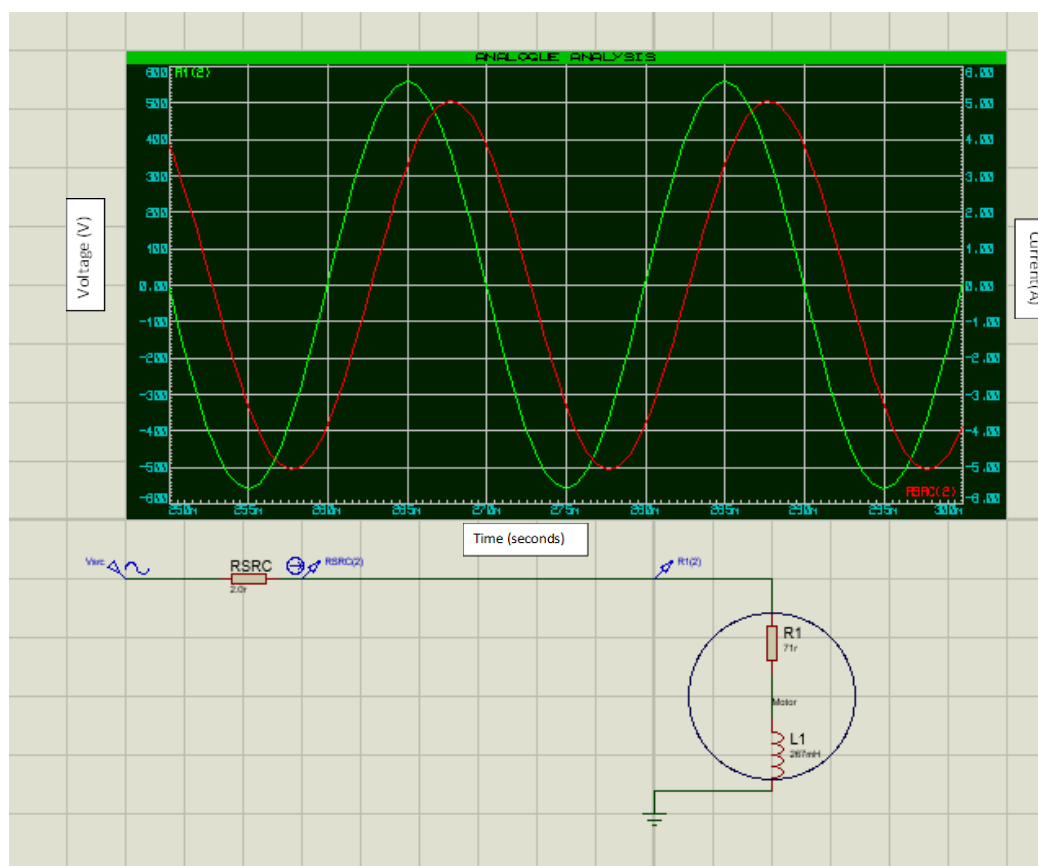


Figure (1)

The analogue graph simulation on proteus (figure 2), using the cursors the simulation illustrates that the current is lagging behind the voltage by 2.77ms, this time can be used to calculate the phase angle between the voltage and current, this will show how many degrees the current is out of phase from the voltage. The calculation used is as follows:

Key :

360 – full cycle of wave

F – frequency

t – time delay between two waveforms

$$360 \times F \times t = \phi$$

$$\phi = 360 \times 50 \times (2.77 \times 10^{-3})$$

$$\phi = 49.86^\circ$$

Initial analogue graph of the relationship of voltage to current before P.F.C

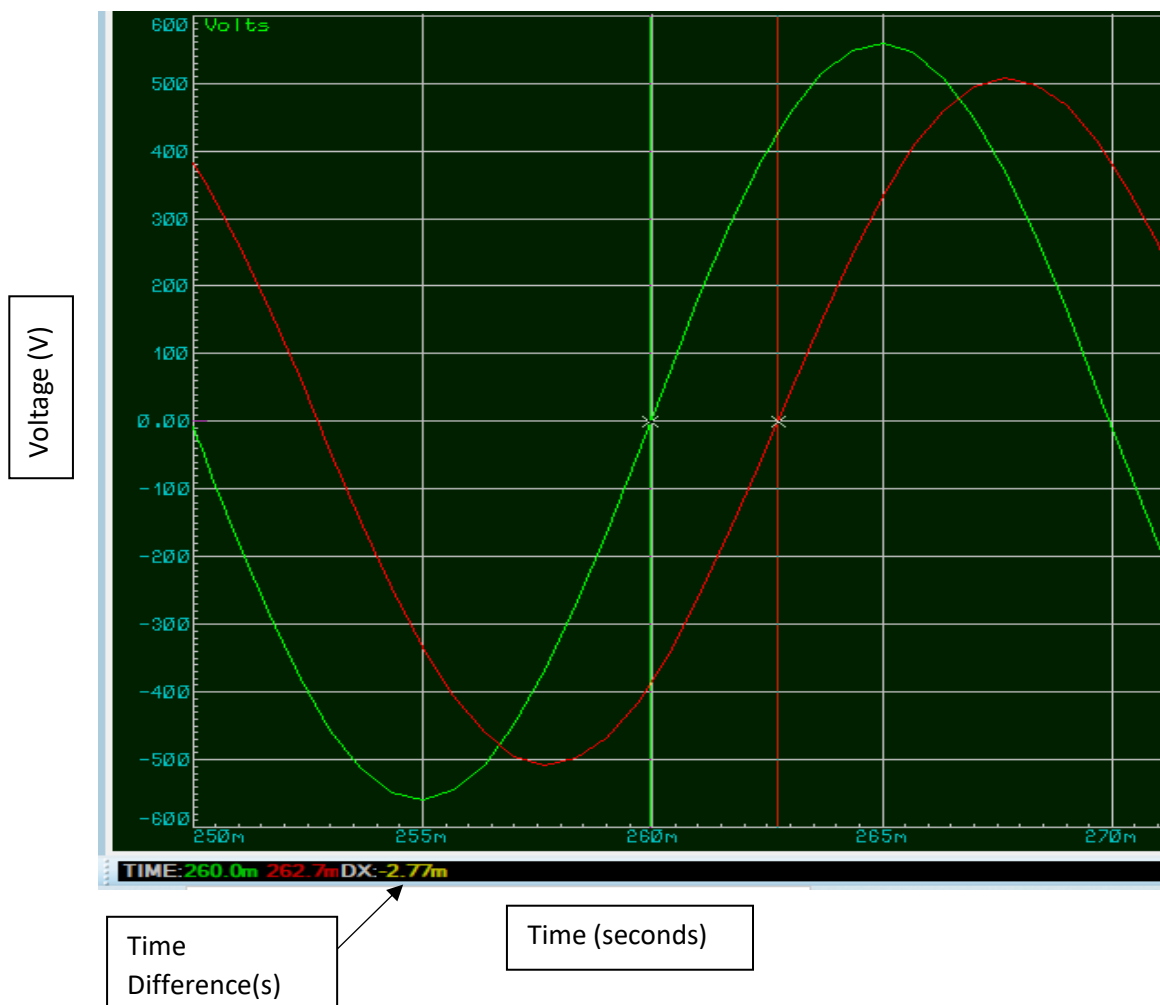


Figure 2

To calculate the power factor of this circuit a few more values must be found.

Firstly, the new rms values of the voltage and current that the load will be using, these values would of changed from the original input values as the upfront resistor is acting as an accumulation of losses from what would have been caused by the transformer, cables etc. To calculate the new rms values proteus can be used by moving the cursors to the peak of each wave as shown in figure (3).

Analogue analysis of voltage and current prior to P.F.C



Figure 3

As illustrated in figure 3 the peak voltage is now 559V and the peak current is 5.07A. To obtain the rms values the following calculations must now be calculated.

$$V_{rms} = V_m \div \sqrt{2}$$

$$I_{rms} = I_m \div \sqrt{2}$$

So,

$$V_{rms} = 559 \div \sqrt{2}$$

$$V_{rms} = 395.27V_{rms}$$

$$I_{rms} = 5.07 \div \sqrt{2}$$

$$I_{rms} = 3.59A(I_{rms})$$

To check that out I_{rms} is correct there are a few calculations that can be made. The inductive reactance (X_L) of the inductor must be calculated to then proceed onto being able to work with the overall impedance of the resistor and the inductor, this then used with division against the V_{rms} to show the I_{rms} true value. The calculations follows:

Key:

- f - Frequency
- L – Inductor value
- Π – Pi

$$X_L = 2 \times \pi \times f \times L$$

$$X_L = 2 \times \pi \times 50 \times (267 \times 10^{-3})$$

$$X_L = 83.88\Omega$$

Now that the Inductive reactance has been found the overall impedance can be calculated:

Key:

- Z = Impedance

$$Z = \sqrt{R^2 + X_L^2}$$

$$Z = \sqrt{(71^2 + 83.88^2)}$$

$$Z = 109.90 \Omega$$

The current (Irms) can now be calculated by using Ohm's law.

$$I = \frac{V}{Z}$$

$$I = \frac{395.27}{109.90}$$

$$I = 3.59 \text{ A}$$

This has now proven that the Irms is correct and that this value can be used to support further calculations. As the XL value is now known the Phase angle (ϕ) can now be checked with the following:

$$\phi = \tan^{-1} \frac{X_L}{R}$$

$$\phi = \tan^{-1} \frac{83.88}{71}$$

$$\phi = 49.75^\circ$$

The following calculations will show what the inductor voltage (VL) and resistor voltage (Vr) will be, following on from these the Vrms can then be proven to be correct.

$$V_L = I_{rms} \times X_L$$

$$V_L = 3.59 \times 83.88$$

$$V_L = 301.13 \text{ V}$$

$$V_r = I_{rms} \times R$$

$$V_r = 3.59 \times 71$$

$$V_r = 254.90$$

$$V_{rms}'' = \sqrt{V_r^2 + V_L^2}$$

$$V_{rms}'' = \sqrt{254.90^2 + 301.13^2}$$

$$V_{rms}'' = 394.53 \text{ Vrms}$$

Though the above Vrms is not the same as the original they are very close, this now means the real power (P), Reactive power (Q) and apparent power can now be calculated, by calculating these values it will then be possible to calculate the power factor of the existing circuit.

$$\text{Real Power}(P) = VI\cos\phi = 394.52 \times 3.59 \times \cos(49.75) = 920.58W$$

$$\text{Reactive power}(Q) = VI\sin\phi = 394.52 \times 3.59 \times \sin(49.75) = 1076VAr$$

$$\text{Apparent Power}(S) = 394.52 \times 3.59 = 1416.33 VA$$

$$\text{Power Factor}(P.F) = \frac{\text{RealPower}}{\text{Apparent Power}} = \frac{920.58}{1416.33} = 0.65$$

$$\text{Check Phase Angle}(\phi) = \cos^{-1}(P.F) = \cos^{-1}(0.65) = 49.46^\circ$$

These calculations have now proven the voltages, current and phase angle of the circuit as it is. The power factor will need to be improved to 0.95, so this the phase angle of the new power factor will need to be known, a phasor diagram can then be made to visualise how much the current is lagging the voltage and how much it will need to be corrected by.

$$\text{New P.F} = 0.95 = \cos^{-1}(0.95) = 18.2^\circ$$

This now explains that the new phase angle between the voltage and current will need to be at 18.19° in order to have a P.F of 0.95.

$$\phi_1 = 49.46^\circ$$

$$\phi_2 = 18.2^\circ$$

A Phasor diagram can now be made.

Phasor diagram of voltage, current's and phase angles from task 1

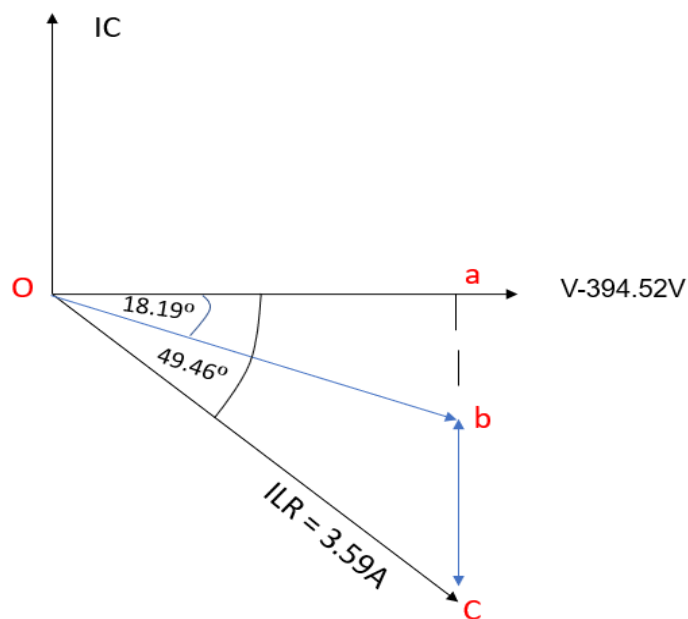


Figure (4)

The phasor diagram (figure 4) illustrates the existing phase angle, it shows the current is lagging as Oc is the current and Oa is the voltage. It also shows the new Current Ob and what the new phase angle will be like after power factor correction. There are some values that will need to be calculated in order to find the right capacitor value.

The Irms of the new circuit can be calculated using trigonometry with the aid if the phasor diagram (figure 4). Firstly the 'opposite' value can be calculated.

$$\text{From triangle } Oac = \cos(49.46) = \frac{Oa}{3.59}$$

$$Oa = 3.59 \times \cos(49.46)$$

$$Oa = 2.33A$$

Now the new supply current can be calculated using the new phase angle and the value of Oa using trigonometry.

$$\text{From triangle } Oab = \cos(18.19) = \frac{Oa}{Ob} = \frac{2.33}{I}$$

$$\text{New supply current} = I = \frac{2.33}{\cos(18.19)} = 2.45A$$

The new supply current is the phasor sum or Ic and ILR . This meaning if,

$$I = IC + = ILR$$

Then the capacitor current can be calculated,

$$IC = I - ILR.$$

So,

$$Ic = 2.45 \angle -18.19 - 3.59 \angle -49.46$$

$$Ic = [2.45 \cos(-18.19) - 2.45 \sin(-18.19)] \\ - [3.59 \cos(-49.46) - 3.59 \sin(-49.46)]$$

$$Ic = (2.33 - j - 0.76) - (2.33 - j - 2.73)$$

$$Ic = 0j3.49$$

$$Ic = 3.49 \angle 90^\circ A$$

$$Ic = \frac{V}{Xc} = \frac{V}{\frac{1}{2\pi f Xc}} = 2\pi f Xc V$$

$$C = \frac{Ic}{2\pi f XV} = \frac{3.49}{2\pi \times 50 \times 394.52} = 0.0000281583f$$

$$\underline{\text{Capacitor Value} = 28.16\mu f}$$

Resonant frequency is when the circuit has minimum impedance and there is maximum current. The resonant frequency can now be calculated.

fr = resonant frequency

$$f_r = \frac{1}{2\pi} \times \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

$$f_r = \frac{1}{2\pi} \times \sqrt{\frac{1}{(267 \times 10^{-3}) \times (28.15 \times 10^{-6})} - \frac{71^2}{(267 \times 10^{-3})^2}}$$

Resonant frequency (fr) = 39.74Hz

5. Results

The image below in figure (5) illustrates the new circuit containing the P.F.C capacitor.

Circuit after integrating P.F.C capacitor within

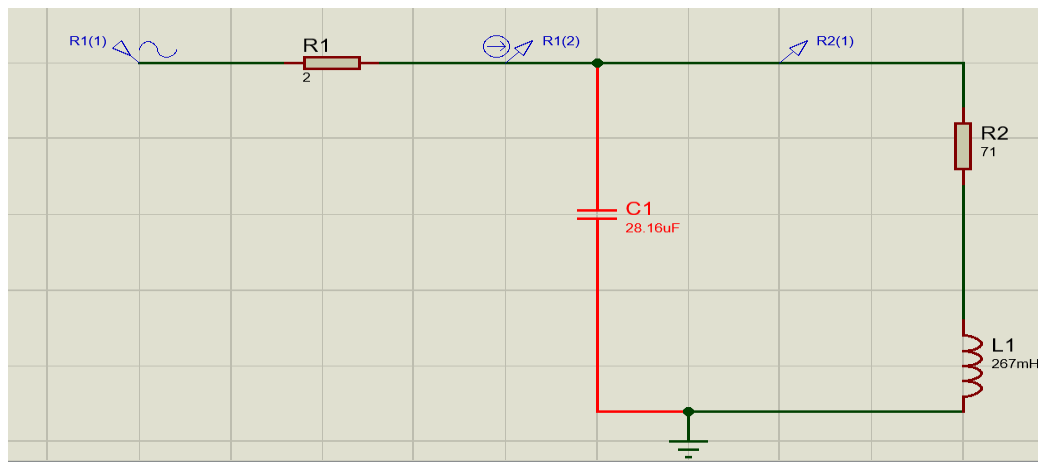


Figure (5)

The image below figure (6) illustrates the wave form after the P.F.C has been applied, it shows that the voltage now lags behind the current for 1.02m seconds.

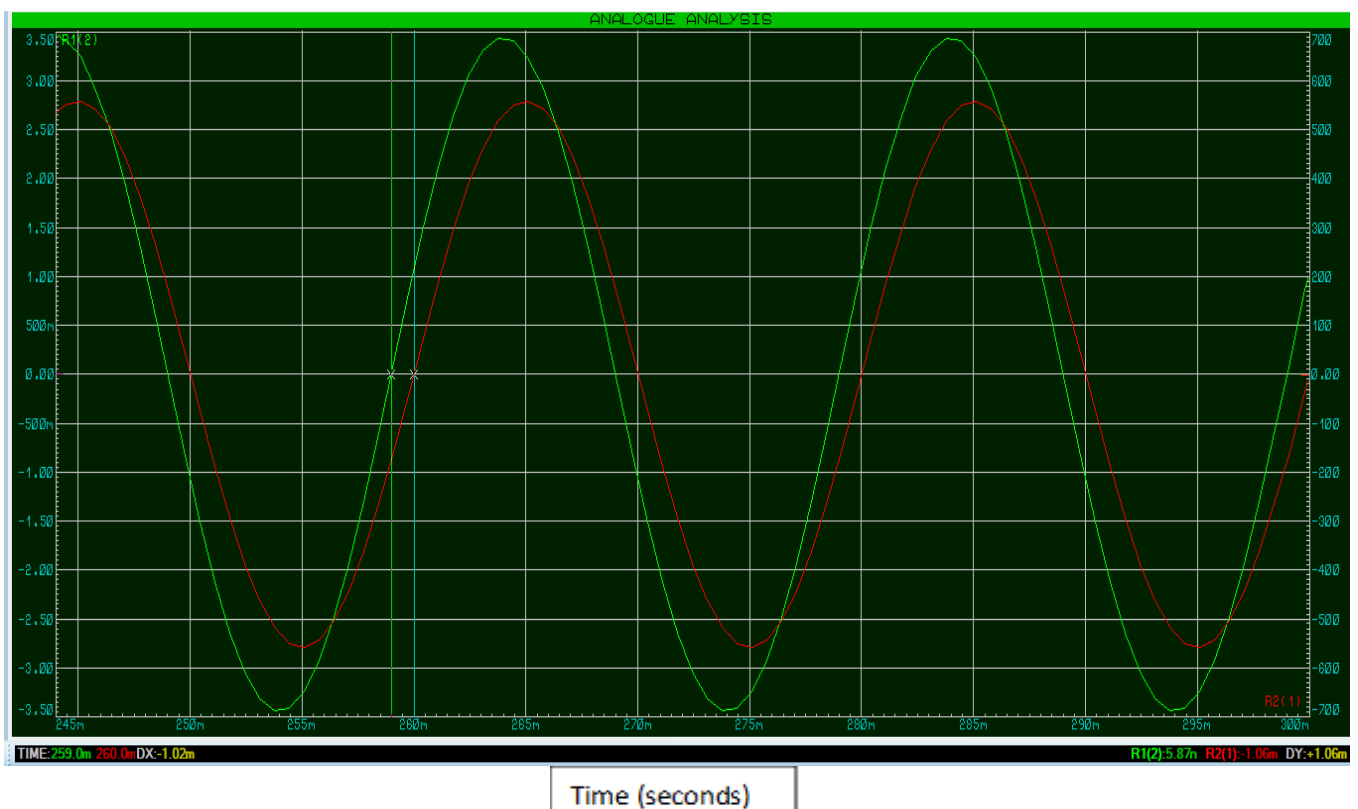
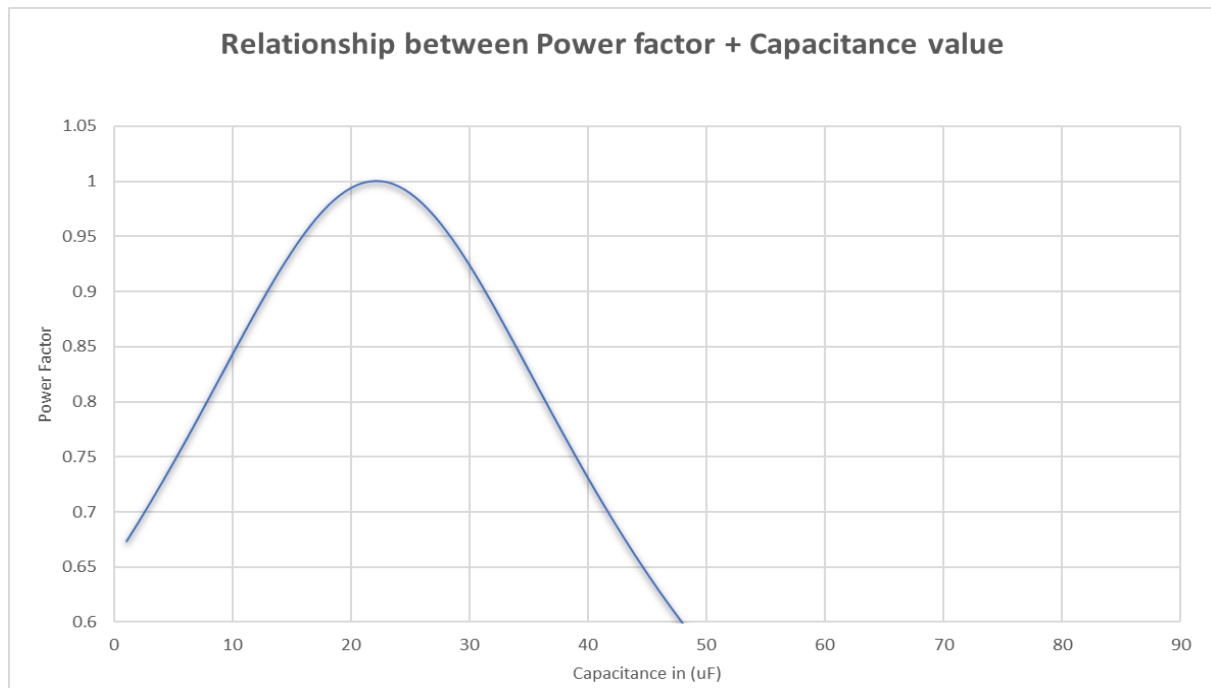


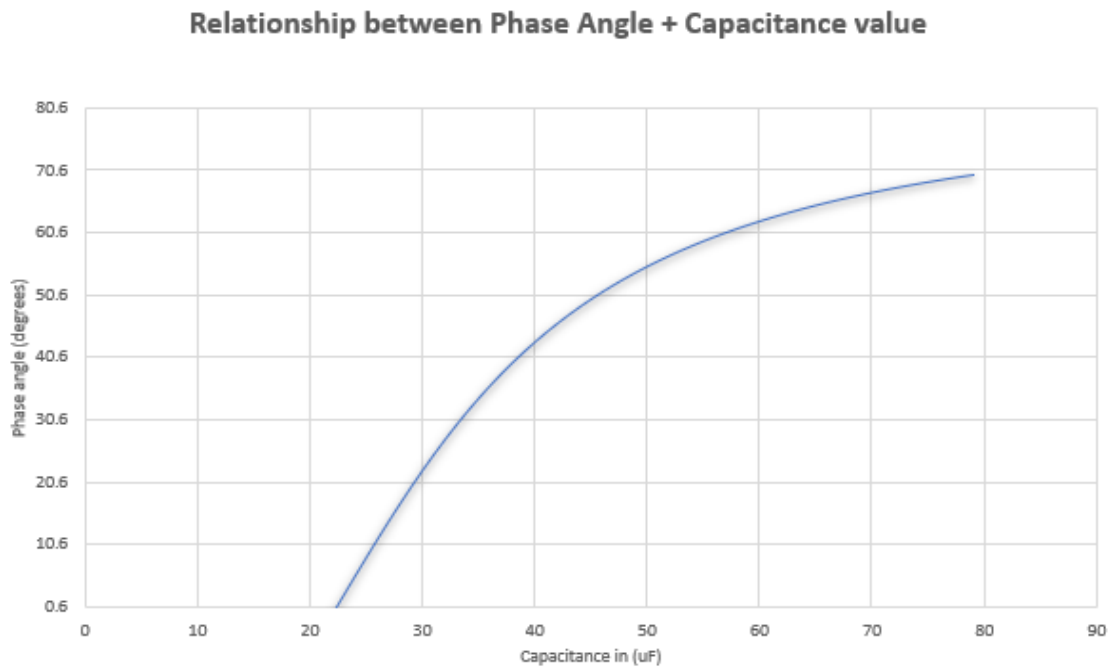
Figure (6)

The image below in figure (7) shows the relationship between power factor and capacitance value. The image illustrates that at a certain capacitance value the power factor will be 1. At power factors close to 1, the energy stored in the capacitor and inductor oscillates between the two components and the energy is continuously exchanged between them. This results in a constant flow of energy between the two components and the energy is not lost as heat. Therefore, the total energy stored in the capacitor and inductor remains constant and is not dissipated, making the system more efficient. therefore at a power factor of 1 the voltage and current will both be in phase.



Figure(7)

The image below in figure (8) shows the relationship between Phase angle and capacitance value. The image shows when the capacitance value starts get less the phase angle grows greater, meaning that the voltage is lagging the current as the phase angle is becoming greater positively.

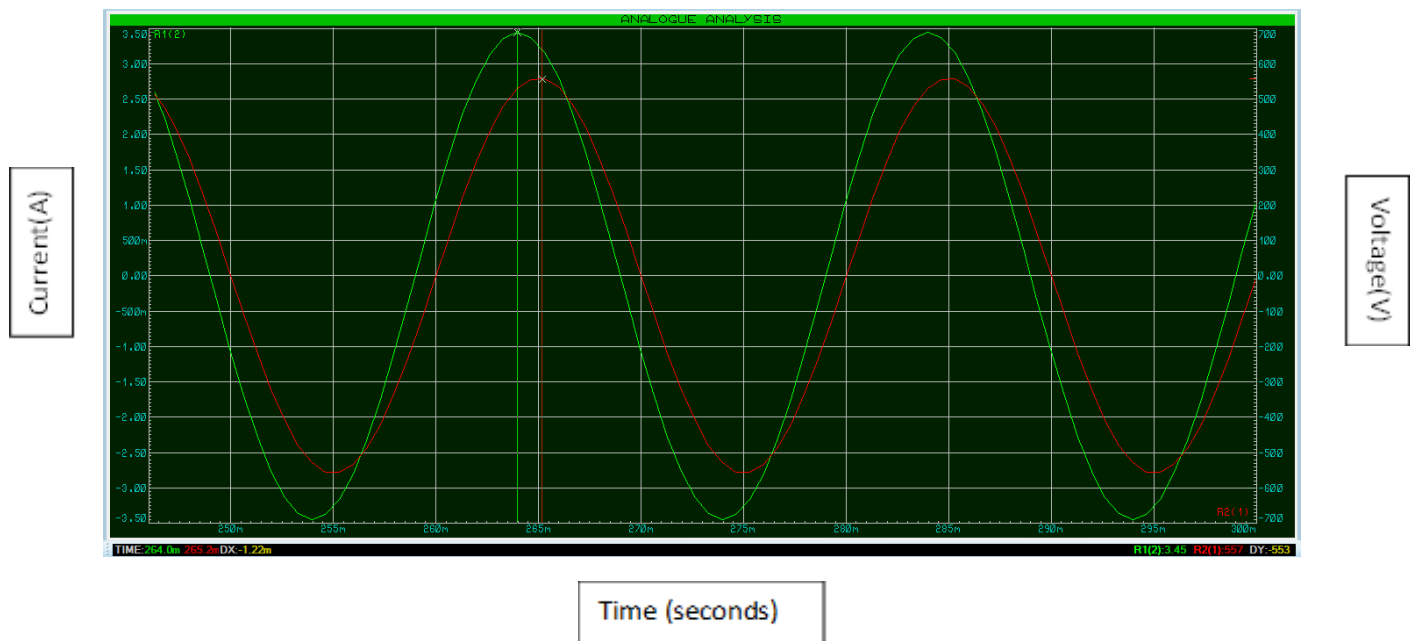


Figure(8)

To create phasor diagrams with different input frequencies of 50Hz, resonant frequency, 25Hz below resonant frequency and 25Hz above resonant frequency the same calculations were used as in the method, the results for these are as follows.

The image below in figure (9) illustrates the analogue graph of the Ac voltage and current when the P.F.C capacitor is included in the circuit and it is at the working frequency of 50Hz. Figure (10) illustrates the phasor diagram to support this.

Analogue analysis of voltage to current with a frequency of 50Hz.



Figure(9)

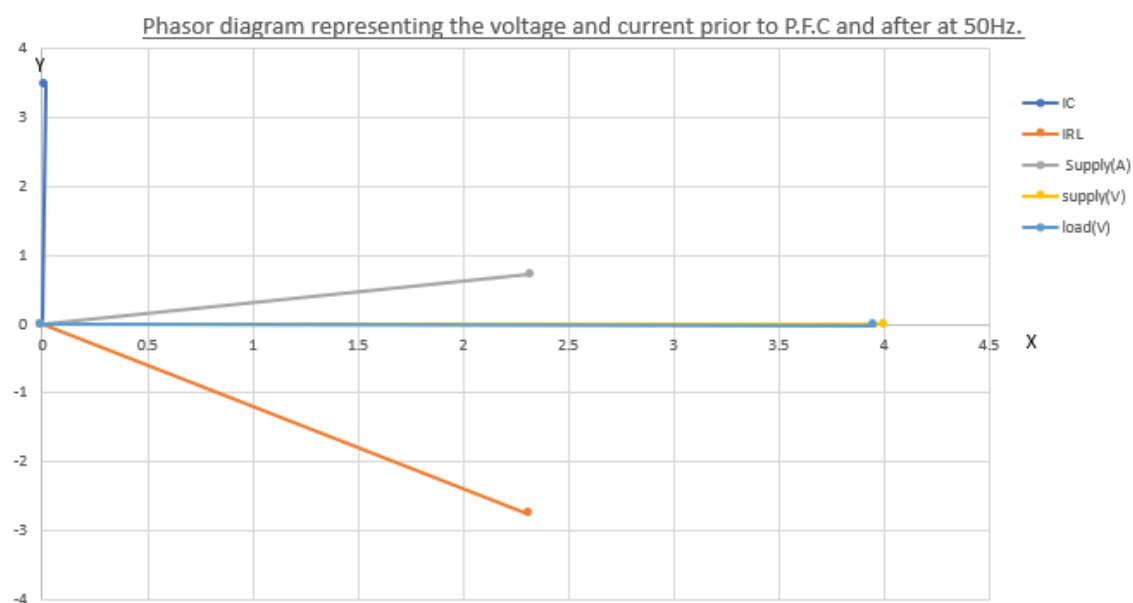
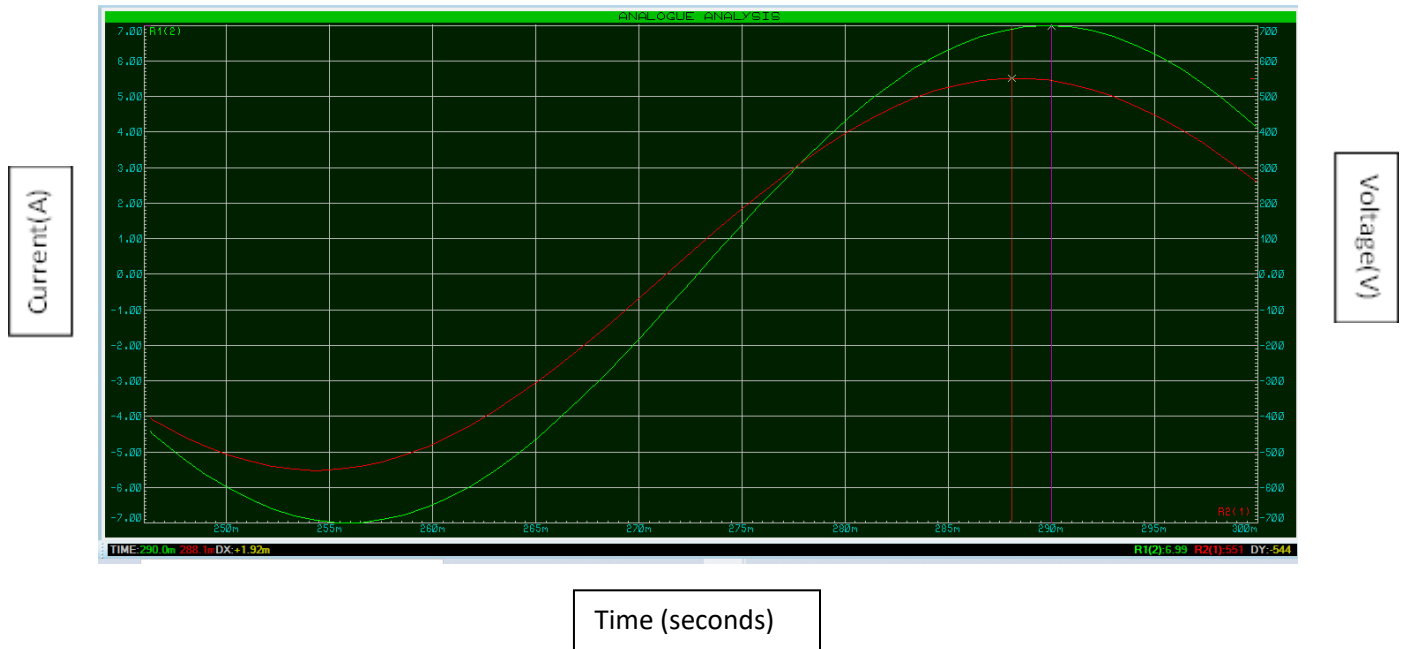


Figure (10)

The below figure (11) represents the voltage and current at 25 Hz (14.74Hz) below resonant frequency, the voltage peaks at 551V and the current peaks at 6.99A . Figure (12) shows the phasor diagram to support this.

Analogue analysis of voltage to current with a frequency of 25Hz below resonance.



Figure(11)

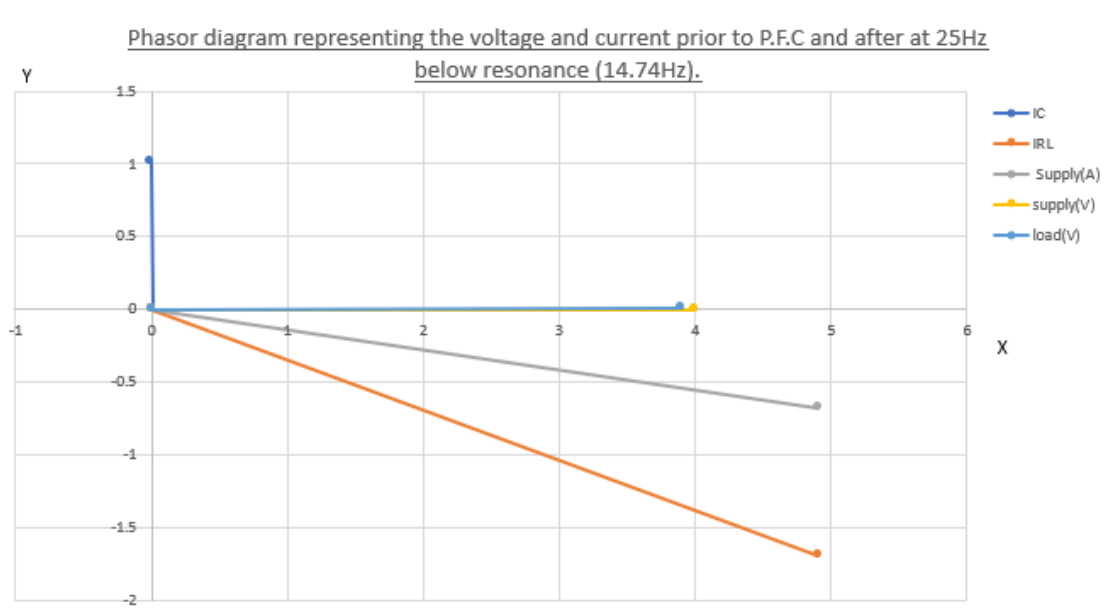


Figure (12)

The below figure (13) represents the voltage and current at resonant frequency (39.74Hz), the voltage peaks at 557V and the current peaks at 4.15A . Figure (14) shows the phasor diagram to support this.

Analogue analysis of voltage to current when frequency is at resonance.

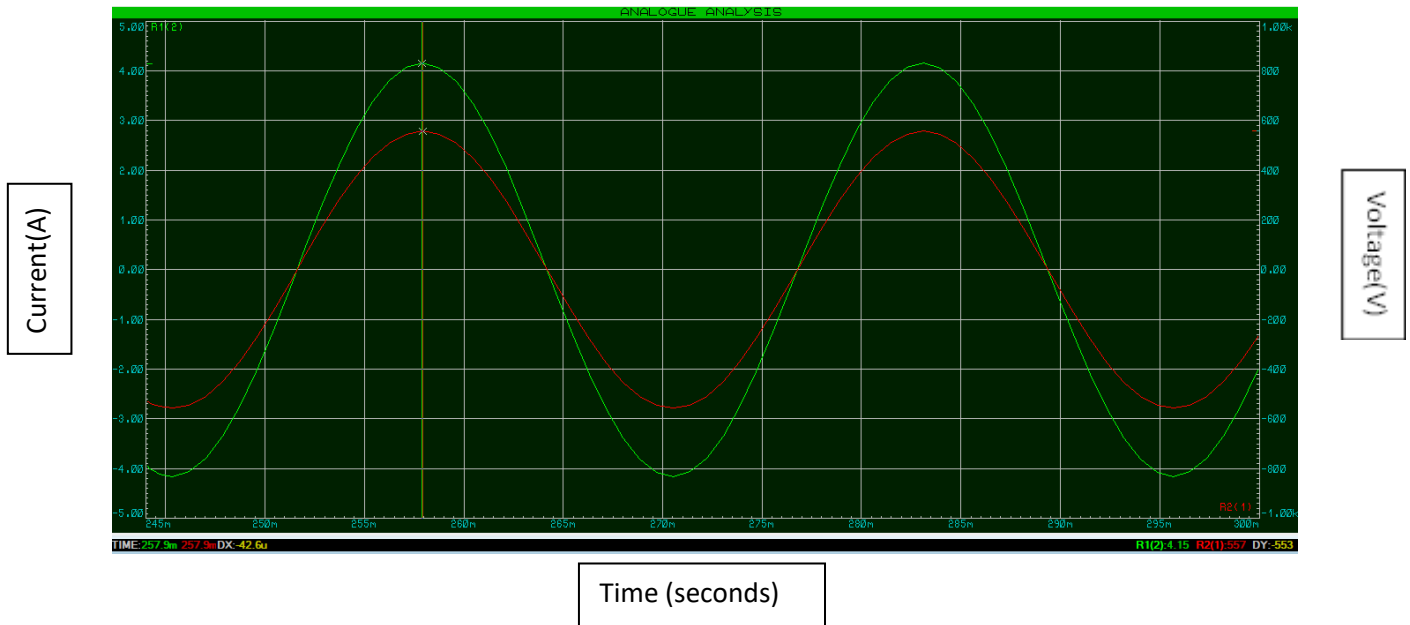


Figure (13)

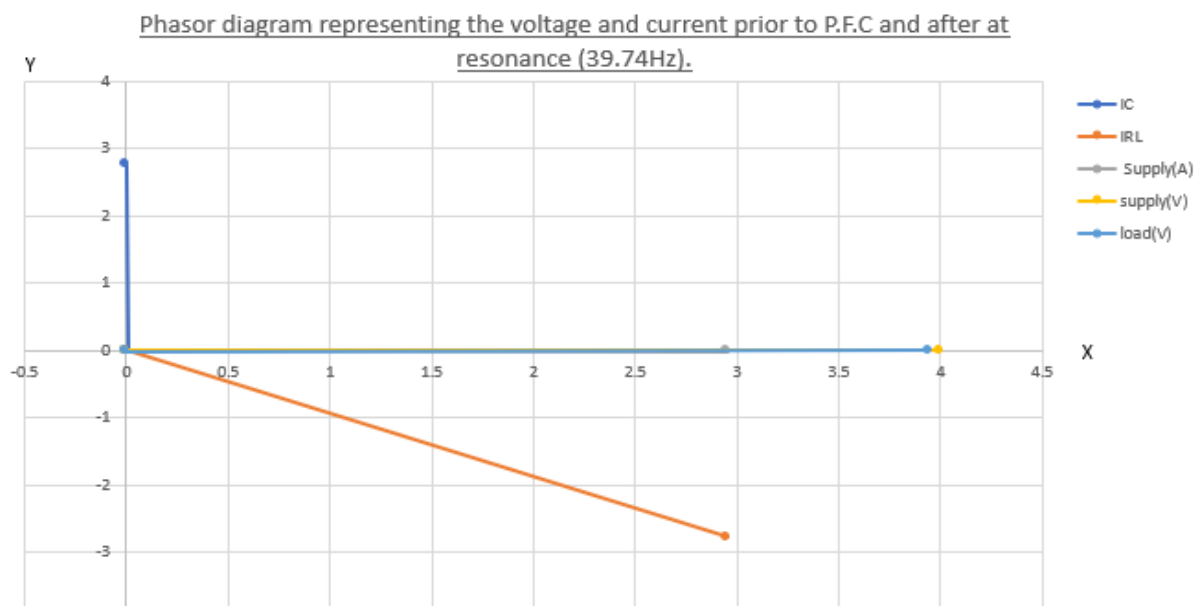


Figure (14)

The below figure (15) represents the voltage and current at 25 Hz (64.74Hz) above resonant frequency, the voltage peaks at 560V and the current peaks at 3.66A . Figure (16) shows the phasor diagram to support this.

Analogue analysis of voltage to current with a frequency of 25Hz above resonance.

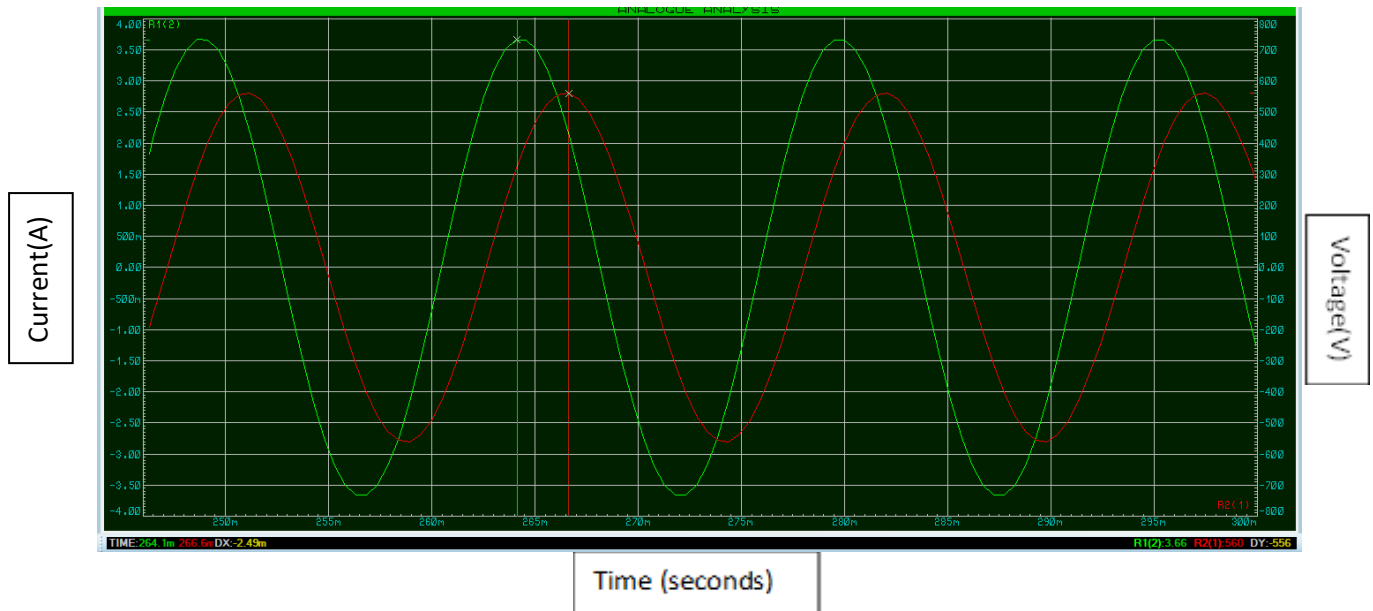


Figure (15)

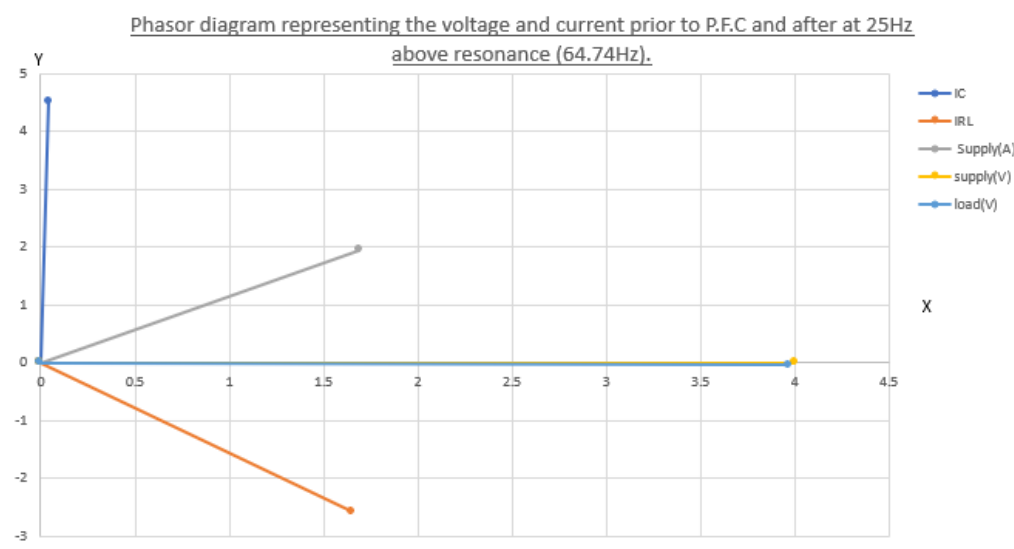


Figure (16)

6. Discussion

In discussion of the outcome of this circuit, firstly the power factor correction capacitor corrected the phase angle as to the specification of a power factor ≥ 0.95 . The final phase angle resulted in the voltage lagging the current as shown in figures 9 and 10. The resultant capacitor value could have been a different value but still had a power factor of 0.95, in this case it would of meant the impedance value was slightly higher than the capacitance value causing the current to lag the voltage by -18.2° . If the inductance value was greater than the capacitance value then the current would lag the voltage, this is because when voltage is applied to an inductor it generates an electromagnetic field, the electromagnetic field which then tries to oppose the change in current flow. According to [3] All About Circuits, an inductor reacts to changes in voltage by generating a back EMF that is proportional to the rate of change of the current through the inductor. This EMF opposes the applied voltage, causing the current to lag behind the voltage. Furthermore if the capacitance value and the inductance values were equal as explained previously this would be when a power factor of 1 would have been achieved resulting in an efficient system. In relation to resonance being achieved, this would not been achieved perfectly due to the impedances not being equal to one another. As calculated the impedance of the resistor and the inductor was 109.90 Ohms. The inductance of the capacitor is $\frac{1}{2\pi f X_C} = \frac{1}{2\pi \times 50 \times (28.16 \times 10^{-6})} = 113.03\Omega$, Even though the two impedances are close in value they are still not equal, this is down to the power factor also being calculated to be at 0.95. In regards to maximum power transfer, according to [4] James W. Nilsson & Susan A. Riedel, gaining a power factor of ≥ 0.95 implies that the amount of active/real power delivered to the load is higher, this is important for achieving maximum power transfer because power is lost as heat during transmission, and the lower the power factor, the more power is lost as reactive power. Therefore, a higher power factor reduces the amount of wasted power, resulting in more efficient power usage and maximum power transfer.

In relation to the results of when the input frequency was changed, the outcome for each different frequency differed. Firstly when the frequency was adapted to the resonant frequency (39.74Hz) figure 13 illustrated that the voltage and current were in phase and that there was no delay, for the circuit to be working at resonance this would mean that the maximum current is flowing while the impedance within the circuit is low, as explained the inductance and capacitance would have to be equal which would make the circuit a pure resistive circuit. In a pure resistive circuit there is no lag on voltage or current and they both stay in phase, which is what is shown in figure 14.

Furthermore figure 15 is illustrating the relationship of voltage and current at 25 Hz above resonance (64.74). In figure 15 it shows that the voltage is lagging the current, this is due to when the circuit is 25Hz above resonance the impedance of the inductor increases which results in the current decreasing (3.66A), this is illustrated in figure 16 as the new current is positively out of phase from voltage.

Finally figure 11 illustrates that the that the current is lagging the voltage, it also shows that the I_m have increased. The current gain is gained from the capacitive reactance of the circuit when the frequency is 25 Hz below resonance. At this frequency, the capacitive reactance is higher than the inductive reactance, resulting in a current gain(6.99A), figure 12 illustrates this showing the positive current while the voltage a lagging.

5. Conclusion

In conclusion of this report the following have been obtained:

- Power factor correction capacitor value
- New circuit resonance value
- New power factor of ≥ 0.95
- More efficient circuit
- New Phase angle

6. References

- [1] John Bird, *Electrical and electronic principles and technology, fourth edition*, (2010)
Available at:
- [2] Barry Kennedy & Melly Leap, *Industrial power factor analysis guidebook*, (March 1995),
Available at: [654078.pdf \(unt.edu\)](#)
- [3] All About Circuits, *Inductors and Inductance*
Available at: <https://www.allaboutcircuits.com/textbook/alternating-current/chpt-3/inductors-inductance/>
- [4] James W. Nilsson & Susan A. Riedel, *Electric Circuits 9th Edition*,
Available at: [Electric Circuits \(9th Edition\) by James W. Nilsson, Susan Riedel \(1\) | Guilherme Michelin Muller - Academia.edu](#)